FORSCHUNGSZENTRUM
ROSSENDORF e.V.

FZR

FZR-42
May 1994
Preprint

I. Rotter

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I. Rotter

Technische Universität Dresden, Institut für Theoretische Physik, D-01062 Dresden, Germany
and
Forschungszentrum Rossendorf, Institut für Kern- und Hadronenphysik, D-01314 Dresden, Germany

Abstract

A model for the description of an open quantum mechanical many-particle system is formulated. It starts from the shell model and treats the continuous states by a coupled channels method. The mixing of the discrete shell model states via the continuum of decay channels results in the genuine decaying states of the system. These states are eigenstates of a non-Hermitean Hamilton operator the eigenvalues of which give both the energies *and* the widths of the states. All correlations between two particles which are caused by the two-particle residual interaction, are taken into account including those via the continuum.

In the formalism describing the open quantum mechanical system, the coupling between the system and its environment appears nonlinearly. If the resonance states start to overlap, a redistribution of the spectroscopic values ("trapping effect") takes place. As a result, the complexity of the system is reduced at high level density, structures in space and time are formed. This redistribution describes, on the one hand, the transition from the well-known nuclear properties at low level density to those at high level density *and* fits, on the other hand, into the concept of selforganization.

1 Introduction

As is well known, the states of many quantum systems have a finite lifetime. An example are the states of nuclei most of which decay. In older papers, either the discrete states of the system or the decay products are described explicitely while the respective other part of the configuration space is taken into account in a much worse approximation. An example is the description of nuclear reactions in the framework of the shell model together with the R-matrix theory. In these calculations, the nuclear structure of the discrete states is relevant. There is no feedback at all of the decay products onto the shell model states. Another example is the unified theory of nuclear reactions in which the open decay channels are the relevant part while the nuclear structure is described by means of statistical assumptions.

Recently, the properties of quantum systems with decaying states are investigated with a renewed interest in the framework of different models by taking into account the feedback of the decay products onto the discrete states [1 – 14]. In most cases studied, the number N of resonance states is much larger than the number K of open decay channels. One of the results obtained is the trapping effect which appears if the average width $\bar{\Gamma}$ of the resonances is of the same order of magnitude as the average distance $\bar{D}$ of two neighboured resonance states. In this case, a redistribution takes place inside the nucleus which results in the formation of K short-lived resonance states together with $N - K$ long-lived ones. The time-scales of both types of states are well separated from each other.

The trapping effect has been observed theoretically in different models. It is shown [1 – 14] to occur in realistic many-body quantum systems such as nuclei and also, e.g., in problems of quantum chemistry. In nuclei at low level density, the nuclear spectroscopic properties are relevant, while at higher level density, the properties of nuclei are described well by the unified theory of nuclear reactions where the open decay channels are relevant. It is exactly this transition which is described by the redistribution taking place inside the nucleus at the critical degree $\bar{\Gamma}/\bar{D} \approx 1$ of resonance overlapping [5].

The trapping effect is the result of the interference between N overlapping resonances. The physical interpretation of the trapping effect as the formation of "structures in space and time" by selforganization is traced in some papers [5, 13, 14]. It seems to be supported by many results obtained. In any case, the number of relevant degrees of freedom of the system is reduced as a consequence of the trapping effect. The system becomes less complex.

Selforganization in *classical systems* appears due to nonlinearities in the rate equations by which the coupling of the system to its environment is characterized. As a consequence, the superposition principle does not hold and the decay occurs according to a power law [15].

In *quantum mechanical systems* embedded into the environment of decay channels, selforganization is the result of non-trivial interferences [5, 8, 13]. The relation to non-linearities in the basic equations is not investigated thoroughly, up to now.

