FORSCHUNGSZENTRUM
ROSSENDORF e.V.

FZR

FZR-40
May 1994
Preprint

E.E. Kolomeitsev, D.N. Voskresensky and B. Kämpfer

KAON POLARIZATION IN NUCLEAR MATTER

KAON POLARIZATION IN NUCLEAR MATTER

E.E. Kolomeitsev[a] [*], D.N. Voskresensky[b, 1], B. Kämpfer[a,c]

[a] *Institut für Kern- und Hadronenphysik, Forschungszentrum Rossendorf e.V., PF 50119, D-01314 Dresden, Germany*

[b] *Gesellschaft für Schwerionenforschung, PF 110552, D-64220 Darmstadt, Germany*

[c] *Institute for Theoretical Physics (KAI e.V.), TU, Mommsenstr 13, D-01062 Dresden, Germany*

Abstract

The kaon-nucleon interaction in nuclear matter is considered by taking into account tree graphs, p-wave interaction, pionic intermediate states, kaon fluctuations and some residual interaction. The latter one is constrained by Adler's consistency condition. The $K^-, K^+, K^o, \bar{K}^o$ polarization operators are calculated in cold nuclear matter with arbitrary isotopic composition. An extra s-wave repulsion is found, which probably shifts the critical point of a K^- condensation with vanishing kaon momentum to large nucleon densities. Oppositely, an extra p-wave attraction is obtained, which may lead to a K^- condensation at vanishing temperatures and densities $\rho \geq \rho_c^- \sim (4-6)\rho_o$. The spectrum of the kaonic excitations in nuclear matter is analyzed and a new low-lying branch in the K^- (and also $\bar{K}^o$) spectrum is found. Its presence may lead to interesting observable consequences, such as the enhancement of the K^- yields in heavy-ion reactions. At $\rho \geq \rho_c^-$ the frequency of this low-lying branch becomes negative at non-vanishing momentum; that signals the onset of inhomogeneous K^- condensation. The K^- condensate energy is calculated in the approximation of a small KK coupling constant. Accordingly, neutron matter may undergo a first-order phase transition to proton matter with K^- condensate at $\rho > \rho_c^-$. The temperature dependence of the most important terms of the K^- polarization operator is discussed. In a rather wide temperature region $0 < T < \frac{1}{2}m_\pi$ a growing temperature enlarges the K^-N attraction and promotes the kaon condensation. The possibility of $\bar{K}^o$ condensation is also considered. The question is qualitatively discussed whether proton matter with K^- condensate or neutron matter with $\bar{K}^o$ condensate is energetically more favorable.

[1]Permanent address: Moscow Engineering Physical Institute, Kashirskoe shosse 31, 115409 Moscow, Russia

1 Introduction

Possible manifestations of in-medium effects in nuclear reactions are under intensive investigation in the last two decades. After focusing on the properties of pion-like excitations in nuclei and in neutron stars and in the course of heavy-ion collisions (for reviews see refs. [1, 2, 3, 4]) now one turns to the properties of strangeness in a nuclear environment [5]. The growing body of experimental information does certainly inspire the future efforts in this field.

First, there exist experimental data on the $K^{\pm}$-nucleon and $K^{\pm}$-nucleus scattering and on kaonic atoms, from which one extracts the information about the optical potential of K^- in atomic nuclei [6, 7, 8, 9, 10]. We mention here also the empirical data on the enhancement of the cross sections of K^+ mesons scattered off carbon and calcium nuclei in comparison with those on nucleons, which indicate certain in-medium effects [11].

Second, the yields of K^+, K^- mesons have been measured in some nucleus-nucleus collisions for a wide range of energies [12, 13]. In order to describe the kaon production in the collisions of nuclei it is required to know, how kaons interact with excited nuclear matter. The $K^{\pm}$ mesons probe different stages of the collision, e.g. the K^+ mesons have a rather long mean free path and carry the information about an earlier stage of the collision, whereas the K^- mesons with shorter interaction length probe somewhat later stages.

Third, on the theoretical side the interest in a possible connection between the low-energy sector of QCD and the phenomenological theory of strong interaction results in the development of improved effective theories, which are widely related to the chiral symmetry. One of the interesting properties of chiral Lagrangians is the hypothesis of the universal in-medium scaling of hadron masses [14]. Among the topics on hadrons in dense and hot nuclear matter [15] the investigation of the kaon properties can be considered as intriguing test of our knowledge on strangeness degrees of freedom in the strongly interacting many-body system. The opinion has been put forward that at a density several times larger than the nuclear saturation density, $\rho_o \simeq 0.17 fm^{-3}$, a s-wave K^- condensation might happen [16, 17, 18, 19, 20, 21, 22]. Such a phenomenon would lead to interesting consequences also in the physics of neutron stars [18, 19].

Up today the KN interaction in nuclear matter has been considered mainly in relation to the K^- condensation. This problem has been raised first by Kaplan and Nelson, who use for this aim a SU(3)×SU(3) model Lagrangian. In ref. [16] and in the subsequent more detailed investigations [17, 18, 20, 23] only the s-wave terms of the KN interaction are included into consideration. Within the chiral-symmetric tree expansion it is shown that the s-wave interaction is attractive and grows with increasing nucleon density. Hence, it is argued that at the density $\rho > \rho_c \sim (3-4)\rho_o$ a K^- condensate appears. The role of the p-wave KN interaction is discussed in refs. [21, 22], where it has been concluded that K^- condensation may arise due to the s-wave attraction, and only at appreciably larger density a K^- condensate with non-zero momentum may set in.

In the present paper we consider the problem of KN interaction in nuclear matter and calculate the K^- polarization operator. We argue for a rather strong p-wave attraction. We show that due to this p-wave interaction a new low-lying branch of excitations appears in the spectrum of K^- mesons. The presence of this additional branch may substantially affect the properties of K^- mesons in a nuclear medium as well as the kaon yields in nucleus-nucleus collisions. To describe the s-wave interaction we take into account new graphs connected with pionic intermediate states, loop contributions and Adler's consistency condition. Due to the latter one the form of the s-wave part of the off-shell K^- polarization operator becomes different from the on-shell polarization operator and an additional repulsion arises (a similar conclusion is drawn also in ref. [24]). Therefore the zero-momentum K^- condensation probably does not occur up to rather high nucleon density. On the contrary we show that, in spite of some uncertainties in the s-wave description, the K^- condensation is likely to be retained due to the p-wave K^-N attraction and appears in an inhomogeneous state with finite momentum at densities as large as $\rho_c^- \sim (4-6)\rho_o$ (at vanishing temperature). We also argue that at approximately the same densities a condensation of $\bar{K}^o$ mesons may appear. (Such a possibility was mentioned in ref. [25] but has not been considered in the sufficient details.) As a result new particularities in the KN interaction may lead to interesting consequences in the description of K-nucleus scattering, kaon production in nucleus-nucleus collisions and the neutron star structure.

Our paper is organized as follows. In sect. 2 we calculate the p-wave part of the K^- polarization operator. The $\Lambda\bar{p}$, $\Sigma^o\bar{p}$ and $\Sigma^-\bar{n}$ intermediate states are considered. In sect. 3 we calculate the regular part of the K^- polarization operator. Besides the previously discussed graphs, which are obtained in the tree expansion of a chiral Lagrangian, we consider contributions from $\Lambda\bar{p}$, $\Sigma^o\bar{p}$, $\Sigma^-\bar{n}$ loops at vanishing kaon momentum $k=0$ and frequency $\omega=0$ and also the extra contributions from the pionic intermediate states and from the proper kaon fluctuations. The residual interaction in the K^- polarization operator is fixed by making use of Adler's consistency condition. We extend this analysis then to the $K^+, K^o, \bar{K}^o$ polarization operators. In sect. 4 we obtain the kaon spectrum in nucleonic matter. The appearance of the extra low-lying branch in the K^- spectrum is discussed in detail. In sect. 5 we calculate the K^- condensate energy and find the critical density of the K^- condensate formation. Sect. 6 is devoted to the discussion of finite temperature effects. In sect. 7 we consider the possibility of the $\bar{K}^o$ condensation and raise the question which phase of dense nuclear matter is energetically more favorable, e.g., proton matter with K^- condensate or neutron matter with $\bar{K}^o$ condensate. In sec. 8 we draw our conclusions and discuss some perspectives.

2 P-wave part of the kaon polarization operator

Let us start with the consideration of the K^- mesons. The system K^- meson - nucleon has the strangeness $S = -1$. Hence, the intermediate states of $K^-N \to K^-N$ scattering can consist only of the particles with the same strangeness $S = -1$, i.e., $\Lambda, \Sigma, \Lambda^*, \Sigma^*$ etc. We restrict ourselves to the consideration of the lightest of them since just these intermediate states correspond to the most sensitive dependence of the kaon polarization operator on frequency and momentum in the region of frequencies $|\omega| \leq m_K$ and momenta $k \ll m_N$, which are of interest here ($m_K \simeq 3.5m_\pi$, $m_N \simeq 6.7m_\pi$ are the kaon and nucleon masses, respectively). By this reason we do not introduce the form factors in the vertices, which are important at somewhat larger frequencies and momenta. Some graphs, which depend somewhat more smoothly on frequency and momentum, are also presented in explicit form, whereas the residual part of the kaon polarization operator is extracted from the experimental data as well as from the current algebra relations. Such an approach to fix the polarization operator has been used in the case of pions in ref. [3].

