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R. Wünsch

Self-Consistent Solutions of the Semibosonized Nambu & Jona-Lasinio Model

Abstract

We consider a two-flavor Nambu & Jona-Lasinio model in Hartree approximation involving scalar-isoscalar and pseudoscalar-isovector quark-quark interactions. Average meson fields are defined by minimizing the effective Euklidean action. The fermionic part of the action is regularized within Schwinger's proper-time scheme. The meson fields are restricted to the chiral circle and to hedgehog configurations. The only parameter of the model is the constituent quark mass M which simultaneously controls the regularization.

We evaluate meson and quark fields self-consistently in dependence on the constituent quark mass. It is shown that the self-consistent fields do practically not depend on the constituent quark mass. This allows us to define a properly parameterized reference field which for physically relevant constituent masses can be used as a good approximation to the exactly calculated one. The reference field is chosen to have correct behaviour for small and large radii.

To test the agreement between self-consistent and reference fields we calculate several observables like nucleon energy, mean square radius, axial-vector constant and delta-nucleon mass splitting in dependence on the constituent quark mass. The agreement is found to be fairly well.

Contents

1 Introduction

The model of Nambu & Jona-Lasinio (NJL) [1] has been used quite successfully as effective chiral theory for low and medium energy hadronic phenomena. First it has been applied to vacuum and meson properties as well as medium effects (for reviews c. f. [2, 3, 4]). Later on it turned out that also baryonic systems (nucleons and hyperons) can be described within this model (for a review c. f. [5]). Starting from a semi-bosonized version [6] with scalar-isoscalar and pseudoscalar-isovector interaction and treating the meson fields classically various authors have shown that for constituent quark masses $M \gtrsim 350\,MeV$ it is possible to get self-consistent solitonic solutions with baryon number B=1 consisting of 3 valence quarks in addition to the polarized Dirac sea [7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Because of the non-renormalizability of the Nambu & Jona-Lasinio model the sea contribution diverges and has to be regularized. The parameters of the model can be fixed to the physics of the meson and vacuum sector, mainly to the weak pion decay constant f_π and the pion mass m_π. In doing so only one free parameter remains open for the baryonic sector for which we take the constituent quark mass M.

The self-consistent determination of the meson fields is a time-consuming numerical procedure. Changing the parameters of the model or the regularization scheme the procedure has to be repeated. So it might be helpful to look for an analytic parametrization of the selfconsistent profile function $\Theta(r)$ of the solitonic solution which approximates the exact $\Theta(r)$ as well as possible. Within the restrictions to hedgehog configurations [17] and to the chiral circle the meson fields are uniquely described by the profile function $\Theta(r)$. In the course of our calculations we noticed to a very large extent an independence of this profile function on the constituent quark mass M. It is the aim of this paper to investigate this dependence quantitatively and to look for a general function which may approximate the profile function, if possible independently of M.

In section 2 we review the main ideas of the semi-bosonized and regularized Nambu & Jona-Lasinio model for two flavors and introduce observables characterizing the quark and meson configuration. The procedure of getting self-consistent meson and quark fields is explained and illustrated in sect. 3. We investigate the dependendence of the self-consistent meson profiles $\Theta(r)$ on the constituent quark mass M within a wide range ($350\,MeV \leq M \leq 1000\,MeV$) and compare them with a reference profile $\Theta^{Ref}(R;r)$ obtained from an asymptotic expansion of the equation of motion at $r \to 0$ and $r \to \infty$. In sect. 4 we calculate several observables like nucleon mass, mean-square radius, axial-vector coupling constant and delta-nucleon mass splitting using both the self-consistently determined profiles and the standard profiles. The comparison of both values illustrates the quality of the reference profile.

2 The regularized and bosonized Nambu & Jona-Lasinio model and its observables

The details of the following section can be found in ref. [18, 15, 5, 16]. Here we shortly review these parts of the formalism which make this paper self-contained.

We consider a two-flavor NJL lagrangian

$$\mathcal{L}_{NJL}(\bar{q}q) = \bar{q}[i\partial\!\!\!/ - m]q + \frac{G}{2}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] \tag{1}$$

for the quark fields $q(x)$ (u and d quarks of $N_c = 3$ colours). Here $\hat{\tau}$ is the vector of Pauli-matrices and m is the average current mass of the light quarks $m = (m_u + m_d)/2$. The chiral invariant combination of a scalar-isoscalar and a pseudoscalar-isovector quark-quark interaction with a constant strength G is assumed to describe effectively the quark interaction at low energy mediated by gluons. High-energy interaction processes will be excluded by a regularization procedure.

The theory with only quark degrees of freedom is converted into an effective quark-meson theory by means of standard path-integral bosonization [18]. We introduce an isoscalar field

$$\sigma = \frac{m}{g} - \frac{g}{\lambda^2}\bar{q}q \tag{2}$$

and an isotriplet of fields

$$\hat{\pi} = -\frac{g}{\lambda^2}\bar{q}i\gamma_5\hat{\tau}q. \tag{3}$$

The parameters λ and g are related to G via

$$G = \frac{g^2}{\lambda^2}. \tag{4}$$

The resulting semi-bosonized theory is described by an effective (Euklidean) action

$$\mathcal{A}_{eff}\left[\sigma,\hat{\pi}\right](\mu) = \mathcal{A}^q\left[\sigma,\hat{\pi}\right](\mu) + \mathcal{A}^m\left[\sigma,\hat{\pi}\right] \tag{5}$$

which consists of a quark part

$$\begin{aligned}\mathcal{A}^q\left[\sigma,\hat{\pi}\right](\mu) &= -Log\int D\bar{q}\,Dq\,e^{-\int d^4x_E\,\bar{q}(\not{D}_E-\mu\beta)q} = \\ &= -Log\,Det\left(\not{D}_E - \mu\beta\right) = -Sp\,Log\left(\not{D}_E - \mu\beta\right)\end{aligned} \tag{6}$$

with the Euklidean Dirac operator

$$\not{D}_E = \beta\left(\frac{\partial}{\partial\tau} + h\right) \tag{7}$$

and the quark hamiltonian

$$h = \vec{\alpha}\vec{p} + g\,\beta\left(\sigma + i\gamma_5\hat{\tau}\hat{\pi}\right), \tag{8}$$

and of a meson part

$$\mathcal{A}^m\left[\sigma,\hat{\pi}\right] = \frac{\lambda^2}{2}\int d^4x_E\left[\left(\sigma - \frac{m}{g}\right)^2 + \hat{\pi}^2\right] = T\frac{\lambda^2}{2}\int d^3r\left[\left(\sigma - \frac{m}{g}\right)^2 + \hat{\pi}^2\right] \tag{9}$$

with the Euklidean space-time vector $x_E^\mu = (\tau, \vec{r})$ and its volume element $d^4x_E = d\tau\, d^3r$. Here we have assumed the meson fields to be time-independent and classical fields, i. e. σ and $\hat{\pi}$ are ordinary functions of the space vector $\vec{r}$ and the integral in eq. (9) is proportional to the Euklidean time intervall T. The chemical potential μ for quarks has been introduced in order to adjust the baryon number to a definite value. The symbol Sp indicates functional and matrix (spin, isospin, colour) trace

$$Sp\,\mathcal{O} \equiv N_c\,tr_\gamma\,tr_\tau\int d^4x_E\,\langle x_E|\,\mathcal{O}\,|x_E\rangle\,. \tag{10}$$

In the limit $T \to \infty$ (zero temperature) the quark action (6) for time-independent and classical meson fields can be written

$$Sp\,Log\,(\not{D}_E - \mu\beta) \,=\, T\,N_c \int \frac{d\omega}{2\pi} \sum_\alpha Log(-i\omega + \varepsilon_\alpha - \mu) \,=\, T\frac{N_c}{2}\left[\sum_\alpha |\varepsilon_\alpha - \mu| + const\right] \tag{11}$$

with the real eigenvalues ε_α of the hamiltonian (8) defined by

$$h\,\Phi_\alpha(\vec{r}) \,=\, \varepsilon_\alpha\,\Phi_\alpha(\vec{r}) \tag{12}$$

and normalized as

$$\int d^3r'\,\Phi^\dagger_\alpha(\vec{r}\,')\Phi_{\alpha'}(\vec{r}\,') \,=\, \delta_{\alpha\alpha'}. \tag{13}$$

