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Abstract

The pion self-energy and propagator in a pion gas at temperature $T \sim m_\pi$ are calculated within Hartree approximation. The pion-pion interaction is described by Weinberg's Lagrangian. The modification of the pion spectrum consists in the replacement of the free pion mass by an effective one which increases with growing temperature. The thermodynamical quantities can be described by a quasi-particle representation and turn out to be smaller than in a free pion gas.

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In ultrarelativistic collisions of heavy nuclei a dense and hot hadronic system is formed. For large enough nuclei and high enough energies such a hadron gas might even achieve local equilibrium irrespectively whether it emerges from a previously deconfined matter state. The conjectured increase of nuclear transparency at high bombarding energy provides the colliding nuclei to go through each other, and a meson cloud is produced in the middle region between the receding nuclei. In present experiments at CERN-SPS already pion multiplicities of a few hundreds are reached, and in future lead beam experiments one expects thousands of secondaries, which are even more frequently produced at RHIC and LHC energies. So, due to the recent development of high-energy heavy-ion physics [1] one gets a possibility to investigate rather extended meson systems. This new opportunity stimulated a series of theoretical works devoted to the investigation of properties of such systems [2, 3, 4, 5, 6, 7, 8].

Even when assuming local equilibrium, one is faced with the complexity of the strongly interacting many-body system, and a careful analysis relying on known features of meson interactions is needed. The pion-pion scattering amplitude is considered in Ref. [2] within some approximation, where the pion eigenstate itself is assumed as unmodified by the surrounding medium. In Ref. [3], which focus on the phenomenology of the phase transition of the meson gas to quark-gluon plasma, the medium influence on the mesons is taken into account rather schematically by adding to the free meson spectrum a positive correction independent of the meson momentum and proportional to the particle density. Recently, other possible phase transitions are dealt with, e.g., the transition from a homogeneous pion medium to unisotropic states of various kinds, such as pion droplets [4], or a state with boson paring [5], or the formation of isotopically asymmetric matter with boson condensation [6]. The pion optical potential in medium is analyzed in Ref. [7] by using the forward-scattering amplitude of free pions, and the modification of the pion spectrum is found to be practically negligible as compared to free pions. A thorough selfconsistent calculation of the pion propagator and the pion excitation spectrum is performed in Ref. [8], where an effective pion-pion interaction is used, which is originally constructed in order to describe pion-pion scattering in the energy region around 1 GeV [9]. A sizeable decrease of the pion energy at low momentum is obtained in the latter calculation which is at variance with the results of Ref. [7]. Due to this apparent difference one is forced to reinvestigate the excitation spectrum of pion matter in a selfconsistent way by relying on an irreducible interaction.

Unlike to the mentioned work [8, 7], in our approach the considered system of interacting pions is described by Weinberg's pion field Lagrangian which takes the form in the lowest order in the coupling constant $f_{\pi\pi}^{-1}$ (or with an accuracy up to terms being proportional to the square of pion momenta) (see Ref. [10])

$$\mathcal{L} = \mathcal{L}^0 + \mathcal{L}', \quad \mathcal{L}' = \lambda(-\partial_\mu \pi\, \partial^\mu \pi + \frac{1}{2}\pi^2 m_\pi^2)\, \pi^2, \qquad (1)$$

where $\lambda = f_{\pi\pi}^{-2}/4$ ($f_{\pi\pi} = 93$ MeV). $\mathcal{L}^0 = \partial_\mu \pi \, \partial^\mu \pi + \frac{1}{2}\pi^2 m_\pi^2$ is the free pion Lagrangian, and $\mathcal{L}'$ stands for the interaction part. We use pionic units $m_\pi = c = \hbar = 1$. The prefactor of the second term in $\mathcal{L}'$, violating the chiral symmetry for real pions, is introduced according to Ref. [11]. The Lagrangian (1) is well known to describe the $\pi\pi$ scattering at low and medium energy [12]. Since we consider temperatures $T \sim m_\pi$, where the pion gas can be quite dense but the density of genuine heavy mesons is low, the pion energy is in a suitable range so that this interaction can be applied. The Lagrangian $\mathcal{L}'$ is understood as irreducible and describing pointlike pion-pion interaction, therefore, it does not suffer modifications in medium. (Note that the investigation in Ref. [8] employs an interaction which is entirely due to various heavy meson exchange and does not include the irreducible interaction (1). According to the use of explicit heavy meson degrees of freedom such an effective potential can hardly be considered as unchangeable in medium, because the rho meson itself might be modified substantially when including in turn the two-pion state.) The physical picture assumed in our approach relies on thermal and chemical equilibrium, as well as on vanishing chemical potential and isospin symmetry.