The terms of the Lagrangian corresponding to the p-wave interaction

$$\mathcal{L}_{KN\Lambda} = f_{KN\Lambda}\bar{\Lambda}\gamma^\mu\gamma_5(\partial_\mu K^T)N \tag{1}$$

$$\mathcal{L}_{KN\Sigma} = f_{KN\Sigma}\bar{\Sigma}^\alpha\gamma^\mu\gamma_5(\partial_\mu K^T)\tau_\alpha N \tag{2}$$

are quite similar to those of πNN interaction. Here N, Λ, Σ are the bispinors of nucleons, lambda and sigma particles, respectively. $K^T \equiv (K^+, K^o)$ is the isotopic spinor of kaons, γ_μ denote the Dirac matrices and τ^α are the Pauli matrices. The values of the coupling constants are determined in ref. [9] by the fitting of hyperon-nucleon scattering within the Bonn boson exchange model performed by the Jülich group as

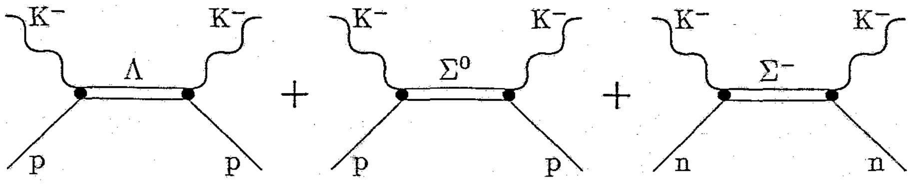
$$f_{KN\Lambda} \approx -1.17/m_\pi, \quad f_{KN\Sigma} \approx 0.22/m_\pi, \tag{3}$$

which are comparable to those obtained in the framework of a SU(3)×SU(3) model [22]

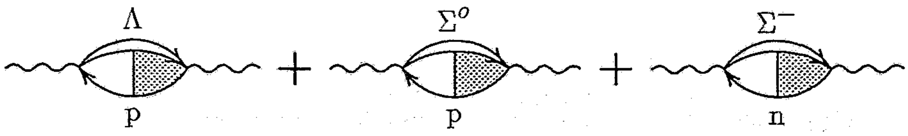
$$f^{\rm chir}_{KN\Lambda} \approx -0.88/m_\pi, \quad f^{\rm chir}_{KN\Sigma} \approx 0.26/m_\pi. \tag{4}$$

Below for applications we use the values of coupling constants given by eq. (3).

The amplitude of the p-wave KN scattering is determined by the following graphs

$A^{K^-N} =$  (5)

which directly correspond to following terms of the K^- polarization operator

$\Pi^{-,P} =$  (6)

Here it is assumed that there are only Fermi seas of neutrons and protons and no Fermi seas of strange particles. The line "$\longleftarrow$" corresponds to the nucleon quasi-hole ($\bar{p}$ or $\bar{n}$) and the double line "$\Longrightarrow$" denotes the strange baryons. The hatched vertices should be calculated by taking into account the in-medium baryon-baryon correlations. In principle this can be done in analogous way as for pions [3]. However because of the lack of the empirical data and without detailed analyses one can say very little not only about the numerical values of the correlation factors, but also even about their sign. Therefore we shall not consider this correlation factors in the present work and postpone their study, bearing in mind that they might have an influence on the final results as it is found for pions [1, 2, 3].

An explicit form of the polarization operator (6) can be simply obtained in the non-relativistic limit of the expressions (1, 2), which is applicable for frequencies and momenta of interest $|\omega|, k \ll m_N$. Then in the direct analogy to the π^- meson case [3] we get

$$A^{K^-N}_{\Lambda(\Sigma)} = \frac{f^2_{KN\Lambda(\Sigma)}(m_{\Lambda(\Sigma)} + m_N)\vec{k}\vec{k}'}{s - m^2_{\Lambda(\Sigma)}} \approx \frac{1}{2}\left(1 + \frac{m_{\Lambda(\Sigma)}}{m_N}\right) \frac{f^2_{KN\Lambda(\Sigma)}\vec{k}\vec{k}'}{\omega - \tilde{\omega}_{\Lambda(\Sigma)} - \dfrac{\vec{k}\vec{p}}{m_N} - \dfrac{t}{2m_N}}, \quad (7)$$

where $s = (p+q)^2$ and p is the 4-momentum of nucleon and $q = (\omega, \vec{k})$, m_Λ, m_Σ denote the masses of the strange baryons and $\vec{k}$ and $\vec{k}'$ are the momenta of the K^- meson before and after scattering, respectively,

$$t = \vec{k}^2 - \omega^2 = -q^2, \quad \tilde{\omega}_{\Lambda(\Sigma)} = \omega_{\Lambda(\Sigma)}\left(1 + \frac{\omega_{\Lambda(\Sigma)}}{2m_N}\right), \quad \omega_{\Lambda(\Sigma)} = m_{\Lambda(\Sigma)} - m_N.$$

Here and in the following we neglect the finite widths of hyperons, suggesting the validity of the quasi-particle approximation. The generalization is straightforward.

The K^- polarization operator is defined as follows

$$\Pi_\Lambda^{-,P} + \Pi_\Sigma^{-,P} = \int \frac{2d^3p}{(2\pi)^3}\left(n^p(p)A_\Lambda^{K^-p} + n^p(p)A_{\Sigma^o}^{K^-p} + n^n(p)A_{\Sigma^-}^{K^-n}\right) \quad (8)$$

where $n^p(p), n^n(p)$ are the occupation numbers of protons and neutrons, respectively.

By inserting the expression for the scattering amplitude (7) into eq. (8) and by integrating over the phase space we obtain ($|\vec{k}| = |\vec{k}'|$)

$$\Pi_\Lambda^{-,P} = -\frac{3}{4}\left(1 + \frac{m_\Lambda}{m_N}\right) f_{KN\Lambda}^2 \frac{\vec{k}\vec{k}'}{\tilde{\omega}_\Lambda(t)} \rho_p \Phi_{\Lambda p}(\omega, k), \quad (9)$$

where

$$\Phi_{\Lambda p}(\omega, \vec{k}) = \frac{\tilde{\omega}_\Lambda(t)}{k^3 v_{F,p}^3}\left\{\frac{a^2 - k^2 v_{F,p}^2}{2} ln \frac{a + k v_{F,p}}{a - k v_{F,p}} - a k v_{F,p}\right\} \quad (10)$$

is the Λ particle–proton-hole Lindhard's function with

$$a = \omega - \tilde{\omega}_\Lambda(t), \qquad \tilde{\omega}_\Lambda(t) = \tilde{\omega}_\Lambda + \frac{t^2}{2m_N}$$

($v_{F,p} = p_{F,p}/m_N$ is the Fermi velocity of protons, $\rho_p \equiv \nu\rho$ is the proton density).

The expression for $\Pi_\Sigma^{-,P}$ can be obtained from eq. (9) by the replacement

$$\Lambda \to \Sigma, \qquad \rho_p \Phi_{\Lambda p} \to \rho_p \Phi_{\Sigma p} + 2\rho_n \Phi_{\Sigma n}, \quad (11)$$

where $\rho_n \equiv (1 - \nu)\rho$ is the neutron density.

In order to derive a simple analytical representation of the polarization operator we use the expansion of the Lindhard functions at $|a| \gg kv$, i.e., at $k \ll |\omega - \omega_{\Lambda(\Sigma)}| m_N p_F^{-1}$. Then we can write

$$\Phi_{\Lambda(\Sigma)}(\omega, k) \simeq \frac{\tilde{\omega}_{\Lambda(\Sigma)}(t)}{\tilde{\omega}_{\Lambda(\Sigma)} - \omega}. \quad (12)$$

Thereby we have

$$\Pi_\Lambda^{-,P} \simeq \frac{A_0 \vec{k}\vec{k}' \nu}{\omega - \tilde{\omega}_\Lambda(t)} \frac{\rho}{\rho_o}, \qquad A_o \simeq 1.1 m_\pi, \quad (13)$$

$$\Pi_\Sigma^{-,P} \simeq \frac{B_0 \vec{k}\vec{k}'(2 - \nu)}{\omega - \tilde{\omega}_\Sigma(t)} \frac{\rho}{\rho_o}, \qquad B_o \simeq 0.1 m_\pi, \quad (14)$$

where we have also used that $t \approx \vec{k}^2$ in the approximation $k \gg \omega$.

As we can see from eqs. (13, 14) the term $|\Pi_\Sigma^{-,P}|$ is, as a rule, much smaller than $|\Pi_\Lambda^{-,P}|$ and can be neglected, except in only some very specific case when $|\omega - \tilde{\omega}_\Sigma(t)| \ll m_\pi$. At $\nu \to 0$, one has $|\Pi_\Sigma^{-,P}| \gg |\Pi_\Lambda^{-,P}|$, however, $|\Pi_\Sigma^{-,P}|$ remains much smaller than the s-wave part of the polarization operator.

Above we use the non-relativistic limit for the vertices (1, 2). Going beyond this approximation and taking into account the minimal relativistic corrections with respect to the parameters $\omega/m_N \ll 1$, $k/m_N \ll 1$, which accounts for the nucleon recoil, instead of eqs. (13, 14) we now find

$$\Pi_{\Lambda}^{-,P} \simeq \left\{ \frac{A_0(\vec{k}\vec{k}' - \omega^2) + A_1\omega\tilde{\omega}_\Lambda(t)}{\omega - \tilde{\omega}_\Lambda(t)} + A_2\omega \right\} \nu \frac{\rho}{\rho_o}, \quad (15)$$

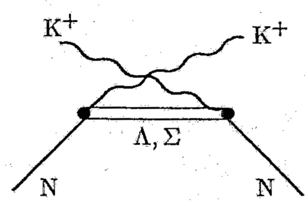
where

$$A_1 \simeq 1.1 m_\pi, \qquad A_2 \simeq 0.7 m_\pi,$$

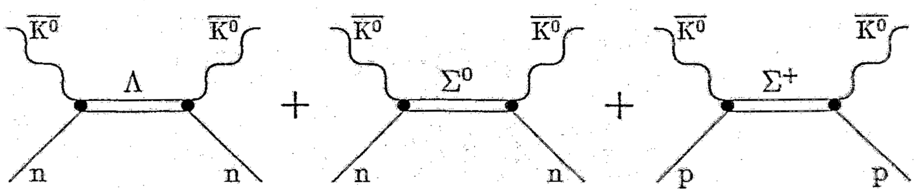
$$\Pi_{\Sigma}^{-,P} \simeq \left\{ \frac{B_0(\vec{k}\vec{k}' - \omega^2) + B_1\omega\tilde{\omega}_\Sigma(t)}{\omega - \tilde{\omega}_\Sigma(t)} + B_2\omega \right\} (2-\nu) \frac{\rho}{\rho_o}, \quad (16)$$

$$B_1 \simeq 0.04 m_\pi, \qquad B_2 \simeq 0.03 m_\pi.$$

The polarization operator of the K^+ meson can be found from the polarization operator of the K^- meson by the replacements $\omega \to -\omega$, $k \to -k$; it corresponds to the consideration of the u-channel amplitude of the K^+N scattering

$A^{K^+N} =$ 

instead of the s-channel amplitudes (5) for the K^-N scattering. Following the suggested approach we can also derive the polarization operator of K^o and $\bar{K}^o$ mesons. In comparing the K^-N scattering amplitude (5) with the amplitude of the $\bar{K}^oN \to \bar{K}^oN$ process given by the graphs

$A^{\bar{K}^oN} =$ 

we conclude that the polarization operator for $\bar{K}^o$ mesons can be obtained from the K$^-$ polarization operator by the replacement $\nu \to 1-\nu$. In order to obtain the K^o-polarization operator we have additionally to replace $\omega \to -\omega$ and $k \to -k$.