The difference between the quark action (6) and its vacuum value $\mathcal{A}^q_V$ calculated at constant σ field ($\sigma(\vec{r}) \equiv \sigma_V$), vanishing $\hat{\pi}$ field and chemical potential $\mu = 0$ is then given by a sum of single-particle contributions

$$\mathcal{A}^q[\sigma,\hat{\pi}]'(\mu) \,\equiv\, \mathcal{A}^q[\sigma,\hat{\pi}](\mu) - \mathcal{A}^q_V \,=\, -T\frac{N_c}{2}\sum_\alpha\left[|\varepsilon_\alpha - \mu| - |\varepsilon^V_\alpha|\right], \tag{14}$$

where ε^V_α are the eigenvalues of the vacuum hamiltonian

$$h_V \,=\, \vec{\alpha}\vec{p} + g\,\beta\sigma_V. \tag{15}$$

We split the total quark action (14) into a valence and a sea-quark contribution

$$\mathcal{A}^q[\sigma,\hat{\pi}]'(\mu) \,=\, \mathcal{A}^q_{val}(\mu) + \mathcal{A}^q_{sea}, \tag{16}$$

where the valence contribution is defined as the difference between the action for a finite value of the chemical potential and the action for vanishing chemical potential

$$\mathcal{A}^q_{val}(\mu) \,\equiv\, \mathcal{A}[\sigma,\hat{\pi}]'(\mu) - \mathcal{A}[\sigma,\hat{\pi}]'(\mu = 0) \,=\, -T\frac{N_c}{2}\sum_\alpha[|\varepsilon_\alpha - \mu| - |\varepsilon_\alpha|]. \tag{17}$$

The remaining part

$$\mathcal{A}^q_{sea} \,\equiv\, \mathcal{A}^q[\sigma,\hat{\pi}]'(\mu) - \mathcal{A}^q_{val}(\mu) \,=\, \mathcal{A}^q[\sigma,\hat{\pi}](\mu = 0) - \mathcal{A}^q_V \,=\, -T\frac{N_c}{2}\sum_\alpha\left[|\varepsilon_\alpha| - |\varepsilon^V_\alpha|\right] \tag{18}$$

does not depend on the chemical potential μ and is called sea contribution. The sea contribution diverges and must be regularized. For stationary meson fields the imaginary part of $Sp\,Log\,\not{D}_E$ vanishes and we have

$$Sp\,Log\,\not{D}_E \,=\, \frac{1}{2} Sp\,Log\,\not{D}^\dagger_E\not{D}_E. \tag{19}$$

Since $\not{D}^\dagger_E\not{D}_E$ is an hermitian operator we can apply Schwinger's proper-time regularization scheme [19] by replacing

$$Sp\,Log\,\not{D}^\dagger_E\not{D}_E \longrightarrow Sp\,Log\,\not{D}^\dagger_E\not{D}_E\Big|_{Reg} \,=\, Sp\int\limits_{1/\Lambda^2}^{\infty}\frac{ds}{s}e^{-s\not{D}^\dagger_E\not{D}_E}, \tag{20}$$

where Λ is the regularization parameter. Applying rule (20) on the sea contribution (18) we get the regularized sea contribution

$$\mathcal{A}_{sea}^{q,Reg} = \frac{1}{2} Sp \int_{1/\Lambda^2}^{\infty} \frac{ds}{s} e^{-s \not{D}_E^\dagger \not{D}_E} = -T \frac{N_c}{2} \sum_\alpha \left[R_E(\varepsilon_\alpha, \Lambda) |\varepsilon_\alpha| - R_E(\varepsilon_\alpha^o, \Lambda) |\varepsilon_\alpha^V| \right] \tag{21}$$

with the regularization function

$$\begin{aligned} R_E(\varepsilon, \Lambda) &= -\frac{1}{\sqrt{4\pi}|\varepsilon|} \int_{1/\Lambda^2}^{\infty} dt\, t^{-3/2} e^{-\varepsilon^2 t} = -\frac{1}{\sqrt{4\pi}} \Gamma\left(-\frac{1}{2}, \frac{\varepsilon^2}{\Lambda^2}\right) \\ &\longrightarrow \begin{cases} 0 & (\varepsilon^2 \gg \Lambda^2) \\ 1 - \frac{\Lambda}{\sqrt{\pi}|\varepsilon|} & (\varepsilon^2 \ll \Lambda^2) \end{cases} \end{aligned} \tag{22}$$

and the incomplete Gammafunction $\Gamma(x,a) = \int_a^\infty dt\, t^{x-1} e^{-t}$. The total regularized quark action is then given by

$$\mathcal{A}^{q,Reg}[\sigma, \hat{\pi}]'(\mu) = \mathcal{A}_{val}^q(\mu) + \mathcal{A}_{sea}^{q,Reg}, \tag{23}$$

with the valence contribution (17) and the regularized sea contribution (21).

The expectation value $K(\mu)$ of an observable $\int d^3r'\, \bar{q}(x'_E)\, \mathcal{K}\, q(x'_E)$ with an operator $\mathcal{K}$, which does not act on the time coordinate τ, calculated for finite chemical potential μ is given by

$$\begin{aligned} K(\mu) &\equiv \left\langle \int d^3r'\, \bar{q}(x'_E)\, \mathcal{K}\, q(x'_E) \right\rangle_\mu = \frac{1}{T'} \left\langle \int d^4x'_E\, \bar{q}(x'_E)\, \mathcal{K}\, q(x'_E) \right\rangle_\mu \\ &= \frac{\int D\bar{q} Dq\, e^{-\int d^4x_E\, \bar{q}(\not{D}_E - \mu\beta) q} \cdot \int d^4x'_E\, \bar{q}(x'_E) \mathcal{K} q(x'_E)}{T' \int D\bar{q} Dq\, e^{-\int d^4x_E\, \bar{q}(\not{D}_E - \mu\beta) q}} \\ &= \frac{d}{T'\, d\kappa} Sp\, Log\, (\not{D}_E - \mu\beta - \kappa\mathcal{K}) \Big|_{\kappa=0} = -\frac{1}{T'} Sp \left[\left(\frac{\partial}{\partial\tau} + h - \mu \right)^{-1} \beta\mathcal{K} \right]. \end{aligned} \tag{24}$$

In analogy to eq. (11) we get for zero temperature, static meson fields and hermitian operators $\mathcal{K}$

$$K(\mu) = -\frac{N_c}{2} \sum_\alpha sign(\varepsilon_\alpha - \mu)\, K_\alpha \tag{25}$$

with

$$K_\alpha = \int d^3r'\, \bar{\Phi}_\alpha(\vec{r}\,')\, \mathcal{K}\, \Phi_\alpha(\vec{r}\,') \tag{26}$$

and the eigenvalues ε_α and eigenfunctions $\Phi_\alpha(\vec{r})$ of the hamiltonian (8) defined in eqs. (14, 15).

A functional integration over the meson fields does not occur in eq. (24) in accordance with the classical approximation for these fields. Like the effective quark action (14) the expectation value (25) consists of a sum of single-particle contributions as a consequence of the mean-field approximation. Again we seperate a valence and a sea contribution

$$K(\mu) = K_{val}(\mu) + K_{sea} \tag{27}$$

with

$$K_{val}(\mu) \equiv K(\mu) - K(\mu=0) = N_c \sum_{0\le\varepsilon_\alpha\le\mu} K_\alpha \tag{28}$$

and

$$K_{sea} = K(\mu=0) - K_V = -\frac{N_c}{2}\sum_\alpha \left[sign(\varepsilon_\alpha)\, K_\alpha - sign(\varepsilon_\alpha^V)\, K_\alpha^V\right], \tag{29}$$

where we have subtracted the vacuum value K_V calculated according to eqs. (24-26) with the eigenvalues ε_α^V and the eigenstates Φ_α^V of the the vacuum hamiltonian (15).