We carry out our investigation of an interacting pion system by using the temperature Greens functions [13]

$$D(\xi, \mathrm{k}, T) = (\xi^2 - 1 - \mathrm{k}^2 - \Pi(\xi, \mathrm{k}, T))^{-1}. \tag{2}$$

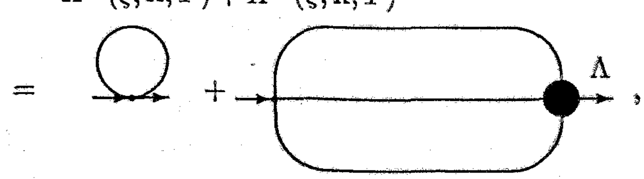
Since in eq. (2) the physical pion mass enters, the self-energy part $\Pi(\xi, \mathrm{k}, T)$ is the difference

$$\Pi(\xi, \mathrm{k}, T) = \tilde{\Pi}(\xi, \mathrm{k}, T) - \tilde{\Pi}(\xi, \mathrm{k}, 0). \tag{3}$$

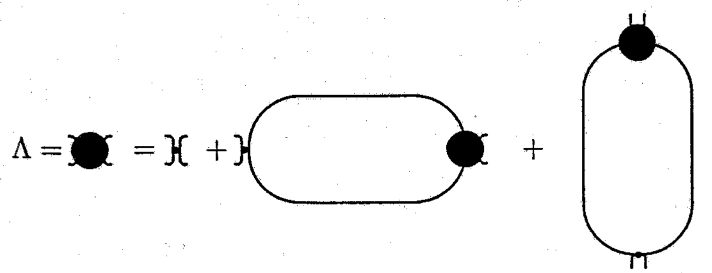
Thereby the infinite contributions to the pion mass operator $\tilde{\Pi}(\xi, \mathrm{k}, 0)$ due to the vacuum pion field fluctuations are removed from $\Pi(\xi, \mathrm{k}, T)$.

The polarization operator of the pion $\tilde{\Pi}(\xi, \mathrm{k}, T)$ in medium at temperature T can be presented graphically in most general form by the set of diagrams [13]

$$\tilde{\Pi}(\xi, \mathrm{k}, T) = \tilde{\Pi}^{1\pi}(\xi, \mathrm{k}, T) + \tilde{\Pi}^{3\pi}(\xi, \mathrm{k}, T) \tag{4}$$



where the thick lines correspond to total propagator D, so that the expression (2) is, in fact, an equation for D (or Π) [13]. The dot represents the irreducible $\pi\pi$-amplitude λ in $\mathcal{L}'(1)$, and the full circle stands for the total $\pi\pi$-amplitude Λ, which is determined by an equation, displayed graphically as follows



(5)

It is too difficult to solve simultaneously the set of eqs. (2, 4, 5) for determining the quantities $\Pi(\xi, \mathrm{k}, T)$ and $D(\xi, \mathrm{k}, T)$. If we take into account only the first term $\Pi^{1\pi}$ in the pion self-energy part (4) we arrive at the Hartree approximation. Such an approximation is reasonable since our investigation is restricted to low temperatures $T \ll m_\rho$. In this case, all our calculations involve the pion field states with sufficiently small energy and momentum $\xi, p \ll 1$ GeV, whereas the second term in eq. (4) influences the pion properties only at large enough values $\xi, p \sim m_{\rho,\sigma}$.

We find that the lengthy calculations performed according to the Greens function method in Ref. [13] result, due to the very form of the interaction $\mathcal{L}'(1)$, in

$$\Pi^{1\pi}(\omega, \mathrm{k}, T) = -10d\lambda + 6d\lambda(\omega^2 - \mathrm{k}^2) + 6d\lambda\frac{1 - 10d\lambda}{1 - 12d\lambda}, \quad (6)$$

$$D(\omega, \mathrm{k}, T) = \gamma(\omega^2 - \mathrm{k}^2 - \tilde{m}_\pi^2)^{-1}, \quad (7)$$

where $\gamma = (1 - 6\lambda d(T))^{-1}$ and the function $d(T)$ is determined by

$$d(T) = \frac{1}{2\pi^2}\int\limits_0^\infty \frac{dk\, k^2\chi(\omega(k))}{\omega(k)(1 - 6d(T)\lambda)}, \quad \chi(\omega) = \left[\exp\left\{\frac{\omega(k)}{T}\right\} - 1\right]^{-1}. \quad (8)$$

Thus, the pion propagator has poles at

$$\omega^2(k) = \mathrm{k}^2 + \tilde{m}_\pi^2, \quad \tilde{m}_\pi^2 = \frac{1 - 10\lambda d(T)}{1 - 12\lambda d(T)} \quad (9)$$

with residues $\gamma/2\omega(k)$. The pion spectrum in the considered approximation gets in medium the simple form (9), i.e., the free pion mass is replaced by the temperature-dependent effective one $\tilde{m}_\pi(T)$. One should have in mind that the pion propagator in the form (7) is valid for energies $\omega \ll 1$ GeV only, since the Lagrangian (1) enables one to describe the pion interaction at low enough energy and momentum.