3 The regular part of the K^- meson polarization operator

3.1 Tree approximation

In ref. [20] the s-wave part of the K^- meson polarization operator is extracted from the expansion of chiral Lagrangian. For this aim the Weinberg counting rule [26] has been used. According to the latter one an amplitude, involving the number E_N of external nucleon lines and E_K external kaon lines, is characterized by the factor $Q^{\bar{\nu}}$ in the amplitude, where Q is the characteristic small momentum scale involved in the process and

$$\bar{\nu} = 2 + 2\ell - \frac{1}{2}E_N + \sum_i (d_i + \frac{1}{2}n_i - 2), \tag{17}$$

where ℓ is the number of the loops. Here the sum over i goes over all vertices, d_i is the number of derivatives acting on the i-th vertex; and n_i is the number of nucleon lines attached to the i-th vertex. In the absence of external fields, the chiral symmetry constrains

$$P_i \equiv d_i + \frac{1}{2}n_i - 2 \geq 0. \tag{18}$$

In application to the KN scattering one sees that the leading term in this counting is given by $\ell = 0$ and $P_i = 0$. This is satisfied for a vertex with $d_i = 1$ and $n_i = 2$ and the amplitude has the index $\bar{\nu} = 1$. In the next order, one has $\ell = 0$ and one $P_i = 1$ vertex with index $\bar{\nu} = 2$. In ref. [20] the chiral expansion was restricted by the contributions up to $\bar{\nu} = 2$, and hence no loops were considered.

The tree terms of the Lagrangian corresponding to $\bar{\nu} = 1$ and $\bar{\nu} = 2$ are equal to

$$\mathcal{L}_{\bar{\nu}=1} = -\frac{i}{8f^2}\left[3(\bar{N}\gamma^\mu N)(\bar{K}\overleftrightarrow{\partial}_\mu K) + (\bar{N}\vec{\tau}\gamma^\mu N)(\bar{K}\vec{\tau}\overleftrightarrow{\partial}_\mu K)\right], \tag{19}$$

$$\mathcal{L}_{\bar{\nu}=2} = \frac{\Sigma_{KN}}{f^2}(\bar{N}N)(\bar{K}K) + \frac{C}{f^2}(\bar{N}\vec{\tau}N)(\bar{K}\vec{\tau}K) + \frac{\tilde{\mathcal{D}}}{f^2}(\bar{N}N)(\partial_\mu\bar{K}\partial^\mu K) + \frac{\tilde{\mathcal{D}}'}{f^2}(\bar{N}\vec{\tau}N)(\partial_\mu\bar{K}\vec{\tau}\partial^\mu K), \tag{20}$$

where

$$N^T = (p, n), \qquad \bar{K}\overleftrightarrow{\partial}_\mu K \equiv \bar{K}\overleftarrow{\partial}_\mu K + \bar{K}\overrightarrow{\partial}_\mu K,$$

and f is assumed to be equal to the pion decay constant $f_\pi \simeq 93$ MeV. $\vec{\tau}$ denotes Pauli's isospin matrix, Σ_{KN} is the so-called sigma term, and C, $\tilde{\mathcal{D}}$, $\tilde{\mathcal{D}}'$ are some constants calculated by taking into account experimental information. In the limit $k \to 0$ eqs. (19, 20) recover those obtained in ref. [20]. For the regular part of the polarization operator of K^- one finds from eqs. (19, 20)

$$\Pi^{-,reg}(\ell=0) = -\left[\frac{\tilde{\mathcal{D}}-\tilde{\mathcal{D}}'}{f^2} + 2\nu\frac{\tilde{\mathcal{D}}'}{f^2}\right]\rho(\omega^2 - \vec{k}\vec{k}') - \frac{1+\nu}{2f^2}\rho\omega - \frac{\Sigma_{KN}}{f^2}\rho. \qquad (21)$$

In the limit $k \to 0$ eq. (21) approaches to the s-wave part of the polarization operator $\Pi^{-,s}$. The numerical values of the quantities $\tilde{\mathcal{D}}, \tilde{\mathcal{D}}', \Sigma_{KN}$ can vary in rather wide limits. The range of their variation has been evaluated in ref. [20] by some theoretical arguments and the experimental data on the K^+N scattering length. Unfortunately, scarcity of experimental information does not allow to define $\Pi^{-,s}$ completely and the probability of K condensation depends sensitively on the numerical value of the sigma term. The latter one has to be determined by taking into account the modifications of kaonic excitations in nuclear matter [27]. In ref. [20] the following choice of parameters is proposed as the most probable one

$$\tilde{\mathcal{D}} \simeq 0.33/m_K - \Sigma_{KN}/m_K^2, \qquad \tilde{\mathcal{D}}' \simeq 0.16/m_K - C/m_K^2, \qquad (22)$$

$$\Sigma_{KN} \simeq 2m_\pi, \quad C \simeq -0.06m_K, \quad m_B \simeq 2m_K.$$

Inserting the numerical values of these parameters into eq. (21), we obtain

$$\begin{aligned}\Pi^{-,reg}(\text{no loops}) \simeq & - 2.3m_\pi^2\frac{\rho}{\rho_o} - 0.57m_\pi(1+\nu)\omega\frac{\rho}{\rho_o} \qquad (23)\\ & - (0.15\nu - 0.15)(\omega^2 - \vec{k}\vec{k}')\frac{\rho}{\rho_o}.\end{aligned}$$

In ref. [23] a somewhat different fit of the s-wave part to the $K^{\pm}N$ scattering data is performed with taking into account the $\Lambda(1405)$ intermediate state and some loop corrections.

Our main aim here is to demonstrate the new qualitative particularities, but not to fit precisely the experimental data. Therefore in the description of regular part of polarization operator we shall restrict ourselves to the choice of the parameters (22).

3.2 The contributions of $(\Lambda|p^{-1})$, $(\Sigma^o|p^{-1})$, $(\Sigma^-|n^{-1})$ loops

The p-wave part of the K^- polarization operator from eqs. (15, 16), calculated via relativistic vertices (1, 2), consists of a term which does not vanish in the limit $k \to 0$, $\omega \to m_K$. The absolute value of this on-shell term can be even larger than the contribution of the s-wave part given by eq. (21), and the former one has a quite different off-shell frequency dependence. In accordance with the parameter choice (22), fitted to the KN scattering data at $\omega \to m_K$, $k \to 0$, we have to perform the subtraction of the on-shell p-wave part at $k \to 0$. It results in an extra contribution which must be added to the regular part of the K^- polarization operator $\Pi^{-,reg}$, i.e.,

$$\delta\Pi^{-,s} = \left\{\frac{A_0 m_K^2 - A_1 m_K \tilde{\omega}_\Lambda(t=-m_K^2)}{m_K - \tilde{\omega}_\Lambda(t=-m_K^2)} - A_2 m_K\right\}\nu\frac{\rho}{\rho_o} \qquad (24)$$

$$+\left\{\frac{B_0 m_K^2 - B_1 m_K \tilde{\omega}_\Sigma(t=-m_K^2)}{m_K - \tilde{\omega}_\Sigma(t=-m_K^2)} - B_2 m_K\right\}(2-\nu)\frac{\rho}{\rho_o}$$
$$\simeq (1.4\nu + 0.2) m_\pi^2 \frac{\rho}{\rho_o}.$$

Therefore at $\omega = m_K, k = 0$ we have in agreement with eqs. (21, 22) $\Pi^{-,P}(m_K, k = 0) + \delta\Pi^{-,s}(m_K, k = 0) = 0$.

3.3 The contribution of the pionic intermediate states

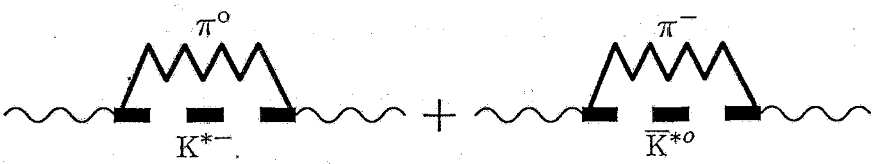
The $K\pi$ interaction is described by the Lagrangian

$$\mathcal{L}_{KK^*\pi} = g_{KK^*\pi} K^{*\mu} \vec{\tau} \{K \partial_\mu \vec{\pi} - \vec{\pi} \partial_\mu K\}, \tag{25}$$

where the value of the coupling constant is [8]

$$\frac{g_{KK^*\pi}}{\sqrt{4\pi}} \simeq 0.86. \tag{26}$$

Taking into account the vertex (25) leads to the appearance of extra terms $\Pi^-_{\pi K^*}$, which are represented graphically



(27)

in the polarization operator. Here the dashed line "▬ ▬ ▬" corresponds to heavy mesons $(\bar{K}^{*o}, K^{*-})$ in the intermediate states $(m_{K^*} \simeq 6.4 m_\pi)$ and the solid saw-seeing line "/\/\/" is related to the exact Greens function $\mathcal{D}_\pi(\omega, k)$ of the pion in nuclear matter. They are in some analogy to the contribution of the pion fluctuations to the pionic polarization operator [28, 29]. Such contributions become important at non-zero temperatures near the critical point of pion condensation. Their influence on the properties of ρ, ω mesons in dense nuclear matter has been recently investigated in ref. [30].