The sea contribution (29) may diverge. In this case we apply the scheme (19, 20) on the operator $(\not{D}_E - \kappa\mathcal{K})$ and get a regularized sea contribution

$$K_{sea}^{Reg} = -\frac{N_c}{2}\sum_\alpha \left[R_m(\varepsilon_\alpha, \Lambda) K_\alpha - R_m(\varepsilon_\alpha^V, \Lambda) K_\alpha^V\right] \tag{30}$$

to the expectation value with the regularization function

$$\begin{aligned} R_m(\varepsilon,\Lambda) &= \frac{\varepsilon}{\sqrt{\pi}} \int_{1/\Lambda^2}^{\infty} dt\, t^{-1/2} e^{-\varepsilon^2 t} = \frac{sign(\varepsilon)}{\sqrt{\pi}}\, \Gamma\!\left(\frac{1}{2}, \frac{\varepsilon^2}{\Lambda^2}\right) \\ &\longrightarrow \begin{cases} 0 & (\varepsilon^2 \gg \Lambda^2) \\ sign(\varepsilon) & (\varepsilon^2 \ll \Lambda^2) \end{cases} . \end{aligned} \tag{31}$$

Now let us apply the scheme displayed in eqs. (24-31) on several observables which characterize a quark configuration. The baryon density

$$\rho(\vec{r};\mu) = \frac{1}{N_c}\left\langle q^\dagger(\vec{r}) q(\vec{r})\right\rangle_\mu \tag{32}$$

can be obtained by means of eqs. (27-29) with the operator $\mathcal{K} = \gamma_0\, \delta^3(\vec{r} - \vec{r}\,')/N_c$ and the matrix elements

$$K_\alpha(\vec{r}) = \frac{1}{N_c}\Phi_\alpha^\dagger(\vec{r})\Phi_\alpha(\vec{r}). \tag{33}$$

In this case, the expectation values (27-29) and the matrix elements (26) depend on the parameter $\vec{r}$. The size of the density distribution is characterized by the mean square radius

$$\bar{R} \equiv \sqrt{\langle R^2 \rangle} = \left[\int d^3r\, r^2\, \rho(\vec{r};\mu)\right]^{1/2} . \tag{34}$$

Taking into account the normalization condition (13) we get the total baryon number

$$B(\mu) \equiv \int d^3r\, \rho(\vec{r};\mu) = \left(\sum_{0\le\varepsilon_\alpha\le\mu} 1\right) - \frac{1}{2}\sum_\alpha \left[sign(\varepsilon_\alpha) - sign(\varepsilon_\alpha^V)\right]. \tag{35}$$

The dependence of the baryon number on μ is used to fix the value of the chemical potential. The valence contribution (first term) to the baryon number equals to the number of levels in the energy region $0 \le \varepsilon_\alpha \le \mu$ (not taking into account the degeneration N_c with respect to the colour quantum number). The sea contribution (second term) counts the number of levels passing from positive to negative energy when switching on the σ and $\hat{\pi}$ fields. A configuration with baryon number 1 can be realized in two different manners. First the meson fields may be strong enough to lower the energy of one of the positive-energy levels so strongly that it gets negative. In this case, an additional level populates the Dirac sea and gives the baryon number 1. The valence-energy region $0 \leq \varepsilon_\alpha \leq \mu$ must be empty ($\mu = 0$). If the meson fields are not strong enough to produce an additional state with negative energy, the sea contribution to the baryon number vanishes and the chemical potential must be chosen such that there is just one level in the valence-energy range. Usually we consider expectation values for configurations with a definite baryon number. In this case we denote expectation values (24) by K instead of $K(\mu)$ and assume the chemical potential to be properly chosen.

Another quantity characterizing a quark configuration is the isoscalar electric form factor $G_E^{T=0}(Q^2)$. It is related to the baryon density (32) via [15]

$$G_E^{T=0}(Q^2) = \int_0^\infty d^3r\, e^{i\vec{q}\vec{r}}\, \rho(\vec{r}), \tag{36}$$

where $Q^2 = -q^2 = |\vec{q}^{\,2}|$ is the negative squared four-momentum transfer in the Breit frame.

The axial density $A_o(\vec{r})$ can be written

$$A_o(\vec{r}) = \left\langle q^\dagger(\vec{r})\,\frac{\sigma_o\tau_o}{2}\,q(\vec{r})\right\rangle \tag{37}$$

and is obtained via eqs. (27-30) with

$$\mathcal{K} = \gamma_0\frac{\sigma_o\tau_o}{2}\,\delta^3(\vec{r}-\vec{r}\,') \qquad \text{and} \qquad K_\alpha(\vec{r}) = \Phi_\alpha^\dagger(\vec{r})\frac{\sigma_o\tau_o}{2}\Phi_\alpha(\vec{r}). \tag{38}$$

Since the sea contribution (29) diverges it has to be replaced by the regularized expression (30). The axial density determines the axial-vector coupling constant of the proton

$$g_A = -2\int d^3r\, A_o(\vec{r}), \tag{39}$$

where an additional factor $(-1/3)$ is incorporated which results from the projection onto the isospin quantum number $T = 1/2$ of the proton [26].

For a self-consistent determination of quark and meson fields we need the expectation values

$$S(\vec{r}) = \langle \bar{q}(\vec{r})\, q(\vec{r})\rangle \tag{40}$$

and

$$\hat{P}(\vec{r}) = \langle \bar{q}(\vec{r})\, i\gamma_5\hat{\tau}\, q(\vec{r})\rangle \tag{41}$$

of the meson field operators (2, 3), which can be obtained by means of eq. (27) with the regularized sea contribution (30) and with

$$\mathcal{K} = \delta^3(\vec{r}-\vec{r}\,') \qquad K_\alpha(\vec{r}) = \bar{\Phi}_\alpha(\vec{r})\,\Phi_\alpha(\vec{r}), \tag{42}$$

and

$$\mathcal{K} = i\gamma_5\hat{\tau}\,\delta^3(\vec{r}-\vec{r}\,') \qquad K_\alpha(\vec{r}) = \bar{\Phi}_\alpha(\vec{r})\, i\gamma_5\hat{\tau}\,\Phi_\alpha(\vec{r}), \tag{43}$$

respectively.

In the limit of classical meson fields the effective Euklidean action (5) agrees with the grand kanonical potential, where the Euklidean time interval T is the inverse of the temperature $\mathcal{T}$. This allows us to calculate the energy of the quark-meson system. At zero temperature ($T \to \infty$) the static meson and sea-quark energies differ from the grand kanonical potential and hence from the corresponding regularized effective action only by a factor T. An additional contribution to the energy results from the valence quarks which depends on the chemical potential μ [27]. The total energy E of a quark-meson configuration is given by

$$E(\mu) = E^q_{val}(\mu) + E^{q,Reg}_{sea} + E^M + E^{CSB} \tag{44}$$

with the valence-quark energy

$$E^q_{val}(\mu) = \frac{1}{T}\left[1 - \mu\frac{\partial}{\partial\mu}\right]\mathcal{A}^q_{val}(\mu) = \frac{\mathcal{A}^q_{val}(\mu)}{T} + \mu N_c B_{val}(\mu) = N_c \sum_{0\le\varepsilon_\alpha\le\mu} \varepsilon_\alpha \tag{45}$$

and the regularized sea-quark energy

$$E^{q,Reg}_{sea} = \frac{1}{T}\mathcal{A}^{q,Reg}_{sea} = -\frac{N_c}{2}\sum_\alpha \left[R_E(\varepsilon_\alpha,\Lambda)\left|\varepsilon_\alpha\right| - R_E(\varepsilon^V_\alpha,\Lambda)\left|\varepsilon^V_\alpha\right|\right]. \tag{46}$$

The meson energy $\left(\mathcal{A}^m[\sigma,\hat{\pi}] - \mathcal{A}^m_V\right)/T$, where we have subtracted the meson energy of the vacuum field, is split into two parts

$$E^M = \frac{\lambda^2}{2}\int d^3r \left[\sigma^2(\vec{r}) + \hat{\pi}^2(\vec{r}) - \sigma_V^2\right], \tag{47}$$

which vanishes on the chiral circle $(\sigma^2(\vec{r}) + \hat{\pi}^2(\vec{r})) = const = \sigma_V^2$, and

$$E^{CSB} = m\frac{\lambda^2}{g}\int d^3r \left[\sigma_V - \sigma(\vec{r})\right]. \tag{48}$$

While E^M is independent of the current quark mass m, E^{CSB} results from the chiral-symmetry-breaking term in the original lagrangian (1) and is proportional to m.

Finally let us consider the parameters of the model. Except the constituent quark mass M we fix them by the properties of the vacuum state and the meson sector. We assume a vacuum with broken chiral symmetry characterized by a finite expectation value $\sigma_V(r) \equiv \sigma_V$ of the σ field. As shown in [15] the vacuum expectation value of the $\hat{\pi}$ field must vanish. The corresponding hamiltonian (15) discribes free quarks with a mass (constituent quark mass)

$$M = g\sigma_V. \tag{49}$$

Its eigenstates are plane waves $\Phi^V_{\vec{k}}(\vec{r})$ labeled by the continuous momentum vector $\vec{k}$ and normalized to a 3-dimensional δ function. The stationary phase condition for the vacuum fields $\delta\mathcal{A}_{eff}/\delta\sigma(r)|_{\sigma_V\equiv f_\pi,\pi(r)\equiv 0} = 0$ gives

$$M = g\sigma_V = m + GN_c\int d^3k\, R_m(\varepsilon_{\vec{k}},\Lambda)\,\bar{\Phi}^V_{\vec{k}}(\vec{r})\,\Phi^V_{\vec{k}}(\vec{r}) = m + G\frac{N_cM^3}{2\pi^2}\Gamma\left(-1,\frac{M^2}{\Lambda^2}\right). \tag{50}$$

Here we have converted the sum over the eigenstates into an integral over all momenta $\vec{k}$ and applied the proper-time regularization. Eq. (50) establishes a relation between cut-off parameter Λ, interaction strength G, current quark mass m and constituent quark mass M.