Hence we see that, in framework of the Hartree approximation, the modification of the pion propagator consists in the change of its pole position and the residue of this pole only, while the analytical properties of $D(\omega, \mathrm{k})$ are not changed. In particular, the quantity $\Pi(\xi, \mathrm{k}, T)$ does not get an imaginary part.

The effective pion mass (9) is displayed in Fig. 1. One observes that the effective mass becomes larger than the vacuum mass at $T > m_\pi$. Thus, unlike the results in

Refs. [7, 8], we obtain for low pion momentum an enhancement of the pion energy in medium as compared to the free pion. Restricting ourselves to temperatures $T \ll 1$ GeV, we do not need the accurate pion spectrum at larger momentum. In particular, the reason for neglecting the second term in eq. (4) is that it influences the spectrum at pion energy in the order of the ρ, σ-meson masses. Such pion states make no sizeable contributions to the thermodynamic quantities at $T < m_\rho$, now we are going to calculate.

A general study of the many-body thermodynamical properties is carried out in Refs. [14, 15], and our further calculations of the thermodynamic quantities are based on the results of these works. The pion polarization operator Π is real, and the propagator D has simple poles only. Therefore, for a spectrum $\omega(k)$ of the form (6, 7, 9), the formulae for the particle density ϱ, and entropy density $\mathcal{S}$ look like for the free pion gas, but with effective mass $\tilde{m}_\pi(T)$. The thermodynamical potential Ω and the energy density $\mathcal{E} = \partial(\Omega/T) \;/\; \partial(1/T)$ can be written after some cumbersome algebra which exploits the stationarity condition $\partial\Omega/\partial\Pi = 0$ (see Refs. [14, 15])

$$\Omega(T) = \tilde{\Omega}(T) - \tilde{\Omega}(0) = \frac{3T}{2\pi^2} \int_0^\infty dk\; k^2 \ln[1 - exp(-\omega(k)/T)] + \Delta(T), \qquad (10)$$

$$\mathcal{E}(T) = \tilde{\mathcal{E}}(T) - \tilde{\mathcal{E}}(0) = \frac{3}{2\pi^2} \int_0^\infty dk\; k^2 \omega(k) \chi(\omega(k)) + \Delta(T), \qquad (11)$$

$$\Delta(T) = -\frac{3\lambda\gamma(T)d(T)}{4\pi^2(1 - 12\lambda d(T))} \int_0^\infty \frac{dk\; k^2}{\omega(k)} \chi(\omega(k)). \qquad (12)$$

The first terms in eqs. (10, 11) are the usual thermodynamical potential and energy density of independent quasi-particles. The additional term $\Delta(12)$, standing for deviations of the quantities $\Omega, \mathcal{E}$ from free quasi-particle ones, turns out to be small (numerically, the Δ contributes less than 10% to $\Omega, \mathcal{E}$). Thus, the system turns out to be described approximately as a free quasi-particles gas, with a quasi-particle spectrum being determined by eqs. (8, 9). Notice the fast convergence of the integrals (10 - 12) at $\omega(k) > T$. Therefore, modifications of the pion spectrum at large momentum k would not influence the quantities ϱ, $\mathcal{S}$, $\mathcal{E}$, Ω at small temperature $T \ll 1$ GeV. The results of numerical calculations are displayed in Fig. 2. It is seen that the onset of modifications of the equation of state due to the $\pi\pi$ interaction $\mathcal{L}'(1)$ appears at $T > m_\pi$. The sizeable deviations of the thermodynamical quantities from the ideal gas set in at $T \sim 1.5 m_\pi$ (see also Fig. 1), i.e., in a range where deconfinement or chiral symmetry restoration is expected. It is worth to note that at $T \sim 2m_\pi$. the modification due to this $\pi\pi$ interaction is of the same order of magnitude as contributions of heavier mesons, however with opposite sign. One striking outcome is the reduction of the energy density compared to the free pion gas. Such a behavior is not found in previous work [3, 7, 8].