The pionic degree of freedom softens at densities $\rho \geq \rho_{c1} \simeq (0.5-0.7)\rho_o$ according to many theoretical studies [3, 4]. Indeed, the quantity $\tilde{\omega}_\pi^2 \equiv -\mathcal{D}_\pi^{-1}(\omega = 0, k)$, which plays the role of an effective pionic gap, gets a minimum at momentum $k = k_{0\pi} \neq 0$, $\rho \geq \rho_{c1}$. At the density $\rho > \rho_{c\pi} \simeq (2-4)\rho_o$ the value $\tilde{\omega}_\pi^2(k_{0\pi}(\rho))$ becomes so small (it would vanish without taking into account the pion fluctuations) that pion condensation can set in by a first-order phase transition [31]. A typical density dependence of the effective pionic gap $\tilde{\omega}_\pi^2(k_{0\pi}(\rho))$ is shown in the fig. B.1. in ref. [4].

The graphs displayed in (27) can be easily calculated in the limit of rather strong softening of the pion mode. Here we should only include the contribution, which consists of the value $\tilde{\omega}_\pi(k_{0\pi})$ characterizing the closeness to the critical point of the π condensation. The regular part of the graphs is contained in the parameters of Π^s fitted to the empirical data.

Assuming $\rho > \rho_{c1}$, $|\omega| \ll m_\pi$, $k \ll m_N$, $k' \simeq k_0 \simeq p_F$, $\tilde{\omega}^2(k_0) \ll m_\pi^2$ we have

$$- \mathcal{D}_\pi^{-1}(\omega', k') \simeq \tilde{\omega}_\pi^2(k_{0\pi}) + \frac{\gamma_\pi}{4k_{0\pi}^2}(k'^2 - k_{0\pi}^2)^2 - i\beta_\pi(k')\omega', \tag{28}$$

where the quantities $\gamma_\pi \sim 1$, $i\beta_\pi(k') = -\frac{\partial \Pi_\pi}{\partial \omega}|_{\omega=0}$ are defined as in ref. [3, 28] and retarded Greens functions are used.

At small frequencies ω and momenta k the Greens function of K^* mesons can be presented in the simple form

$$G_{K^*} = \frac{1}{(\omega - \omega')^2 - m_{K^*}^2 - (\vec{k} - \vec{k}')^2} \approx -\frac{1}{m_{K^*}^2}, \tag{29}$$

since the integration over intermediate states in eq. (27) is characterized by $\omega \sim \tilde{\omega}_\pi(k_{0\pi}) \ll m_K^*, k \sim |\vec{k} - \vec{k}'| \sim k_{0\pi} \ll m_K^*$. Taking this fact into account we can see that the expression for $\Pi_{\pi K^*}^-$ differs from the fluctuation term of the pionic polarization operator [3, 28] only in the coefficients. Making use of the eqs. (28, 29), from eqs. (25, 27) we obtain

$$\begin{aligned} \Pi_{\pi K^*}^-(\omega, k) &\approx \frac{g_{KK^*\pi}^2(\omega^2 - \vec{k}\vec{k}' - k_{0\pi}^2)}{m_{K^*}^2} \\ &\times \int \frac{d^3k'' i d\omega''}{(2\pi)^4}[2\mathcal{D}_{\pi^-}(\rho, \omega'', k'') + \mathcal{D}_{\pi^0}(\rho, \omega'', k'') \\ &-2\mathcal{D}_{\pi^-}(\rho_{c1}, \omega'', k'') - \mathcal{D}_{\pi^0}(\rho_{c1}, \omega'', k'')]\theta(\rho - \rho_{c1}) \\ &= \frac{g_{KK^*\pi}^2(\omega^2 - \vec{k}\vec{k}' - k_{0\pi}^2)}{m_{K^*}^2}(2A_{\pi^-} + A_{\pi^0}), \end{aligned} \tag{30}$$

where $\theta(\rho - \rho_{c1}) = 1$ at $\rho > \rho_{c1}$ and 0 at $\rho < \rho_{c1}$, and

$$A_\pi \equiv \int \frac{d^3k'' i d\omega''}{(2\pi)^4} \left\{\mathcal{D}_\pi(\rho, \omega'', k'') - \mathcal{D}_\pi(\rho_{c1}, \omega'', k'')\right\} \theta(\rho - \rho_{c1}).$$

Then, using the results of ref. [29], we have

$$A_\pi \simeq -C_2(\tilde{\omega}_\pi - m_\pi)\theta(\rho - \rho_{c1}), \tag{31}$$

with $C_2 \simeq 0.1 m_\pi$. Inserting eq. (31) into (30) and assuming that $\mathcal{D}_{\pi^0} \simeq \mathcal{D}_{\pi^-}$ we get the estimate

$$\Pi_{\pi K^*}^-(\omega, k) = g(\omega^2 - \vec{k}\vec{k}' - k_{0\pi}^2), \tag{32}$$

with

$$g = 3\frac{g_{KK^*\pi}^2}{m_{K^*}}A_\pi = -3\frac{g_{KK^*\pi}^2}{m_{K^*}^2}C_2(\tilde{\omega}_\pi - m_\pi)\theta(\rho - \rho_{c1}).$$

Thus the contribution of the graphs (27) with pionic intermediate states corresponds effectively to some attraction at small frequencies. Quantitative estimates at $k \to 0$ $\omega \to 0$ show that this contribution is rather small and can be neglected. This result is analogous to that for pion fluctuations at zero temperature [31]. Namely, quantum fluctuations are small, however, thermal fluctuations can be important at least near the critical point of the π-condensate phase transition [28, 29]. We will return to this question in section 6.

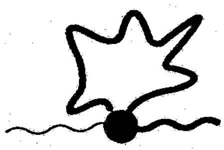
3.4 The proper kaonic fluctuations

Kaon-kaon interaction is described by the following Lagrangian

$$\mathcal{L} = -\frac{\Lambda}{2}(\bar{K}K)^2, \tag{33}$$

where $\Lambda \simeq \frac{m_K^2}{6f^2}$ is the vacuum coupling constant of KK interaction. This vertex produces a fluctuation term in the polarization operator, which can be displayed as

$\Pi^-_{\rm fluct} =$



. (34)

where the fat point is related to the kaon-kaon interaction corrected by the in-medium effects.

The contribution (34) can be calculated in the same way as it has been done for pions [28, 29]. It is determined by eq. (31) with the only difference that at $\rho > \rho_{c1}$ the value $\mathcal{D}_\pi$ should be replaced on the corresponding kaon value at the density when the minimum at $k \neq 0$ arises in the effective kaon gap. The main effect of the graph (34) is that the kaon condensation with $k \neq 0$ sets in as a first-order phase transition. At vanishing temperature the contribution of quantum fluctuations is rather small and can be neglected, whereas the thermal fluctuations are important at least near the critical point of the K condensation phase transition.

3.5 The residual off-shell interaction and the consistency condition of Adler

It is easy to see from eqs. (13, 14, 21, 24, 32, 34), that the polarization operator of K^- mesons at $k \to 0$ differs from that given by eq. (23) in ref. [20] due to the extra graphs taken into account. Only on-shell at $k \to 0$ the expression (23) is approximately valid as before. At this, since the fit of coefficients in eq. (23) to the KN scattering data is not unique, a more detailed analysis is required, which can, in principle, change substantially the corresponding set of parameters. Off-shell (e.g. at $\omega \approx \varepsilon_F \ll m_K$) the regular term of the polarization operator with an additional loop contribution quite differs from that given by eq. (23). Combining the eqs. (13, 14, 21, 24, 32) at $\omega, k \to 0$, we find

$$\tilde{\Pi}^-(\omega = 0, k = 0, \rho) \simeq -\frac{\Sigma_{KN}}{f^2}\rho + \delta\Pi^{-,s} - gk_{0\pi}^2 \approx (1.4\nu - 2.1)m_\pi^2 \frac{\rho}{\rho_o}, \tag{35}$$

where for numerical evaluation we supposed $\Sigma_{KN} \approx 2m_\pi$. So we get an additional repulsion. Therefore, it is allowed to call into question the statements of refs. [16, 17, 18, 19, 20, 21, 22] about the appearance of the s-wave K^- condensation (see also ref. [24]).

One could ask, whether additional new graphs does not change our conclusions? With a lack of experimental data and in the absence of any small parameters in the theoretical analysis such a doubt would be meaningful. However even without an additional experimental information we can get rather important conclusion concerning the off-shell behavior of the kaon polarization operator. For this goal we employ the consistency condition of Adler derived for the πN scattering amplitude in the framework of the SU(2)×SU(2) model [32]. In refs. [4, 33] this condition has been used to fix the off-shell pion propagator. An analogous consistency condition exists for the KN scattering in the framework of the SU(3)×SU(3) symmetry. For the regular part of the KN scattering amplitudes we can write

$$A_{reg}(\omega = \omega' = 0, k = 0, k'^2 = -m_K^2) = 0. \tag{36}$$

Applying eq. (36), one can see that the polarization operator used in the previous paper [20] (e.g., see eq. (23)) does not wittingly satisfy Adler's condition. The polarization operator obtained in our analysis is also still incompatible with the condition (36). This means that we must add some residual term to the kaon polarization operator in order to satisfy the relation (36). Unfortunately, it cannot be done uniquely. One of possible ways to satisfy Adler's relation is to add to the amplitude

$$A_{reg}(q^2 = \omega^2 - k^2, q'^2 = \omega'^2 - k'^2, qq' = \omega^2 - \vec{k}\vec{k}')$$

the vanishing on-shell term $\lambda(\omega^2 - m_K^2 - \frac{k^2+k'^2}{2})$ and to demand the fulfillment of the condition (36). Such a procedure has been applied to the πN scattering [4, 33] [2], where the authors use the expansion of the regular amplitude nearby mass shell ($\omega^2 \simeq m_K^2 + k^2$). As result we obtain the following regular part of the KN scattering amplitude

$$A_{reg} = b + b'\frac{k^2 + k'^2}{2} + c^{reg}\vec{k}\vec{k}' - \lambda(\omega^2 - m_K^2 - \frac{k^2 + k'^2}{2}). \tag{37}$$

[2] A somewhat other variant of going off-shell is recently suggested in refs. [24, 34]. It leads to the same general conclusion that the s-wave amplitude of the KN scattering changes significantly off-shell.