Another relation between the parameters is obtained via the weak pion-decay. Considering fluctuations of the $\hat{\pi}$ field up to second order within proper-time regularization one gets the relation [18]

$$f_\pi^2 = \frac{N_c M^2}{4\pi^2}\Gamma\left(0, \frac{M^2}{\Lambda^2}\right). \tag{51}$$

Applying the PCAC hypothesis to the NJL-lagrangian (1) we get a relation between the current quark mass m and the pion rest mass m_π [18, 15]

$$m = G\frac{m_\pi^2 f_\pi^2}{M} = \frac{g^2}{\lambda^2}\frac{m_\pi^2 f_\pi^2}{M}. \tag{52}$$

Finally we identify the second variation of the effective action (5) with respect to the π field with $-m_\pi^2$ and get by means of eqs. (50, 52)

$$\sigma_V = f_\pi. \tag{53}$$

Using $N_c = 3$ and the experimental values for f_π and m_π the 6 relations (4, 49-53) allows us to determine 6 of the 7 unknown parameters M, g, m, σ_V, λ, G and Λ. We use the constituent quark mass M as the independent parameter and express the other by M.

3 Self-consistent quark and meson fields in mean-field approximation

In classical approximation the meson fields are restricted to those which minimize the effective action (6). Hence they have to fulfil the stationary phase conditions

$$\left.\frac{\delta\mathcal{A}_{eff}[\sigma,\hat{\pi}]}{\delta\sigma(\vec{r})}\right|_{\sigma=\sigma_{cl},\,\hat{\pi}=\hat{\pi}_{cl}} = 0 \qquad \text{and} \qquad \left.\frac{\delta\mathcal{A}_{eff}[\sigma,\hat{\pi}]}{\delta\hat{\pi}(\vec{r})}\right|_{\sigma=\sigma_{cl},\,\hat{\pi}=\hat{\pi}_{cl}} = 0. \tag{54}$$

With the effective action (5) one gets the following equations of motion for the classical meson fields

$$\sigma(\vec{r})|_{cl} = \langle\sigma\rangle = \frac{m}{g} - \frac{g}{\lambda^2}S(\vec{r}) \tag{55}$$

and

$$\hat{\pi}(\vec{r})|_{cl} = \langle\hat{\pi}\rangle = -\frac{g}{\lambda^2}\hat{P}(\vec{r}) \tag{56}$$

with the expectation values $S(\vec{r})$ and $\hat{P}(\vec{r})$ defined in eq. (40) and (41), respectively.

Quark wave-functions $\Phi_\alpha(\vec{r})$ and meson fields are mutually coupled via Dirac equation (12) and eqs. of motion (55, 56). In practice, self-consistent solutions can only be obtained after some additional approximations. A reduction of the degrees of freedom is achieved by restricting the classical meson fields (the index *cl* will be neglected from now) to spherical hedgehog configurations, which can be shown to be a self-consistent symmetry,

$$\sigma(\vec{r}) = \sigma(r) \qquad \text{and} \qquad \hat{\pi}(\vec{r}) = \pi(r)\,\hat{r} \tag{57}$$

with $\hat{r} \equiv \vec{r}/|\vec{r}\,|$, and to the chiral circle

$$\sigma^2(r) + \pi^2(r) = f_\pi^2. \tag{58}$$

The latter constraint turned out to be essential because otherwise no finite solitonic solution exists and the system collapses to a configuration with zero size and energy [20, 21]. Actually condition (58) can be justified from an extended NJL model implementing the trace anomaly of QCD [22, 23, 24].

With the restrictions (57) and (58) the hamiltonians (8) and (15) read

$$h = \vec{\alpha}\vec{p} + g\beta[\sigma(r) + i\pi(r)\gamma_5\hat{\tau}\hat{r}] \tag{59}$$

and

$$h_v = \vec{\alpha}\vec{p} + g\beta f_\pi, \tag{60}$$

respectively. The mesonic fields are uniquely determined by the profile function

$$\Theta(r) = arctan\frac{\pi(r)}{\sigma(r)} \tag{61}$$

according to

$$\sigma(r) = f_\pi\, cos\Theta(r) \qquad \text{and} \qquad \hat{\pi}(r) = f_\pi\, sin\Theta(r)\,\hat{r}. \tag{62}$$

Minimizing the effective action (5) with respect to $\Theta(r)$ one gets the following equation of motion

$$\Theta(r) = arctan\frac{\bar{P}(r)}{\bar{S}(r) - \frac{\lambda^2}{g^2}m}, \tag{63}$$

where $\bar{S}(r)$ and $\bar{P}(r)$ are angular-averaged expectation values

$$\bar{S}(r) = \frac{1}{4\pi}\int d\hat{r}\,\langle\bar{q}(\vec{r})\,q(\vec{r})\rangle = \frac{1}{4\pi}\int d\hat{r}\,S(\vec{r}) \tag{64}$$

and

$$\bar{P}(r) = \frac{1}{4\pi}\int d\hat{r}\,\langle\bar{q}(\vec{r})\,i\gamma_5\hat{\tau}\hat{r}\,q(\vec{r})\rangle = \frac{1}{4\pi}\int d\hat{r}\,P(\vec{r}). \tag{65}$$

The equation of motion (63) can be solved iteratively. Starting from a reasonable profile function $\Theta^0(r)$ we determine eigenfunctions Φ_α^o and eigenvalues ε_α^o by diagonalizing the hamiltonian (59) within an appropriate basis (see below). By means of eqs. (63-65) we get an improved profile function $\Theta^1(r)$. Continuing this procedure to convergence one gets self-consistent meson and quark fields (see fig. 1).

Let us look for spatially restricted fields configurations, i. e. for configurations which differ from the vacuum fields within a finite region characterized by a size parameter R. Then we can introduce a discrete set of basis states by putting the system into a box with radius D and infinitely high walls. The condition $D \gg R$ (in practice $D \approx (3-5)R$) ensures that the artificial wall does not influence the field configuration. A suitable set $|k_n^G lsjt;\, G^\Pi M\rangle$ was introduced in ref. [25]. It is characterized by the angular momentum $l = 0, 1, 2, \ldots$, the spin $s = 1/2$, the total angular momentum $j = l \pm s$, the isospin $t = 1/2$, the grandspin $G = j \pm t = l, l \pm 1$, the parity $\Pi = \pm 1$, and by an additional discrete quantum number (node number) k_n^G $(n = 1, 2, 3, \ldots)$, which results from the introduction of the finite box and depends on the grandspin G. Since the regularization proceduce limits the excitation energy of the quark states taken into account we can restrict the node number to a maximal value $k_{n^{max}}$. Considering cut-off parameters in the region $600\,MeV \leq \Lambda \leq 800\,MeV$, which correspond to constituent quark masses $350\,MeV \leq M \leq 1000\,MeV$, we found $n^{max} = 40$ to be sufficient. For larger values of the grandspin n^{max} can be reduced.

In the general case ($\hat{\pi} \not\equiv 0$), the hamiltonian (8) does not commute with the operators of l, s, j and t. Only Π, G and M are good quantum numbers and characterize the eigenstates Φ_α, which are superpositions of basis states $\left|k_n^G lsjt;\, G^\Pi M\right\rangle$

$$|\Phi_\alpha\rangle \equiv |\Phi^\nu_{G^\Pi M}\rangle = \sum_{k_n^G lj} a\left(k_n^G lj;\, G^\Pi \nu\right) \left|k_n^G lsjt;\, G^\Pi M\right\rangle . \tag{66}$$

There are altogther four combinations $l = G, G \pm 1$ and $j = l \pm 1/2$ which contribute to the sum (66). For $G = 0$ the number of combinations is reduced to 2. The coefficients $a\left(k_n^G lj;\, G^\Pi \nu\right)$ have numerically to be determined by diagonalizing the hamiltonian (59) within the subspace with definite values of parity and grandspin. The size of the subspace is $4(2) \times n^{max}$. The dependence on the projection M is trivial since we describe spherically symmetric objects. The index ν distinguishes the various eigenstates within the subspace. The maximal grandspin G^{max} which has to be taken into account depends on the size of the field. Using a maximal grandspin $G^{max} \approx 80$ we got a satifactory description in all cases up to such radii, where the asymptic formula for the meson fields [15] could be applied.