It is the aim of this paper to write down the formalism for the description of an open quantum mechanical system. In section 2, the method is desribed for solving the original Schrödinger equation with an ansatz containing both discrete and continuous states. This method ("continuum shell model") is formulated several years ago [16] in order to describe experimental nuclear reaction data at low energy. In section 3, the properties of the wavefunctions describing the nuclear states embedded in the continuum of decay channels are investigated analytically. The extraction of rate-like equations and their relation to the original Schrödinger equation is performed in section 4. In section 5, the equation for the motion of one particle of the many-body system is written down. Here, *all correlations between two particles* which are caused by the two-particle residual interaction are taken into account. In section 6, the number of degrees of freedom in an open quantum system is investigated. It is reduced by selforganization at a certain critical value of the degree of overlapping of the resonances. The transition from low to high level density in nuclei is traced in section 7. It is described not only by the model formulated in section 2 but fits also into the concept of selforganization. Some conclusions on the question of the origin of selforganization in the quantum mechanical equations are drawn in section 8.

2 Solution of the time-independent many-particle Schrödinger equation

In the continuum shell model of the nucleus, the Schrödinger equation

$$(H - E)\Psi_E^c = 0 \tag{1}$$

is solved with the ansatz

$$\Psi_E^c = \sum_{R=1}^{N} b_R \Phi_R^{(0)} + \sum_{c'=1}^{\Lambda} \int_{\epsilon_{c'}}^{\infty} dE' a_{E'}^{c'} \chi_{E'}^{c'} \tag{2}$$

containing both discrete and continuous states. The N discrete states are described by the $\Phi_R^{(0)}$ which are antisymmetrized products of A single-particle wavefunctions in bound states (Slater determinants) while the Λ wavefunctions χ_E^c are the channel wavefunctions with $A-1$ particles in bound states of the target (or residual) nucleus and 1 particle in a scattering state. The sum runs over open as well as closed channels. The Hamilton operator is $H = H_0 + V$ where V is the two-particle residual interaction and H_0 describes the central potential in which the particles move. The target nucleus is described by the same Hamilton operator, i.e. the channel wavefunctions χ_E^c contain the residual interaction V between the $A-1$ particles bound in the target nucleus. The residual interaction between the different decay channels is not included into the channel wavefunctions χ_E^c.

A special problem of nuclear physics is the existence of single-particle resonances. They have a large amplitude inside the nucleus like bound states but behave asymptotically like scattering states. In order to make possible spectroscopic investigations, a "cut-off technique" is used for them by subdividing them into two parts: The part from $R = 0$ up to the cut-off radius R_{cut} is treated together with the wavefunctions of the bound states while the part for $R > R_{cut}$ is included into the set of scattering states [16]. R_{cut} is about a factor 2 larger than the nuclear radius. The basic set of single-particle wavefunctions obtained in such a manner is orthogonalized [16].

Then, the total function space is subdivided, by using the projector operator technique, into the two orthogonal subspaces P and Q under the condition

$$P + Q = 1 \,. \tag{3}$$

The subspace Q contains the many-body states of A nucleons formed by products of the wavefunctions of the single-particle bound states and of the single-particle resonances up to the cutoff radius R_{cut}. Therefore, the structural part in the continuum shell model is the same as in the standard shell-model approach,

$$Q = \sum_{R=1}^{N} |\Phi_R^{SM}\rangle \langle \Phi_R^{SM}| \,. \tag{4}$$

The eigenstates Φ_R^{SM} of the Q-projected Hamiltonian H_{QQ},

$$(H_{QQ} - E_R^{SM})\ \Phi_R^{SM} = 0\ , \tag{5}$$

are called [16] "quasibound states embedded in the continuum" (QBSEC). These QBSEC's differ from the "bound states embedded in the continuum" (BSEC) introduced by Mahaux and Weidenmüller [17] by the contribution of the single-particle resonances in the interior of the nucleus ("cut-off procedure").

The subspace P contains the many-body states with $A-1$ nucleons in bound orbits and one nucleon in a scattering state as well as the part of the single-particle resonance wavefunctions beyond the cut-off radius R_{cut}. The coupled-channel wavefunctions ξ_E^c follow from

$$(H_{PP} - E^{(+)})\ \xi_E^{c(+)} = 0\ , \tag{6}$$

where H_{PP} is the P projected part of the Hamiltonian H. It is

$$P = \sum_{c=1}^{\Lambda} \int_{\epsilon_c}^{\infty} dE\ |\xi_E^{c(+)}\rangle\ \langle \xi_E^{c(+)}|\ . \tag{7}$$

In the following, the method for the solution of eq. (1) is represented.