The parameters b, b', c^{reg} have to be extracted from the experiments on $K^{\pm}N$ and $K^{\pm}$ nucleus scattering and on K^--atom data. Unfortunately, as mentioned above, the experimental data are still incomplete, so they do not allow us even to constrain the constant b uniquely and provide no information on the values b' and c^{reg}. That is why the forthcoming experiments, which make accessible the precise values of b', c^{reg}, would be of paramount importance. In the subsequent theoretical analysis, it would be also plausible to use somewhat more complete information on π mesons and consider the π and K mesons together in the framework of the SU(3)×SU(3) symmetry. In the absence of detailed investigation of this problem at present we use the simplest choice of interactions (15, 16, 21, 24, 32) which are compatible with the KN scattering data. In our analysis we obtain the explicit frequency dependence of the regular part of the KN scattering amplitude on the neutron and proton, see eqs. (15, 16, 21). Accordingly to this the coefficients b, b' and c^{reg} in eq. (37) can be considered as functions of frequency and isotopic composition. Thus to take into account the Adler consistency condition we have to add to the resulting polarization operator the extra term

$$\delta\Pi^{-,reg} = \lambda\rho\left(\omega^2 - m_K^2 - \frac{k'^2 + k^2}{2}\right), \tag{38}$$

where

$$\lambda = b' - \frac{2b}{m_K^2}.$$

In our notation we have $b \simeq -\tilde{\Pi}^-(\omega = 0, k = 0, \rho)/\rho$ and the parameter b' remains undefined. Below we assume that the main contribution to the p-wave part of the polarization operator is given by the loops (6), i.e., we put $b' = 0$. One should note here that even without extra graphs (6), which we have derived in the above analysis, the expression (21) together with the condition (36) lead to a substantial change of the polarization operator at $\omega \ll m_K$, $k \to 0$. As result (see below) a s-wave K^- condensation seems not to appear, at least up to rather high nuclear densities $\rho \sim 10\rho_o$.

Finally we note that (*i*) the expression for the s-wave term of the polarization operator of K^- meson is model dependent, and (*ii*) with the extra graph considered above and by taking into account Adler's condition (36) the polarization operator of K^- mesons at $\omega = k = 0$ changes substantially in such a way that the conclusions about the K^- condensation with $k = 0$ remain unsettled.

4 Spectrum of kaons in nuclear matter

The spectrum of K^- quasi-particles is defined by the dispersion equation

$$\omega^2 - m_K^2 - k^2 - \Pi^-(\omega, k, \rho) = 0, \tag{39}$$

where we have finally

$$\Pi^{-}(\omega,k,\rho) = - d\rho - \alpha(1+\nu)\rho\omega - (\beta+\beta'\nu)\rho(\omega^2-k^2) + \delta\Pi^{-s}(\rho,\nu) \quad (40)$$
$$+ \left\{ \frac{A_0(k^2-\omega^2)+A_1\omega\tilde{\omega}_\Lambda(t)}{\omega-\tilde{\omega}_\Lambda(t)} + A_2\omega \right\} \nu\frac{\rho}{\rho_o}$$
$$+ \left\{ \frac{B_o(k^2-\omega^2)+B_1\omega\tilde{\omega}_\Sigma(t)}{\omega-\tilde{\omega}_\Sigma(t)} + B_2\omega \right\} (2-\nu)\frac{\rho}{\rho_o}$$
$$+ g(\omega^2-k^2-k_0^2) + \lambda_{+}\rho\left(\omega^2-m_K^2-k^2\right),$$

with

$$d=\frac{\Sigma_{KN}}{f^2},\quad \alpha=\frac{1}{2f^2},\quad \beta=\frac{\tilde{\mathcal{D}}-\tilde{\mathcal{D}}'}{f^2},\quad \beta'=2\frac{\tilde{\mathcal{D}}'}{f^2},\quad g=3\frac{g^2_{KK^*\pi}}{m^2_{K^*}}A_\pi$$

as the total polarization operator of the K^- meson obtained by combining the expressions (15, 16, 21, 24, 32, 38).

First of all let us analyze a part of the p-wave contribution. Therefore, just for illustration, we consider the simplest form of the s-wave term and also use the simple form of the p-wave part given by the expression (13) (by neglecting the small $K\Sigma$ and $K\pi$ interaction). Thus equation (39) obtains the form

$$\alpha(\omega^2-k^2)-\tilde{m}_K^2-\frac{A_0k^2\nu\rho/\rho_o}{\omega-\tilde{\omega}_\Lambda(t\simeq k^2)}=0, \quad (41)$$

where

$$\tilde{\omega}_\Lambda(t\simeq k^2)\simeq\tilde{\omega}_\Lambda+\frac{k^2}{2m_N},\quad A_0\simeq 1.1m_\pi,\quad \tilde{\omega}_\Lambda\simeq 1.4m_\pi.$$

The value $\tilde{m}_K$ is determined as follows. Without taking the s-wave terms of the polarization operator into account we have $\alpha=1$ and $\tilde{m}_K^2=m_K^2$. The contribution of the s-wave interaction to the polarization operator can be simply involved by adding to m_K^2 an extra term $\tilde{\Pi}^-(\omega=0,k=0,\rho)$ given by eq. (35). However, as discussed above, it seems to be natural to take into account Adler's condition. Then we obtain the dispersion equation (41) with the values

$$\alpha\simeq 1-\lambda\rho, \quad (42)$$
$$\lambda=\frac{2}{\rho}\frac{\tilde{\Pi}^-(\omega=0,k=0,\rho)}{m_K^2},$$

and

$$\tilde{m}_K^2=m_K^2(1-\frac{\lambda}{2}\rho),$$

instead of $\alpha=1$ and $\tilde{m}_K=m_K$.

As can be seen from (41), the spectrum of the charged kaons has three branches of excitations, namely

$$\begin{aligned}\omega_1(k) &\approx m_K^* + \zeta_- \frac{k^2}{2m_K^*}, \\ \omega_2(k) &\approx \tilde{\omega}_\Lambda + \frac{k^2}{2m_N} - \frac{\tilde{A}_0 k^2 \nu\rho/\rho_o}{m_K^{*2} - \tilde{\omega}_\Lambda^2}, \\ \omega_3(k) &\approx -m_K^* - \zeta_+ \frac{k^2}{2m_K^*},\end{aligned} \tag{43}$$

where

$$\zeta_\pm = 1 \mp \frac{\tilde{A}_0 \nu\rho/\rho_o}{m_K^* \pm \tilde{\omega}_\Lambda \mp \dfrac{m_K^{*2}}{2m_N}}, \qquad m_K^{*2} = m_K^2 \frac{1 - \dfrac{\lambda}{2}\rho}{1-\lambda\rho}, \qquad \tilde{A}_0 = \frac{A_0}{1-\lambda\rho}.$$

The selection of the branches is performed in accordance with the sign of the factor $\tilde{\Gamma} = 2\omega - \dfrac{\partial \Pi^-}{\partial \omega}$ on the branch ref. [1]. When the sign of $\tilde{\Gamma}$ is positive, we deal with branch of K^-, whereas it is negative, we get the branch of K^+ after the replacement $\omega \to -\omega$, $k \to -k$. As result, branches 1 and 2 correspond to the K^- meson quasi-particles. The upper branch $\omega_1(k)$ transforms into the vacuum one at $\rho \to 0$. The second branch $\omega_2(k)$ consists mainly of the mixed states of Λ particles and p-holes having the quantum numbers of the K^- meson. It is analogous to the Δ isobar branch of excitations in the spectrum of π^- mesons [3]. The main difference is that in our case branch 2 lies below branch 1. Branch 3 becomes the branch of K^+ mesons after changing $\omega \to -\omega$, $k \to -k$.

The existence of the extra low-lying branch of the excitations is important for the description of (*i*) the K^- nucleus scattering, (*ii*) the kaon yields in the heavy ion collisions, since this branch is occupied with the maximal probability ($\propto \exp(-\omega_2(k)/T)$), and (*iii*) the possibility of the kaon condensation in dense nuclear matter.

The value m_K^* is an effective kaon mass in medium. At low densities we get the density dependence similar to that in refs. [16, 21, 22], i.e.,

$$m_K^{*2}(\rho \to 0) \approx m_K^2 (1 - \frac{\rho}{\tilde{\rho}_c}), \qquad \rho_c = -\frac{2}{\lambda}.$$

On the other hand the effective mass tends to the constant value at high densities

$$m_K^*(\rho \gg \rho_c) \simeq m_K/\sqrt{2}.$$

From the estimate (35) for proton matter ($\nu = 1$) we have $\lambda \approx -0.11/\rho_o$ (for $\Sigma_{KN} = 2m_\pi$). For the somewhat different parameter choice used also in [20] ($\Sigma_{KN} = 1.4m_\pi$) we have $\lambda \approx 0$. Notice that for this particular choice the result becomes independent on the procedure of going off-shell, because Adler's condition (36) is fulfilled identically.

In fig. 1 we present the exact solution of eq. (41) for two nuclear matter densities. As it is seen from eq. (43) at the condition

$$\tilde{A}_0 \nu \rho/\rho_o \geq \frac{m_K^{*2} - \tilde{\omega}_\Lambda^2}{2m_N} \tag{44}$$

a minimum with $k = k_o \neq 0$ appears in the low-lying branch of K^- mesons (see fig. 1a). The minimum value $\omega_2(k_o)$ decreases with the growth of the proton density and equals zero (see fig. 1b) at some value $\nu\rho = \rho_c^-$ ($\rho_c^- \simeq 5.8\rho_o$ for $\Sigma_{KN} = 2m_\pi$ and $\rho_c^- \simeq 4\rho_o$ for $\Sigma_{KN} = 1.4m_\pi$). The dependence $\omega_2(k)$ in the vicinity of the minimum and at ρ nearby ρ_c^- can be obtained from eq. (41) as

$$\omega_2(k) \approx \tilde{\omega}_\Lambda + \frac{k^2}{2m_N} - \frac{\tilde{A}_0 k^2 \nu\rho/\rho_o}{k^2 + m_K^{*2}}.$$

At $k \sim k_0$ this expression can be rewritten as

$$\omega_2(k) \approx \omega_c + \gamma\left(1 - \frac{k^2}{k_0^2}\right)^2, \tag{45}$$

where

$$\omega_c = \tilde{\omega}_\Lambda - \left\{\sqrt{\tilde{A}_0 \nu\rho/\rho_o} - \sqrt{\frac{m_K^{*2}}{2m_N}}\right\}^2, \tag{46}$$

$$k_0^2 = \sqrt{2m_N \tilde{A}_0 \nu(\rho/\rho_o) m_K^{*2}} - m_K^{*2} \geq 0, \tag{47}$$

$$\gamma = \frac{k_0^2}{4m_N m_K^* \sqrt{2m_N \tilde{A}_0 \nu(\rho/\rho_o) m_K^{*2}}}. \tag{48}$$

From eq. (46) we obtain that $\omega_c \leq 0$, if the condition

$$\tilde{A}_0 \nu\rho/\rho_o \geq \left\{\sqrt{\tilde{\omega}_\Lambda} + \sqrt{\frac{m_K^{*2}}{2m_N}}\right\}^2, \tag{49}$$

is fulfilled.