Fig. (1) illustrates the development of the iteration procedure for $M = 400\, MeV$ starting from a linear profile function $\Theta(r \leq 2R) = -\pi(1 - r/2R)$ and $\Theta(r \geq 2R) = 0$ with $R = M^{-1} = 0.49\, fm$. The constituent quark mass corresponds to $\Lambda = 1.592\, M = 636.7\, MeV$, $g = 4.30$, $G = 6.33\, M^{-2} = 39.58\, GeV^{-2}$ and $m = 0.0413\, M = 16.54\, MeV$ according to eqs. (49-53). A box with the radius $D = 15\, M^{-1} = 7.5\, fm$ was large enough not to affected the iteration.

As we can see most effort is necessary to produce the correct asymptotic behavior for $r \longrightarrow \infty$ (exponential), while the correct behaviour at small distances (linear) has been obtained after a few interations already. The lower part of fig. (1) illustrates the changes of the various energies during the iteration up to their finite value for self-consistent fields. One notices that the total energy E converges much faster than its partial contributions. The part E^{CSB} of the meson energy is the most sensitive quantity and we use it to control the iteration procedure.

In order to accelerate the convergence we start the procedure with a reference profile equipped with the correct asymptotic behaviour, which can analytically be determined by means of an asymptotic expansion of the field equation [15]. The reference profile

$$\Theta^{Ref}(R;r) = \begin{cases} -\pi\left(1 - \frac{r}{2R}\right) & \text{if} \quad r \leq R_m \\ -b\, \frac{1+m_\pi r}{r^2}\, e^{-m_\pi r} & \text{if} \quad r \geq R_m \end{cases} \tag{67}$$

interpolates beween the correct asymptotic behaviour at $r \longrightarrow 0$ and $r \longrightarrow \infty$. R is a parameter which characterizes the size of the field and can be chosen freely. The matching point R_m and the amplitude b are uniquely determined by the condition that expression (67) is smooth at $r = R_m$. If $R \ll 1/m_\pi \approx 1.4\, fm$ one gets $R_m \gtrsim 4R/3$ and $b = (\pi/3) R_m^2\, e^{m_\pi R_m}$.

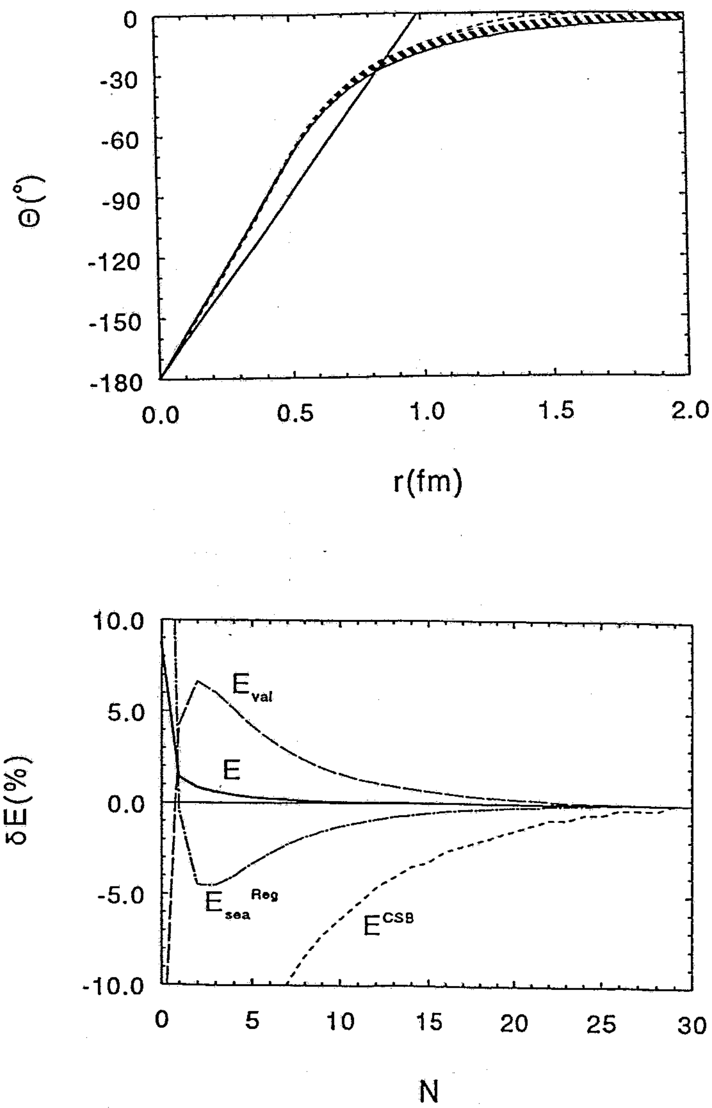

Fig. 1:
Upper part: Profile function during the iteration.
Straight full line: starting profile.
Broken lines: profile functions of different iteration steps.
Full line: finite profile (after 30 iterations)

Lower part: Relative deviations δE of the various energies E, E^q_{val}, $E^{q,Reg}_{sea}$ and E^{CSB} from their finite values in dependence on the number N of iterations.

In fig. 2 we compare the self-consistently determined profile functions $\Theta(r)$ calculated for $M = 350, 365, 400, 450, 465, 500, 600, 700, 730, 800, 900, 930$ and $1000\,MeV$ among each other and with a reference profile (67) with the fixed radius $R = 0.42\,fm$. First we recognize that the actual form of the self-consistent profile is nearly independent of the constituent mass M and hence of the regularization parameter Λ. Moreover the reference profile with the empirically determined size parameter $R = 0.42\,fm$ approximates all the calculated profiles quite well.

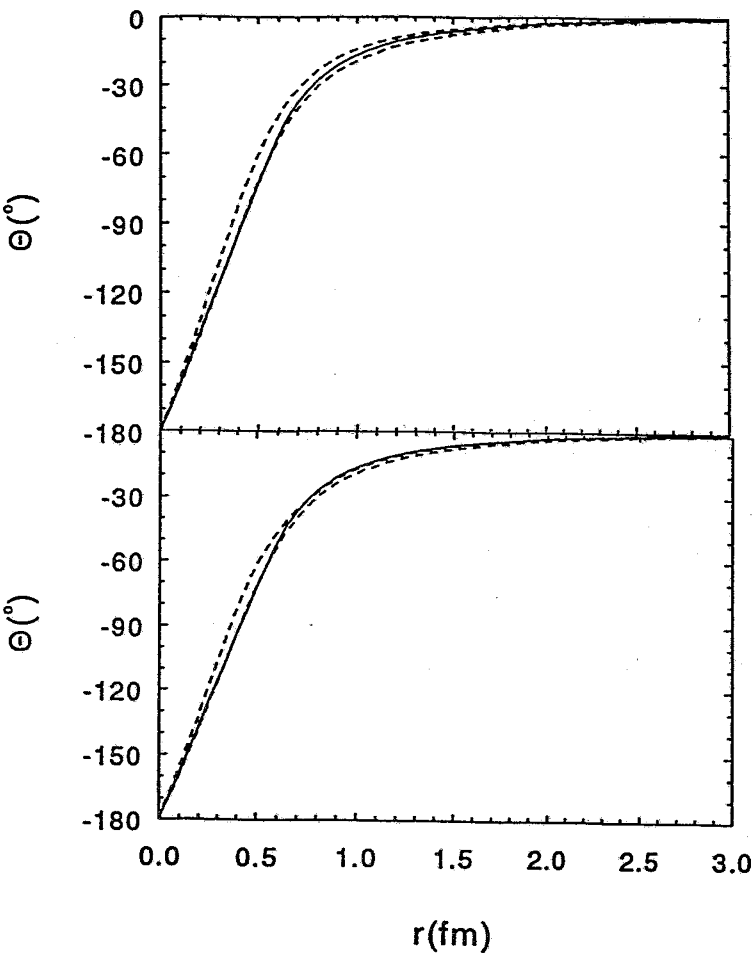

Fig. 2:
Self-consistet profiles in comparison with the reference profile $R = 0.42\,fm$ (*full line*).
All self-consistently calculated profiles fit in the area marked by the *broken lines.*

Upper part: mass region $350\,MeV \leq M \leq 930\,MeV$.

Lower part: mass region $350\,MeV \leq M \leq 600\,MeV$.