Relying on these results, one can estimate the total number of pion excitations

$N(T)$ in a pion cloud at temperature T. Assuming that the volume has a value of $\approx 10^3$ fm^3, the pion number at $T \approx 200$ MeV is $N \approx 300$, while for a free pion gas this value would be ≈ 420. Thus, the $\pi\pi$-interaction suppresses the pion production at given temperature. It opposes simultaneously a fast increase of the energy (as well as entropy and pressure), with growing temperature, because at given excitation energy the interacting pion system has a higher temperature than a free pion gas. So, we are finally left approximately with the same particle numbers, at given excitation energy, both for interacting and noninteracting pion systems. For instance, the number $N \approx 400$ corresponds to excitation energy density $\mathcal{E} \approx 250$ MeV·fm^{-3} for the free as well as interacting pion system.

In summary, we present a quasi-particle description of an interacting pion gas in Hartree approximation. The effective pion mass increases with growing temperature, while the thermodynamical quantities are reduced compared to a free gas. Collective ρ, σ meson degrees of freedom in such a meson system may turn out to be important, even in spite of small genuine heavy mesons densities, providing an additional effective $\pi\pi$ interaction in medium. Therefore, one has to determine self-consistently the π, ρ, σ propagators in the medium, which is the goal of future work.

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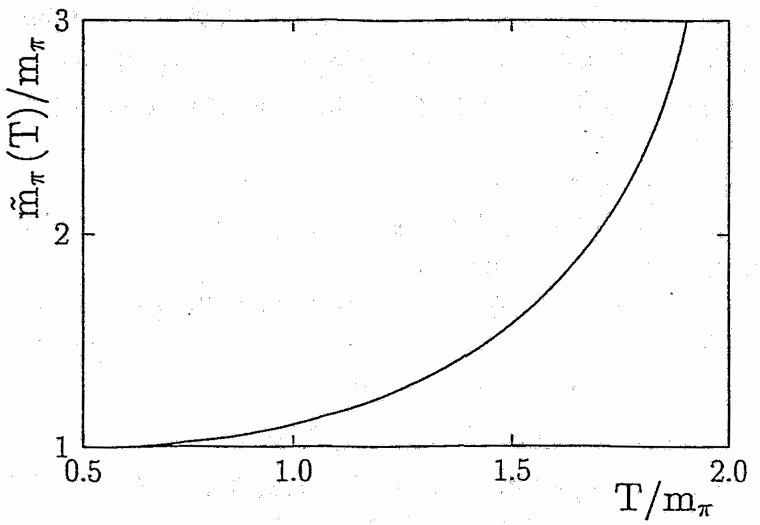

Fig. 1: The temperature dependence of the effective pion mass $\tilde{m}_\pi(T)$.

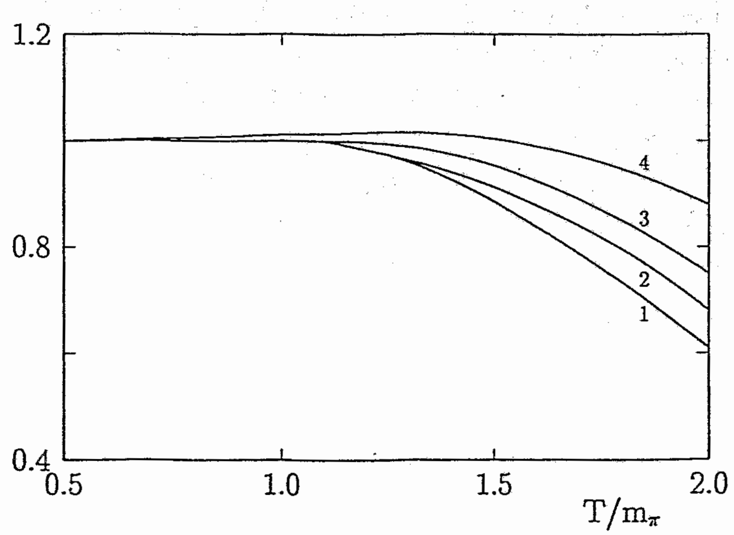

Fig. 2: The temperature dependence of the pion excitation density ϱ (curve 1), energy density $\mathcal{E}$ (curve 2), the entropy density S (curve 3) and pressure P (curve 4). The curves are scaled by corresponding ideal pion gas values. At higher temperatures the contributions of genuine heavier mesons and collective excitations need to be included. At $T \sim m_\pi$ there is a small overshoot of pressure on the free pion gas pressure due to the contribution of Δ.