5 Nuclear matter with kaons at high density

Now let us turn to the question of the possibility of a K^- condensation in dense nucleonic matter. We write the total energy density of nuclear matter with K^- condensate in the form

$$\varepsilon_{\text{tot}} = \varepsilon_B + \varepsilon_{K^-} - \rho_p V - \frac{(\nabla V)^2}{8\pi e^2}, \tag{50}$$

where ε_B is the energy density of the nucleon subsystem,

$$\varepsilon_{K^-} = \omega \frac{\partial \mathcal{L}_{K^-}(\omega - V, k)}{\partial \omega} - \mathcal{L}_{K^-}(\omega - V, k) \tag{51}$$

is the energy density of the K^- condensate, and

$$\mathcal{L}_{K^-} = \left\{(\omega - V)^2 - k^2 - m_K^2 - \Pi^-(\omega - V, k)\right\} |\varphi|^2 - \frac{1}{2}\Lambda |\varphi|^4 \tag{52}$$

is the Lagrangian density of the classical K^- field. In order to take into account the electromagnetic interaction we make the replacement $\omega \to \omega - V$ in the Lagrangian of the K^- mesons, where V is electric potential. The constant Λ describes the kaon self-interaction (see eq. (33)). Further we will consider rather extended systems in which the electric neutrality condition

$$\rho_p \; = \; \rho_{K^-} \; = \; \frac{\partial \mathcal{L}_{K^-}}{\partial \omega} \tag{53}$$

is supposed to be satisfied. Therefore the last term in eq. (50) gives a negligible contribution. Besides the charge neutrality condition the expressions (50-52) have to be completed by the equation for the kaon field

$$\left\{(\omega - V)^2 - k^2 - m_K^2 - \Pi^-(\omega - V, k)\right\} \varphi - \Lambda |\varphi|^2 \varphi = 0. \tag{54}$$

For the sake of simplicity let us assume that Λ is equal to zero. (Finite value of Λ will slightly shift the critical density.) Then from eqs. (50-54) we obtain

$$\varepsilon = \varepsilon_B + (\omega - V)\rho_p. \tag{55}$$

Finally, we must minimize the value ε as function of the kaon momentum k. Noticing that the value $(\omega - V)$ as a function of k is determined by eq. (54) with $\Lambda = 0$ and therefore its minimum with respect to k equals the value ω_c, given by eq. (46), we obtain

$$\varepsilon = \varepsilon_B + \left\{ \tilde{\omega}_\Lambda - \left[\sqrt{\tilde{A}_0 \nu \frac{\rho}{\rho_o}} - \sqrt{\frac{m_K^{*2}(\rho)}{2m_N}} \right]^2 \right\} \nu \frac{\rho}{\rho_o}. \tag{56}$$

The energy density of the baryon subsystem ε_B can be calculated via one of the nuclear matter models.

In order to verify an opportunity of a phase transition with the change of the isotopic composition, caused by the appearance of the K^- condensate, we should extract the isotopically dependent part of the ε_B value. Thereby we have

$$\varepsilon_B = \varepsilon_B^0(\rho) + \varepsilon_B^I(\rho, \nu), \tag{57}$$

where $\varepsilon_B^0(\rho)$ is isotopically independent part of the baryon energy density, while the value of $\varepsilon_B^I(\rho,\nu)$, in the simplest parametrization, is given by

$$\varepsilon_B^I \simeq 4\varepsilon_{\rm sym}\rho(\nu-\frac{1}{2})^2. \tag{58}$$

The value $\varepsilon_{\rm sym} \approx 40$ MeV can be taken, e.g., from the nuclear matter calculation by Fridman and Pandharipande [35].

In fig. 2 we show the values of the isotopic part of the total energy density in the presence of the K^- condensate

$$\Delta\varepsilon(\rho,\nu) = \varepsilon_{\rm tot} - \varepsilon_B^o(\rho) \tag{59}$$
$$= 4\varepsilon_{\rm sym}\rho(\nu-\frac{1}{2})^2 + \left\{\tilde{\omega}_\Lambda - \left[\sqrt{\tilde{A}_0\nu\rho/\rho_o} - \sqrt{\frac{m_K^{*2}(\rho)}{2m_N}}\right]^2\right\}\nu\frac{\rho}{\rho_o}$$

as function of the density ρ for pure neutron matter ($\nu=0$), isospin symmetric nuclear matter ($\nu=\frac{1}{2}$) and proton matter ($\nu=1$). As seen from fig. 2 at $\rho \geq \rho_c^-$ ($\omega_c(\rho_c^-,\nu=1)=0$) the isotopic phase transition

$$\nu = 0 \;\Rightarrow\; \nu = 1 \tag{60}$$

becomes energetically favorable (i.e., the curve with $\nu=1$ is below the curve with $\nu=0$). This result is the direct consequence of the ν-proportionality of the p-wave part of the polarization operator.

Thus neutron star matter may undergo a first-order phase transition at $\rho > \rho_c^-$ into proton matter with the charge compensated by the negative inhomogeneous ($k = k_0 \neq 0$) kaon condensate.

6 Polarization operator of kaons at finite temperature

Having in mind the application of our results to neutron star matter and to heavy-ion collisions we will consider only the temperatures $T \leq m_\pi$. Then we can suppose that the s-wave part of the polarization operator does not depend too strongly on the temperature, because the corresponding diagrams consist of heavy particles in the intermediate states. The temperature dependence of the Lindhard function is connected only with recoil effects (i.e. $\vec{k}\vec{p}/m_N \neq 0$ in the nucleon Greens function, cf. ref. [28]). With the simplified representation of the Lindhard function we have

$$\Phi^i_{\Lambda(\Sigma)}(\omega,k,T) \simeq \frac{\tilde{\omega}_{\Lambda(\Sigma)}(t)}{\tilde{\omega}_{\Lambda(\Sigma)}-\omega}\left\{1+\frac{\pi^2}{12}\frac{(T/\varepsilon_F^i)^2}{\left(\dfrac{\omega-\tilde{\omega}_{\Lambda(\Sigma)}(t)}{kv_F^i}-\dfrac{k}{2p_F^i}\right)^2-1}\right\}. \tag{61}$$

The contribution of the term $\propto A_2$ in eqs. (15,16) is proportional to the density, i.e. it is independent of the temperature. As it is seen from eq. (61) the growing temperature amplifies the p-wave attraction in a wide region of frequencies and momenta for $\omega < \tilde{\omega}_{\Lambda(\Sigma)}$ and enlarges the repulsion at $\omega > \tilde{\omega}_{\Lambda(\Sigma)}$. Since the K^- condensate arises at $\omega_c = 0 < \tilde{\omega}_{\Lambda(\Sigma)}$, the value of the critical density $\rho_c^-(T)$ decreases with growing temperature.

To avoid possible misunderstanding we have to note that eq. (61) obtained in the framework of the perturbation theory is applicable only if the temperature dependent term in the brackets of eq. (61) is smaller than unity. In assuming $\omega \approx 0$ $k \approx p_F$ one can see that the parameter of the expansion is indeed $4T^2/m_\pi^2 \ll 1$. Of course, in the opposite limit $4T^2/m_\pi^2 \gg 1$ one has $\Phi^i_{\Lambda(\Sigma)} \to 0$.

The graphs, which are related to the pion fluctuations, display the most sensitive dependence on temperature (see eq. (27)). Eq. (31) at $T \neq 0$ becomes (see ref. [28])

$$\begin{aligned} A_\pi &= -T \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} [\mathcal{D}_\pi(\rho, \omega_n = 2\pi i n T, k) \\ & \quad - \mathcal{D}_\pi(\rho_{c1}, \omega_n = 2\pi i n T, k)] \theta(\rho - \rho_{c1}). \end{aligned} \tag{62}$$

It is simply calculated in the limiting case $T \gg \tilde{\omega}_\pi(k_{0\pi})$, when only one term $n = 0$ remains in the sum in eq. (62). As result we get

$$A_\pi \simeq C \frac{T}{\tilde{\omega}_\pi} \theta(\rho - \rho_{c1}), \qquad C = \frac{k_{0\pi}^2}{2\pi\sqrt{\gamma_\pi}}. \tag{63}$$

Bearing this in mind from eqs. (40, 63) we can conclude that a growing temperature promotes the attraction in the polarization operator. Near the critical point of the π condensation, when $\tilde{\omega}_\pi$ becomes rather small, the contribution of $|\Pi^-_{\pi K^*}|$ to the K^- polarization operator increases sharply. Thus when $T \neq 0$, the pion softening may promote the kaon condensation. If one supposes a second-order phase transition into a π condensate state, then $\tilde{\omega}_\pi(\rho \to \rho_{c\pi}) \to 0$ and hence $\Pi^-_{\pi K^*}(\omega = 0, k) \to -\infty$, that means the possibility of the growth of the classical kaon field. Therefore the K^- and pion condensation can arise at the more or less same critical density. What kind of condensate arises in reality at $\rho > \rho_{c\pi}(T)$, $\rho > \rho_c^-(T)$ depends on the values of the K^- and π condensate energies and on the interaction between the condensate fields.

As we have mentioned, another temperature dependent term is connected with the kaon fluctuations. Its contribution is determined by the closeness of the effective kaon gap of the kaon propagator at $\omega \approx 0$, $k \simeq k_0 \neq 0$ to zero.