Before testing the quality of the approximation of the self-consistent profiles by the reference profile we show the radial dependence of the (scalar) baryon density (32) and of the axial density (37). Figs. 3 and 4 illustrate their behaviour in two different regions. At $M = 400\,MeV$ the main contribution to both quantities stems from the valence quarks confirming their dominating role within our model for constituent quark masses $M < 600\,MeV$. However, there are details, like the asymptotic behaviour of the axial density at large radii, which are determined by the sea quarks. At $800\,MeV$ the 0^+ valence level has joined the Dirac sea and does not give a seperate valence contribution. Nevertheless it continues to give the dominating contribution to the observables. The sum of valence and sea contribution depends smoothly on the valence energy. The value $sign(\varepsilon)$ of the regularization function (31) guarenties a smooth behaviour of the sum at $\varepsilon = 0$.

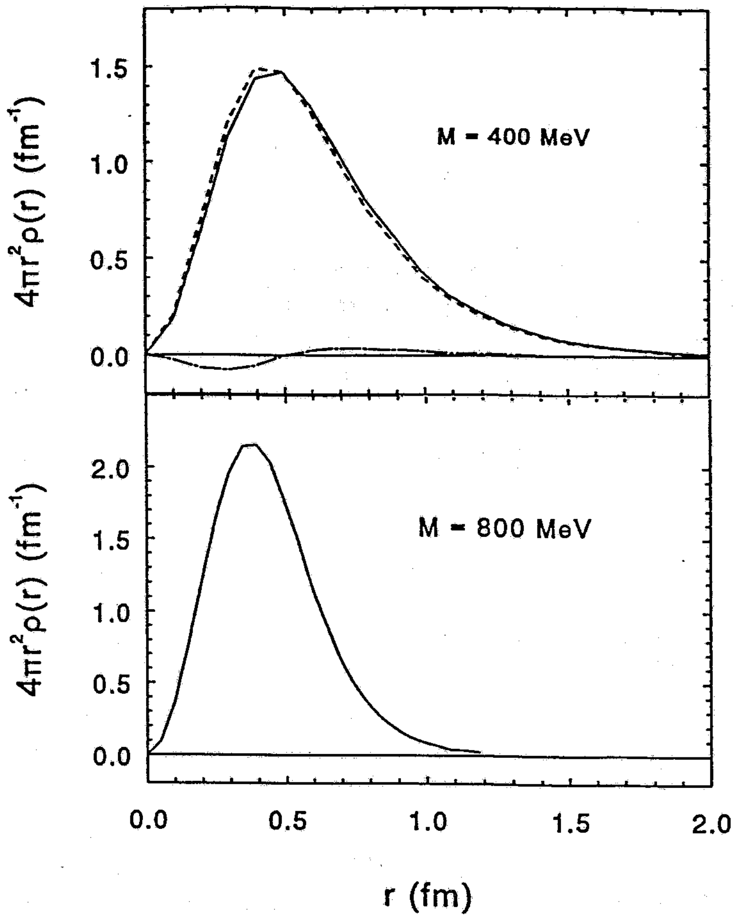

Fig. 3:
Radial distribution of the baryon density $4\pi r^2\,\rho(\vec{r})$ for $M = 400\,MeV$ (*upper part*) and $M = 800\,MeV$ (*lower part*) calculated for self-consistently determined meson fields.

Broken line:
Valence contribution.

Dash-dotted line:
Sea contribution.

Full lines: Sum of valence and sea contributions.

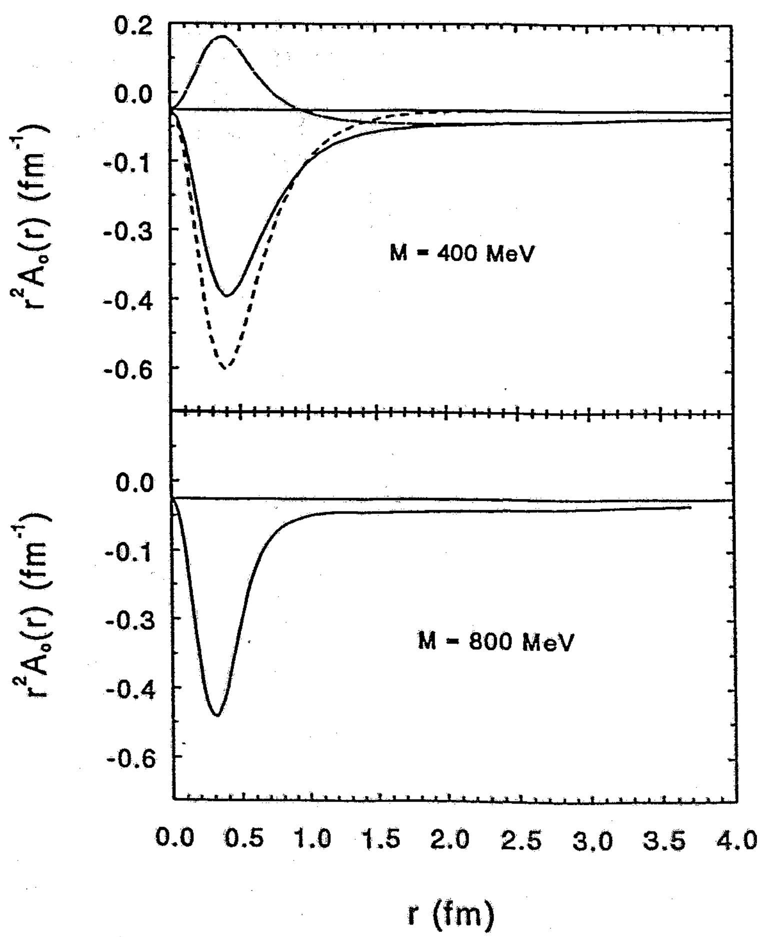

Fig. 4:
Radial distribution of the axial density $r^2\,A_o(\vec{r})$ for $M = 400\,MeV$ (*upper part*) and $M = 800\,MeV$ (*lower part*) calculated for self-consistently determined meson fields.

Broken line:
Valence contribution.

Dash-dotted line:
Sea contribution.

Full lines:
Sum of valence and sea contributions.

4 Testing the reference profile on nucleon observables

As shown in the last section the self-consistently determined meson profiles agree visually quite well with the reference profile defined in eq. (67) with $R = 0.42\,fm$. In order to test the quality of the approximation of the self-consistent profiles by the reference profile we calculate several nucleon observables. Fig. 5 shows the total energy (44) and their components (45, 46, 48), the mean square radii 34) of the baryon density, including their valence and sea-quark contribution, and the axial-vector coupling constant (39) calculated

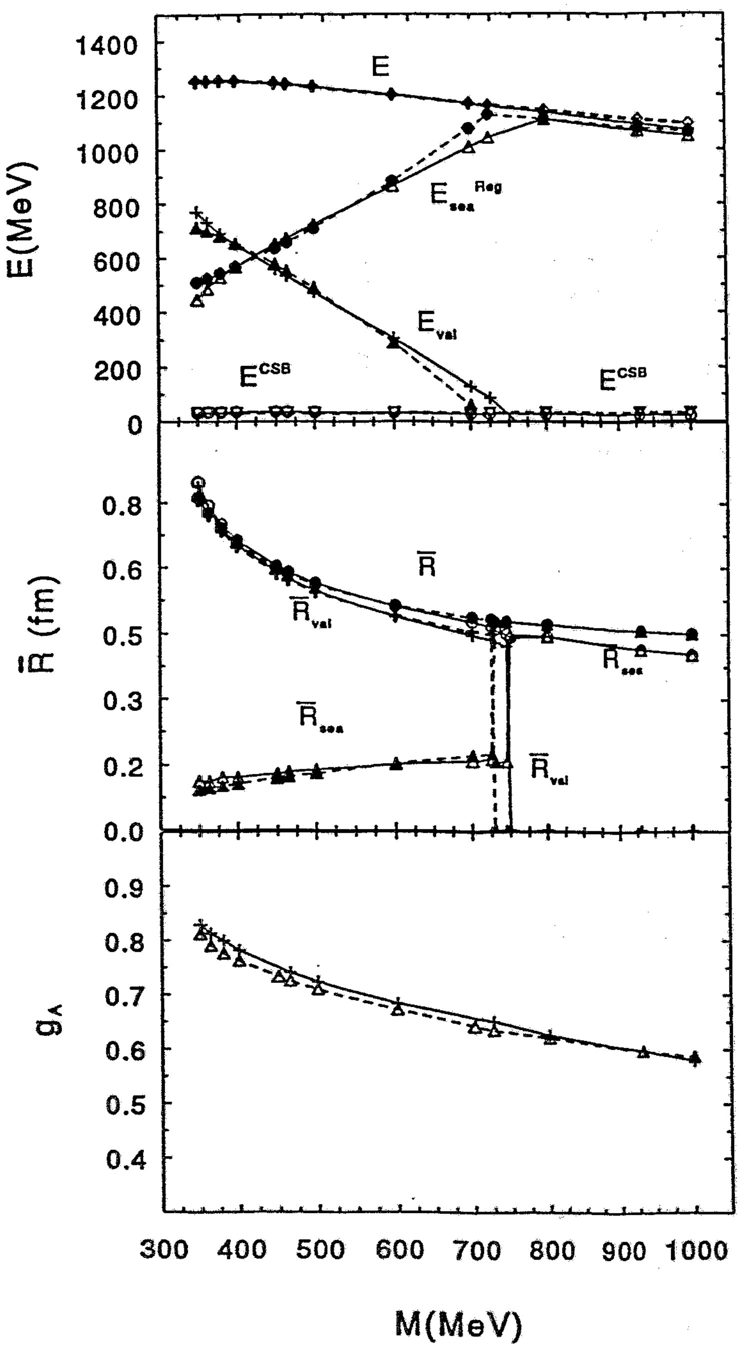

Fig. 5:

Nucleon observables in dependence on the constituent quark mass M calculated with self-consistently determined profiles (*full lines*) in comparison to the reference profile $\Theta^{Ref}(R;r)$ defined in eq. (67) with $R = 0.42\,fm$. (*broken lines*)

Upper part:
Energy E and its components E_{val}, E_{sea}^{Reg} and E^{CSB}.

Central part:
mean square radius $\bar{R}$ and its valence ($\bar{R}_{val}$) and sea contribution ($\bar{R}_{sea}$).

Lower part:
axial-vector coupling constant g_A (total value only).

Here and in the following figures signs like o • △+ indicate the points, were the calcultations have actually been performed. The lines interpolate between these points.

with either profile. The kink in the valence and sea contributions results from the definition of the valence-energy region ($0 \leq \varepsilon \leq \mu$). At the critical mass $M_{crit} \approx 750\, MeV$ the valence level leaves this region and joins the Diract sea. The behaviour of the regularization functions (22, 31) at $\varepsilon \rightarrow 0$ guarantees that the sum of valence and regularized sea contributions is a smooth function of the constituent quark mass M. Fig. 5 illustrates that nicely.

The only noticeable difference between the values for self-consistent and reference profile appears in the valence and sea contributions in the vicinity of M_{crit}. For the self-consistent profiles, the valence level dips into the Dirac sea at $M \approx 750\, MeV$. This point is shifted to $M \approx 725$ using the reference profile. This deviation is another evidence for the more sensitive dependence of valence and sea contributions on details of the profile function, while their sum is quite insensitive to them. One should note, however, that the physically relevant region for the constituent mass is around $M = 400\, MeV$, where nucleon observables get reproduced by the reference profile.

The calculated nucleon observables are in sufficient agreement with similar calculations [15, 26, 16]. The too small value of the axial-vector coupling constant ($g_A \approx 0.6 \sim 0.8$) in comparison to the experimental value ($g_A^{exp} \approx 1.25$) is a lack shared by many chiral models of the nucleon. However it is rather the aim of this paper to compare between two theoretical approaches than to reproduce the experimental values.

To complete our check of the reference profile we evaluate energy corrections to the static hedgehog configuration. They have been introduced to equip the static hedgehog with a definite value of spin and isospin and to make its center-of-mass momentum vanish. Applying the pushing approach to the center-of-mass motion one gets the correction [28]

$$\Delta E_{\vec{P}=0} = -\frac{\langle \vec{P}^2 \rangle}{2E}, \tag{68}$$

where $\langle \vec{P}^2 \rangle$ is the expectation value of the square of the total quark momentum, which can be calculated and regularized by means of eqs. (27, 28, 30) with

$$\mathcal{K} = -\gamma_0 \nabla^2 \qquad \text{and} \qquad K_o = -\int d^3r'\, \Phi_\alpha^\dagger(\vec{r}\,')\, \frac{\partial^2}{\partial \vec{r}\,'^2} \Phi_\alpha(\vec{r}\,'). \tag{69}$$

E is total static hedgehog energy (44). The cranking approach [29, 30, 31] applied to the iso-rotational degrees of freedom of the static hedgehog configuration gives an energy correction

$$\Delta E_T = \frac{T(T+1) - \frac{9}{4}}{2I} \tag{70}$$

for the restoration of isospin T. The term $-\frac{9}{4}$ results from the isospin $T = \frac{1}{2}$ of the 3 single valence quarks. Due to the restriction to hedgehog configurations with grandspin $G = 0$, spin and isospin are equal and eq. (70) describes the energy correction for a semiclassically quantized hedgehog with isospin T and spin $S = T$. According to eq. (70) the difference between the total energy (rest mass) of the Δ isobar ($S = T = 3/2$) and the nucleon ($S = T = 1/2$) is given by

$$E_\Delta - E_N = \frac{3}{2I}, \tag{71}$$

where I is the (iso-)rotational moment of inertia. It can be calculated by adding a rotational energy $-\vec{\omega}\vec{\tau}/2$ to the Euklidean action (5) and expanding the regularized sum with respect to the imaginary rotational frequency $\vec{\omega}$. Applying the proper-time scheme one gets [31]

$$I = I_{val} + I_{sea} \tag{72}$$

with the valence contribution

$$I_{val} = \frac{N_c}{2} \sum_{0 \leq \varepsilon_\alpha \leq \mu} \sum_{\varepsilon_\beta \neq \varepsilon_\alpha} \frac{\langle \Phi_\alpha | \tau_3 | \Phi_\beta \rangle \ \langle \Phi_\beta | \tau_3 | \Phi_\alpha \rangle}{\varepsilon_\beta - \varepsilon_\alpha} \tag{73}$$

and the regularized sea contribution

$$I_{sea} = \frac{N_c}{2} \sum_{\alpha\beta} R_I(\varepsilon_\alpha, \varepsilon_\beta; \Lambda) \frac{\langle \Phi_\alpha | \tau_3 | \Phi_\beta \rangle \ \langle \Phi_\beta | \tau_3 | \Phi_\alpha \rangle}{\varepsilon_\beta - \varepsilon_\alpha}. \tag{74}$$

The regularization function is given by

$$\begin{aligned} R_I(\varepsilon_\alpha, \varepsilon_\beta; \Lambda) &= \\ &= \frac{1}{2} \frac{1}{\sqrt{4\pi}} \int_{1/\Lambda^2}^{\infty} ds\, s^{-\frac{3}{2}} \frac{1}{\varepsilon_\beta + \varepsilon_\alpha} \left[e^{-s\varepsilon_\alpha^2} - e^{-s\varepsilon_\beta^2} + s\,(\varepsilon_\beta - \varepsilon_\alpha) \left(\varepsilon_\alpha e^{-s\varepsilon_\alpha^2} + \varepsilon_\beta e^{-s\varepsilon_\beta^2} \right) \right] \\ &= \frac{1}{4} \left[sign(\varepsilon_\beta)\, erfc\left(\frac{|\varepsilon_\beta|}{\Lambda} \right) - sign(\varepsilon_\alpha)\, erfc\left(\frac{|\varepsilon_\alpha|}{\Lambda} \right) - \frac{2}{\sqrt{\pi}} \frac{e^{-\left(\frac{\varepsilon_\alpha}{\Lambda}\right)^2} - e^{-\left(\frac{\varepsilon_\beta}{\Lambda}\right)^2}}{(\varepsilon_\alpha + \varepsilon_\beta)/\Lambda} \right] \\ &\longrightarrow \begin{cases} 0 & (\varepsilon^2 \gg \Lambda^2) \\ \frac{1}{4}\left[sign(\varepsilon_\beta) - sign(\varepsilon_\alpha) \right] & (\varepsilon^2 \ll \Lambda^2). \end{cases} \end{aligned} \tag{75}$$

In the limit $\Lambda \to \infty$ one gets the well-known Inglis formula [32] for the moment of inertia. The incomplete error-function is given by $erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\, dt$.

There is almost no difference between energy corrections calculated for self-consistent and reference profiles up to masses $M = 600$, in particular for the physically relevant mass $M \approx 400\, MeV$ (fig. 6). At larger mass parameters the correction reaches half the total energy E. Such a large correction is in conflict with the perturbation expansion of the effective action and the observed deviations are not very relevant.