By multiplying (1) with P and Q, respectively, from the left, it follows

$$(H_{PP} - E)\ P\Psi_E^c = -H_{PQ}\ Q\Psi_E^c \tag{8}$$

and

$$(H_{QQ} - E)\ Q\Psi_E^c = -H_{QP}\ P\Psi_E^c\ . \tag{9}$$

Here, $H_{PQ} \equiv PHQ$ and so on. By using the definition (6) of the coupled-channel wavefunctions $\xi_E^{c(+)}$ and the Green function $G_P^{(+)}$ in the P subspace,

$$G_P^{(+)} = P(E^{(+)} - H_{PP})^{-1} P\ , \tag{10}$$

one obtains from (8)

$$P\Psi_E^{c(+)} = \xi_E^{c(+)} + G_P^{(+)} H_{PQ}\ Q\Psi_E^{c(+)}\ . \tag{11}$$

From (9) and (11),

$$(H_{QQ} - E)\ Q\Psi_E^{c(+)} = -H_{QP}\xi_E^{c(+)} - H_{QP}\ G_P^{(+)}\ H_{PQ}\ Q\Psi_E^{c(+)} \tag{12}$$

and

$$(H_{QQ} + H_{QP}\, G_P^{(+)}\, H_{PQ} - E)\, Q\Psi_E^{c(+)} = -H_{QP}\xi_E^{c(+)} \,. \tag{13}$$

Defining the operator

$$\begin{aligned} H_{QQ}^{eff} &= H_{QQ} + H_{QP}\, G_P^{(+)}\, H_{PQ} \\ &= H_{QQ} + V_{QP}\, G_P^{(+)}\, V_{PQ} \,, \end{aligned} \tag{14}$$

eq. (13) can be written as follows

$$Q\Psi_E^{c(+)} = (E - H_{QQ}^{eff})^{-1}\, H_{QP}\xi_E^{c(+)} \,. \tag{15}$$

The ansatz

$$Q\Psi_E^{c(+)} = \sum_R B_R^{(+)}\Phi_R^{SM} \tag{16}$$

with the shell model wavefunctions Φ_R^{SM} obtained from (5) leads to

$$\sum_R B_R^{(+)}\Phi_R^{SM} = (E - H_{QQ}^{eff})^{-1}\, H_{QP}\, \xi_E^{c(+)} \tag{17}$$

and

$$B_R^{(+)} = \sum_{R'} \langle \Phi_R^{SM} | (E - H_{QQ}^{eff})^{-1} | \Phi_{R'}^{SM} \rangle \, \langle \Phi_{R'}^{SM} | V | \xi_E^{c(+)} \rangle \,. \tag{18}$$

From (11),

$$\begin{aligned} \Psi_E^{c(+)} &= P\Psi_E^{c(+)} + Q\Psi_E^{c(+)} \\ &= \xi_E^{c(+)} + (G_P^{(+)} H_{PQ} + 1)\, Q\Psi_E^{c(+)} \end{aligned} \tag{19}$$

and with (16),

$$\Psi_E^{c(+)} = \xi_E^{c(+)} + \sum_R (1 + G_P^{(+)} H_{PQ}) B_R^{(+)} \Phi_R^{SM} \,. \tag{20}$$

We define now the wavefunctions ω_R as solutions of the coupled channel equations with source term

$$(H_{PP} - E^{(+)})\, \omega_R^{(+)} = -H_{PQ}\Phi_R^{SM} \,. \tag{21}$$

Then,

$$\begin{aligned} \Psi_E^{c(+)} &= \xi_E^{c(+)} + \sum_{R,R'} \Omega_R^{(+)} \times \\ &\quad \langle \Phi_R^{SM} | (E - H_{QQ}^{eff})^{-1} | \Phi_{R'}^{SM} \rangle \, \langle \Phi_{R'}^{SM} | V | \xi_E^{c(+)} \rangle \end{aligned} \tag{22}$$

where

$$\begin{aligned}\Omega_R^{(+)} &= \Phi_R^{SM} + \omega_R^{(+)} \\ &= (1 + G_P^{(+)}\, H_{PQ})\, \Phi_R^{SM} \,. \end{aligned} \tag{23}$$