7 The $\bar{K}^o$ meson condensation

As mentioned in sect. 2, the polarization operator of $\bar{K}^o$ mesons can be easily obtained from the polarization operator of K^- mesons by performing the replacement $\nu \to 1-\nu$.

This means that the spectrums of K^- mesons presented in fig. 1 for the case of proton matter ($\nu = 1$) can also be interpreted as the spectrum of $\bar{K}^o$ for neutron matter ($\nu = 0$) at the same baryon density. The vanishing of $\omega_2(k_0, \rho)$ at the density $\rho = \rho_c^-$ is now the reason for the instability of the $\bar{K}^o$ field with respect to the reaction

$$n \quad \rightarrow \quad n + \bar{K}^o. \tag{64}$$

In the mean field approximation the phase transition to the $\bar{K}^o$-condensed state is expected to be of a second-order. With taking into account the kaon fluctuations it becomes a first-order, but at $T = 0$ the fluctuation contribution is rather small and can be neglected.

The value of the condensate field can be found from eq. (54) at $\omega - V \rightarrow \omega = 0$, $k = k_0$, $\Pi^-(\omega - V, k) \rightarrow \Pi^0(0, k_0)$. Making use of the expansion of the $\bar{K}^o$ condensate field $\varphi^2_{\bar{K}^o}$ in the vicinity of the critical point in analogy to π^0 case ref. [1] we have

$$\varphi^2_{\bar{K}^o} = -\frac{1}{\Lambda}\left(\frac{\partial \Pi^0}{\partial \rho}\right)_{\rho_c^-}(\rho_n - \rho_c^-). \tag{65}$$

The energy density gain is given by

$$\Delta\varepsilon_{\bar{K}^o} \simeq -\beta\frac{(\rho_n - \rho_c^-)^2}{2}, \tag{66}$$

$$\beta = -\frac{1}{2\Lambda}\left(\frac{\partial \Pi^0}{\partial \rho}\right)_{\rho_c^-}.$$

As for the case of π^0 condensation [1], the system with $\bar{K}^o$ condensate becomes stable only due to the repulsion $\Lambda \neq 0$, whereas in the case of K^- mesons the system is stable even at $\Lambda = 0$ because of the electromagnetic coupling. Therefore if the values $\Lambda_{\bar{K}^o}$ and Λ_{K^-} are essentially the same, then the $\bar{K}^o$ condensate would be energetically favorable. Besides, the transition to $\bar{K}^o$ state takes probably the shorter time, since in the mean field approximation it arises by a second-order transition, whereas the K^- transition is of a first-order. Nevertheless in order to draw more definite conclusion one has to calculate carefully the values $\Lambda_{\bar{K}^o}$ and Λ_{K^-} and to investigate thoroughly the dynamics of both phase transitions. This is, however, rather complicated problem. Thus, despite that the energy of a $\bar{K}^o$ neutron core may be indeed a bit smaller than the one of the K^- proton core, both phase transitions may manifest themselves in the neutron star dynamics [36] in dependence on the initial configuration and on the time evolution scenario.

8 Concluding remarks

In summary, we calculate the $K^\pm, K^o, \bar{K}^o$ polarization operators in a dense nuclear matter medium of various isotopic compositions by taking into account the s-and p-wave interactions and also the residual KN interaction obtained by some procedure of going off-shell. The low-energy theorem for the meson-nucleon scattering has been applied in the framework of a chiral Lagrangian. Some new extra graphs are explicitly calculated such as hyperon - particle-hole - loops and graphs with pionic intermediate states. Thereby an extra s-wave repulsion and p-wave attraction are obtained. We argue that a temperature increase enforces the KN attraction in a rather wide range of temperatures. Although, because of a lack of more detailed experimental data, the polarization operator which we obtain remains model dependent, some essential conclusions can nevertheless be drawn.

A new low-lying branch of excitations is found in the spectrum of K^- and $\bar{K}^o$ mesons. We have reconsidered the possibility of the kaon condensation with respect to the new particularities mentioned above. We show that kaon condensation is likely to arise not due to the s-wave interaction but mainly due to the p-wave interaction. We show that at $\rho > \rho_c^- \simeq (4-6)\rho_o$ (at $T=0$) neutron matter may undergo a first-order phase transition to proton matter with electric charge compensated by the K^- field. At nearby the same density ρ_c^- the neutron matter may also undergo a phase transition into a $\bar{K}^o$ condensate state. This transition is of first-order at $T \neq 0$ and can be considered approximately as a second-order transition at vanishing temperature. Some arguments are given that the $\bar{K}^o$ condensate is energetically more favorable than the K^- condensation. At $T \neq 0$ a pion softening may promote the kaon condensation.

The particularities of the KN interaction that we find can manifest themselves in (i) experiments on kaon-nucleus scattering, (ii) kaonic atoms, (iii) heavy-ion collisions, and (iv) in different neutron star phenomena.

Indeed the strong p-wave attraction obtained in the above analysis as well as the presence of the extra low-lying branch in the K^- spectrum can result in an enhancement of the K^- nucleus cross-section at the corresponding frequencies and momenta. It also modifies the description of the K^- atoms. In forthcoming investigations one should focus on the possibility to extract from experiments the coefficient ζ_- proportional to k^2 for the upper branch of the K^- spectrum, i.e., $\omega_1(k) \simeq m_K^* + \zeta_- \frac{k^2}{2m_K^*}$ (see eq. (43) as well as on manifestations of the low-lying K^- branch $\omega_2(k)$.

Also from our p-wave propagator one can extract the imaginary part of the kaon frequency and compare it with that given by experiments on the $K^{\pm}$-nucleus scattering.

In heavy-ion collisions the presence of the low-lying branch as well as the in-medium modification of the upper branch can manifest themselves in an enhancement of the K^-, $\bar{K}^o$ yields.

New possibilities of the kaon condensate formation may affect the neutron star dynamics so that a newly born neutron star undergoes either a first-order phase transition into the K^- condensate state with a proton-dominated core or a first-order ($T \neq 0$) phase transition into a $\bar{K}^o$ condensate state with a neutron-enriched core. The final state depends on the dynamical features of the corresponding phase transitions and on the time evolution scenario. Such transitions may result in an extra heating-up of the star core and in a neutrino burst. In addition, rather old neutron stars might undergo a K condensation phase transition, when its interior density would be enlarged up to ρ_c^* given by the corresponding Maxwell construction for the phase equilibrium curve. This value ρ_c^* is somewhat smaller then ρ_c^-. The interior density may increase because of the accretion of the matter in binary systems or by some other processes.

A more detailed theory of kaon-nucleon interaction as well as its manifestations in different physical phenomena are planned to be considered in subsequent work.

Acknowledgement: One of us (D.N.V.) thanks the hospitality and support rended to him at Rostock University and GSI Darmstadt.

References

[1] A.B. Migdal, Rev. Mod. Phys. **50** (1978) 107.

[2] G.E. Brown, W. Weise, Phys. Rep. **27** (1976) 1;
T.O. Ericson, W. Weise, Pions and nuclei, Clarendon Press. Oxford (1988).

[3] A.B. Migdal, E.E. Saperstein, M.A. Troitsky, D.N. Voskresensky, Phys. Rep. **192** (1990) 179.

[4] D.N. Voskresensky, Nucl. Phys. **A555** (1993) 293.

[5] R. Stock, Phys. Rep. **135** (1986) 259;
P. Koch, B. Müller, J. Rafelski, Phys. Rep. **142** (1986) 167;
P. Koch, Prog. Part. Nucl. Phys. **26** (1990) 253;
W. Cassing, U. Mosel, Prog. Part. Nucl. Phys. **25** (1990) 235;
G.E. Brown, C.M. Ko, K. Kubodera, Z. Phys. A **341** (1992) 301.

[6] E. Friedman, A. Gal, C.J. Batty, Phys. Lett. **308B** (1993) 6;
C.J. Batty, Nucl. Phys. **A372** (1981) 418.

[7] P.B. Siegel, W. Weise, Phys. Rev. **C38** (1988) 221.

[8] A. Müller-Groeling, K. Holinde, J. Speth, Nucl. Phys. **A513** (1990) 557;
R. Büttgen, K. Holinde, A. Müller-Groeling, J. Speth, P. Wyborny, Nucl. Phys. **A506** (1990) 586.

[9] K.Holzenkamp, K. Holinde, J. Speth Nucl. Phys. **A500** (1989) 487.

[10] T. Barnes, E.S. Swanson, MIT-CTP-2169/ORNL-CCIP-92-15(December 1992).

[11] P.B. Siegel, W.B. Kaufman, W.R. Gibbs, Phys. Rev. **C31** (1985) 2184;
R. Sawafta, R. Weiss, J. Aclander et al., Phys. Lett. **307B** (1993) 293.

[12] S. Schnetzer et al., Phys. Rev. Lett. **49** (1982) 989;
S. Shor et al., Phys. Rev. Lett. **48** (1982) 1597;
J.W. Harris et al., Phys. Rev. Lett. **47** (1981) 229;
J. Julien et al., Phys. Lett. **264B** (1991) 269;
E. Grosse, Prog. Part. Nucl. Phys. **30** (1993) 89.

[13] E. Barasch, Phys. Lett. **161B** (1985) 265;
J. Carrol, Nucl. Phys. **A488** (1988) 203c.

[14] G.E. Brown, M. Rho, Phys. Rev. Lett. **66** (1991) 2720.

[15] C. Adami, G.E. Brown, Phys. Rep. **234** (1993) 1.

[16] D. Kaplan, A. Nelson, Phys. Lett. **175B** (1986) 57; Nucl. Phys. **A479** (1988) 273c;
A. Nelson, D. Kaplan, Phys. Lett. **192B** (1987) 143.

[17] H.D. Politzer, M.B. Wise, Phys. Lett. **273B** (1991) 156.

[18] G.E. Brown, K. Kubodera, M. Rho, V. Thorsson, Phys. Lett. **291B** (1992) 355.

[19] G.E. Brown, H. Bethe, preprint Caltech-92.

[20] G.E. Brown, C.H. Lee, M. Rho, V. Thorsson, Nucl. Phys. **A567** (1994) 937.