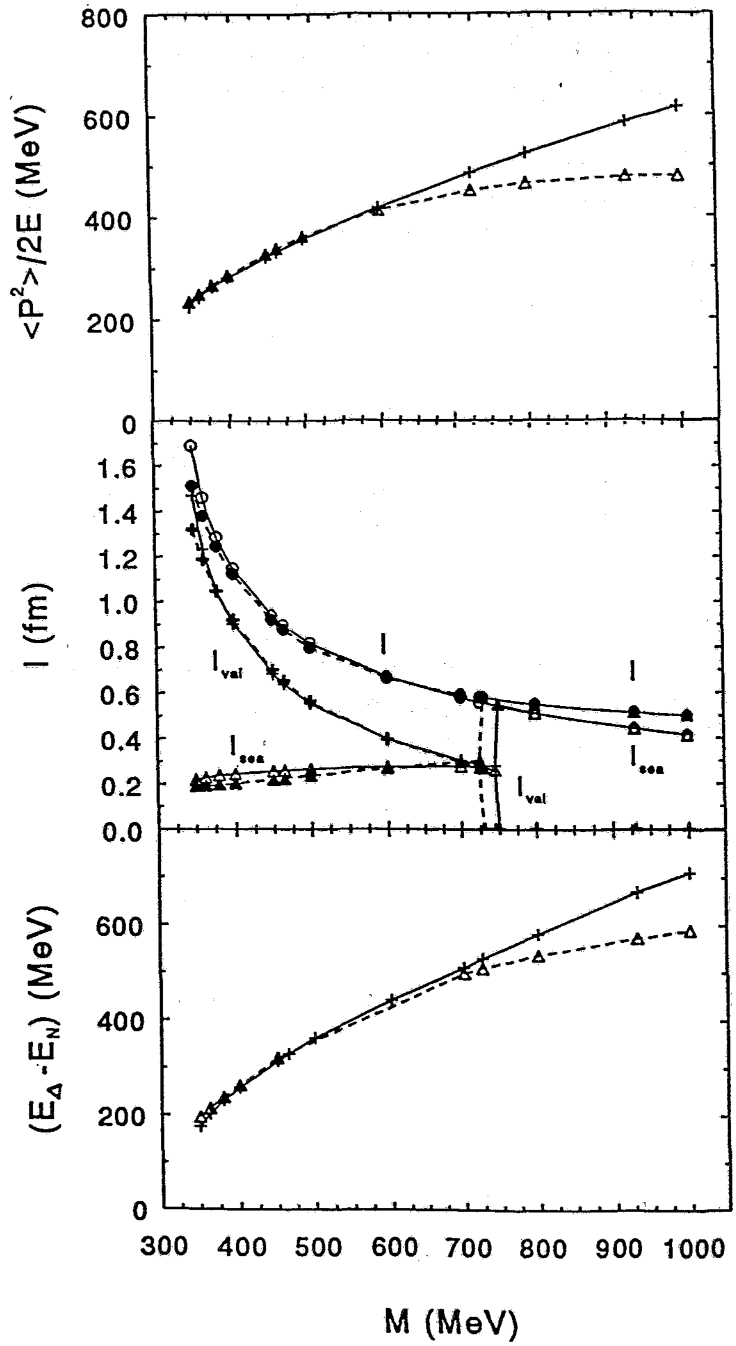

Fig. 6:

Energy corrections to the static hedgehog energy calculated with self-consistently determined profiles (*full lines*) and with the reference profile (*broken lines*) in dependence on the constituent quark mass M.

Upper part:
Center-of-mass energy (68).

Central part:
(Iso)rotational moment of inertia I (72) and its valence and sea contributions I_{val} (73) *and* I_{sea} (74).

Lower part:
Delta-nucleon mass splitting (71).

5 Conclusions

We have self-consistently calculated average meson fields for the SU(2) Nambu & Jona-Lasinio model with scalar-isoscalar and pseudoscalar-isovector couplings in Hartree approximation. The fields are restricted to the chiral circle and to hedgehog configurations. Infinite quark contributions are regularized within Schwinger's poper-time scheme.

The numerically determined self-consistent profile functions turn out to be nearly independent of the constituent quark mass. The profile function, which describes the meson fields, can be approximated by a reference profile with a simple analytic form, which interpolates smoothly between the correct asymptotic behaviour for small and large radii. The reference profile does not only approximate the self-consistent profiles but also reproduces the relevant observables of the quark and meson configuration quite well.

We conclude that many of the properties of the Nambu & Jona-Lasinio Lagrangian can be studied using the reference profile instead of applying the time-consuming determination of the self-consistent profile. Changing the constituent quark mass M mainly the strength g of the quark-meson coupling is changed, while the meson fields are approximately independent of M. If an accurate determination of the self-consistent profile turns out to be necessary, the reference profile may serve as a suitable starting profile for an iteration procedure.

The reference profile plays a similar role as the harmonic oscillator or Woods-Saxon potential in the description of the average nuclear field. Most of the nuclear properties are sufficiently well described by these potentials which rather distinguish themself by their formal simplicity than by their confirmation in a Hartree or Hartree-Fock procedure.

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References

[1] J. Nambu, G. Jona-Lasinio *Phys. Rev.* **122** (1961) 345; **124** (1961) 246

[2] U. Vogl, W. Weise *Prog. Part. and Nucl. Phys.* **27** (1991) 195

[3] S. Klevansky *Rev. Mod. Phys.* **64** (1992) 649

[4] T. Hatsuda, T. Kunihiro *Phys. Rep.* (1993) to be published

[5] Th. Meißner, A. Blotz, E. Ruiz Arriola, K. Goeke *Rep. Prog. Phys.* (1993) to be published

[6] T. Eguchi *Phys. Rev.* **D14** (1976) 2755

[7] D. Dyakonov, V. Petrov, P. Pobylitsa *Nucl.Phys.* **B306** (1988) 809

[8] Th. Meißner, E. Ruiz Arriola, F. Grümmer, K. Goeke, H. Mavromatis *Phys. Lett.* **B214** (1988) 312

[9] H. Reinhardt, R. Wünsch *Phys. Lett.* **B215** (1988) 577; **B230** (1989) 93

[10] Th. Meißner, F. Grümmer, K. Goeke *Phys. Lett.* **B227** (1989) 296

[11] D. Dyakonov, V. Petrov, P. V. Pobylitsa, M. Praszalowicz *Nucl. Phys.* **B323** (1989) 53

[12] M. Praszalowicz *Phys. Rev.* **D42** (1990) 216

[13] R. Alkofer *Phys. Lett.* **B236** (1990) 310

[14] Th. Meißner, F. Grümmer, K. Goeke *Ann. Phys.* **202** (1990) 297

[15] Th. Meißner and K. Goeke *Nucl. Phys.* **A524** (1991) 719

[16] M. Wakamatsu and H. Yoshiki *Nucl. Phys.* **A524** (1991) 561

[17] A. Chodos, B. Thorn *Phys. Rev.* **D12** (1975) 2733

[18] D. Ebert, H. Reinhard *Nucl. Phys.* **B271** (1986) 188

[19] J. Schwinger *Phys. Rev.* **82** (1951) 664

[20] P. Sieber, Th. Meißner, F. Frümmer and K. Goeke *Nucl. Phys.* **A547** (1992) 459

[21] Watabe and H. Toki *Prog. Theor. Phys.* **87** (1992) 651

[22] G. Ripka and M. Jaminon *Ann. Phys. (NY)* **218** (1992) 51

[23] Th. Meißner, G. Ripka, R. Wünsch, P. Sieber, F. Grümmer and K. Goeke *Phys. Lett.* **B299** (1993) 183

[24] C. Weiss, R. Alkhofer and H. Weigel *Mod. Phys. Lett.* **A8** (1993) 79

[25] S. Kahana, G. Ripka *Nucl. Phys.* **A419** (1984) 462

[26] Th. Meißner, K. Goeke *Z. Phys.* **A339** (1991) 513

[27] Chr. V. Christov, E. Ruiz Arriola and K. Goeke *Nucl. Phys.* **A510** (1990) 689

[28] P. V. Pobylitsa, E. Ruiz Arriola, Th. Meißner, F. Grümmer, K. Goeke, W. Broniowski *J. Phys.* **G18** (1992) 1455

[29] G. S. Adkins, C. R. Nappi, E. Witten *Nucl. Phys.* **B228** (1983) 552

[30] T. D. Cohen, W. Broniowski *Phys.Rev.* **D34** (1986) 3472

[31] H. Reinhardt *Nucl.Phys.* **A503** (1989) 825

[32] D. R. Inglis *Phys. Rev.* **96** (1954) 1059