[21] T. Muto, Prog. Theor. Phys. **89** (1993) 415.

[22] T. Muto, R. Tamagaki, T. Tatsumi, Prog. Theor. Phys. Suppl. **112** (1993) 159.

[23] C.H. Lee, H. Jung, D.-P. Min, M. Rho, SNUTP-93-81(1993);
C.H. Lee, G.E. Brown, M. Rho, SNUTP-94-28 (1994).

[24] H.Yabu, S. Nakamura, F. Myhrer, K. Kubodera, Phys. Lett. **315B** (1993) 17.

[25] H. Forkel, M. Rho, N.N. Scoccola, Nucl. Phys. **A509** (1990) 673.

[26] S. Weinberg, Phys. Lett. **251B** (1990) 288; Nucl. Phys. **B363** (1991) 2.

[27] M.J. Musolf, M. Burkhardt, Z. Phys. C **61** (1994) 433.

[28] D.N. Voskresensky, I.N. Mishustin, Pis'ma v ZETF **34** (1981) 317; Yad. Fiz. **35** (1982) 1139.

[29] A.M. Dygaev, Pis'ma v ZhETF **35** (1982) 341; Yad. Fiz **38** (1983) 1131;
Zh. Eksp. Theor. Fiz. **83** (1982) 1005.

[30] G.G. Bunatian, B. Kämpfer, preprint FZR-92-08, (1992).

[31] A.M. Dygaev, Pis'ma v ZhETF **22** (1975) 181.

[32] S.L. Adler, Phys. Rev. **137** (1965) B1022; Phys. Rev. **139** (1965) B1638; J.J. Sakurai, Currents and Mesons, University of Chicago Press 1967.

[33] M.A. Troitsky, N.I. Chekunaev, Yad. Fiz. **33** (1981) 1300.

[34] H. Yabu, S. Nakamura, K. Kubodera, Phys. Lett. **317B** (1993) 269.

[35] B. Friedman, V.R. Pandharipande, Nucl. Phys. **A361** (1981) 502.

[36] B. Kämpfer, Phys. Lett. **153B** (1985) 121.

Figure captions

Fig. 1: The spectrum of branches of K$^-$ excitations $\omega_i(k)$, $i = 1, 2, 3$ (a: $\nu\rho = 3\rho_o$, b: $\nu\rho = 5.8\rho_o$) for $\Sigma_{KN} = 2m_\pi$.

Fig. 2: Isotopic part of the energy density per nucleon with K^- condensate plotted as function of nucleon density for different isotopic compositions of the nuclear matter ($\Sigma_{KN} = 2m_\pi$).

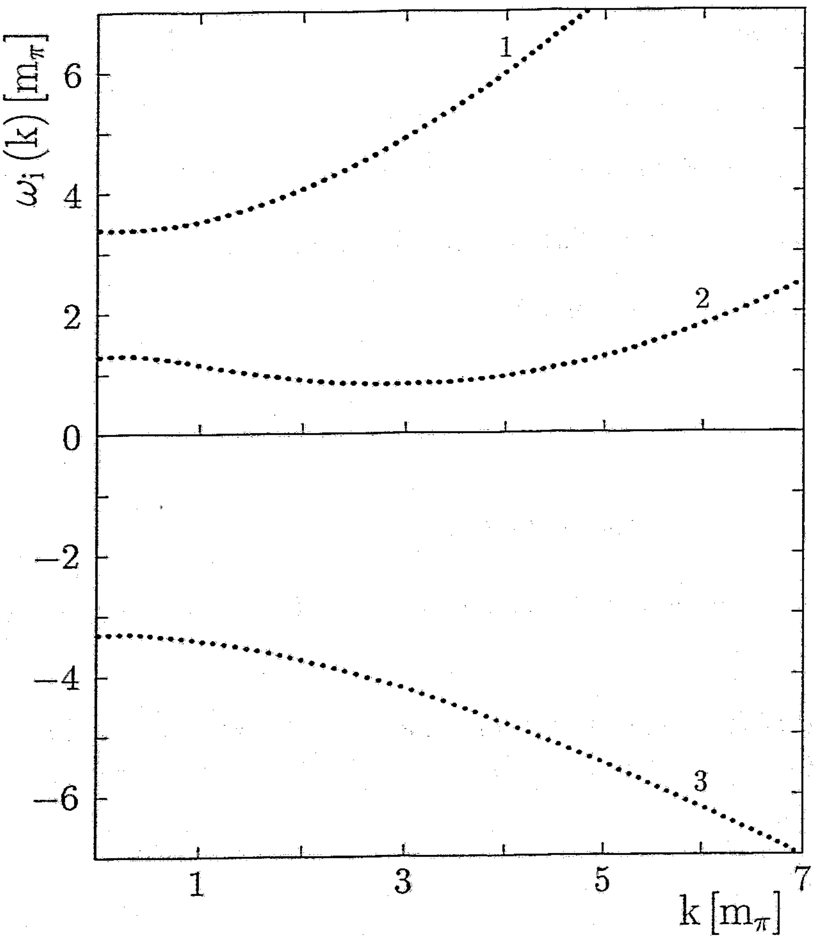


Fig. 1a

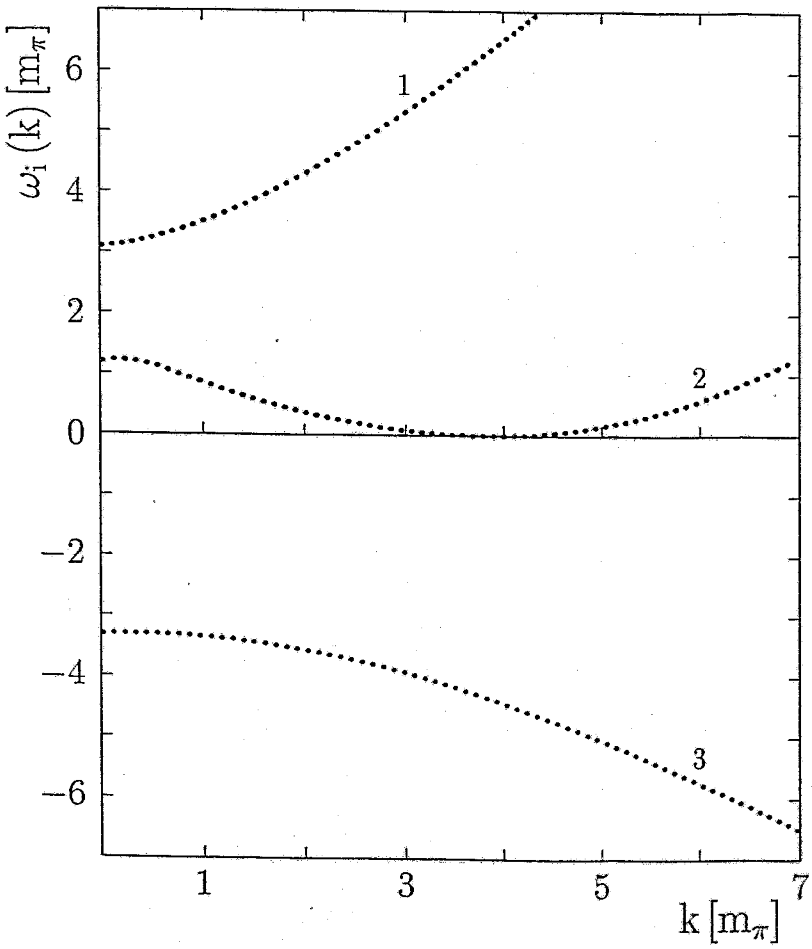


Fig. 1b

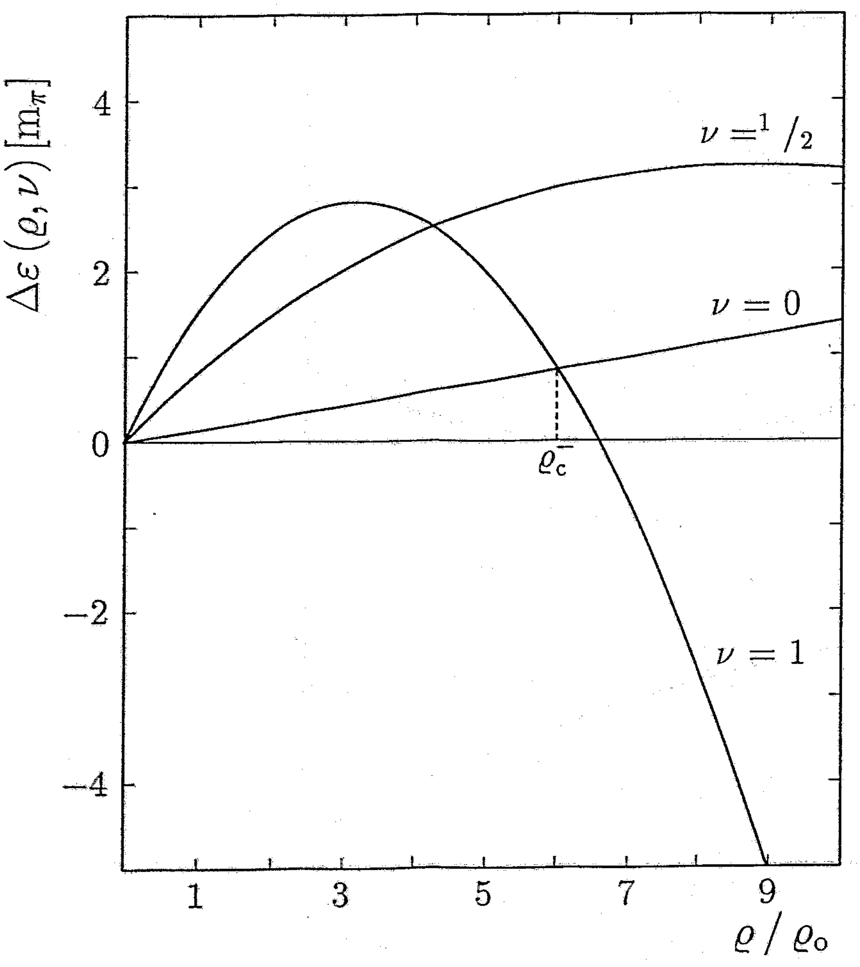


Fig. 2