The diagonalisation of the operator H_{QQ}^{eff},

$$H_{QQ}^{eff}\, \tilde{\Phi}_R^{(+)} = (\tilde{E}_R - \frac{i}{2}\tilde{\Gamma}_R)\, \tilde{\Phi}_R^{(+)} \,, \tag{24}$$

with the orthogonal matrix O leads to

$$\Psi_E^{c(+)} = \xi_E^{c(+)} + \sum_R \tilde{\Omega}_R^{(+)}\, (E - \tilde{E}_R + \frac{i}{2}\tilde{\Gamma}_R)^{-1}\, \langle \tilde{\Phi}_R^{(-)} | V | \xi_E^{c(+)} \rangle \,, \tag{25}$$

where the identity

$$\langle \Phi_{R'}^{SM} | O^{-1} (O H_{QQ}^{eff} O^{-1})^{-1} O | \Phi_R^{SM} \rangle = \langle \Phi_{R'}^{SM} | (H_{QQ}^{eff})^{-1} | \Phi_R^{SM} \rangle \tag{26}$$

is used. Further,

$$\begin{aligned}\tilde{\Omega}_R^{(+)} &= \tilde{\Phi}_R^{(+)} + \tilde{\omega}_R^{(+)} \\ &= (1 + G_P^{(+)}\, H_{PQ})\, \tilde{\Phi}_R^{(+)} \end{aligned} \tag{27}$$

is the wavefunction of the resonance state R while $\tilde{E}_R$ and $\tilde{\Gamma}_R$ determine the energy and width of it. $\tilde{\Omega}_R^{(+)}$ as well as $\tilde{E}_R$ and $\tilde{\Gamma}_R$ are energy dependent functions. The identification of $E_R = \tilde{E}_R(E = E_R)$, $\Gamma_R = \tilde{\Gamma}_R(E = E_R)$ and $\Omega_R = \tilde{\Omega}_R^{(+)}(E = E_R)$ with the spectroscopic values energy, width and wavefunction of the resonance state R is meaningful since the cut-off technique for the single-particle resonances is used in the continuum shell model [16].

The wavefunctions $\tilde{\Phi}_R^{(+)}$ can be represented as

$$\tilde{\Phi}_R^{(+)} = \sum_{R,R'} a_{RR'}^{(+)}\, \Phi_{R'}^{SM} \tag{28}$$

with complex energy dependent coefficients $a_{RR'}$ and $\sum_{R'} a_{RR'}^2 = 1$.

It is

$$\begin{aligned}\xi_E^{c(+)} &= \chi_E^{c(+)} + G_P^{(+)}\, V \chi_E^{c(+)} \\ &= (1 + G_P^{(+)} V)\, \chi_E^{c(+)} \,. \end{aligned} \tag{29}$$

Together with (23) and (27), it follows [18]

$$\langle \Omega_R^{(-)} | V | \chi_E^{c(+)} \rangle = \langle \Phi_R^{SM} | V | \xi_E^{c(+)} \rangle \tag{30}$$

and

$$\langle\tilde{\Omega}_R^{(-)}|V|\chi_E^{c(+)}\rangle \;=\; \langle\tilde{\Phi}_R^{(-)}|V|\xi_E^{c(+)}\rangle\;. \tag{31}$$

The matrix elements

$$\tilde{\gamma}_{Rc}^{1/2} = (2\pi)^{1/2}\,\langle\tilde{\Omega}_R^{(-)}|V|\chi_E^{c(+)}\rangle \tag{32}$$

are energy dependent complex functions. The values $\gamma_{Rc}^{1/2} = \tilde{\gamma}_{Rc}^{1/2}(E = E_R)$ are the amplitudes of the partial widths. They are complex if the resonances overlap.

The S-matrix reads [5]

$$\begin{aligned} S_{cc'} &= e^{2i\delta_c} - 2i\pi\langle\chi_E^{c'(-)}|V|\Psi_E^{c(+)}\rangle \\ &= e^{2i\delta_c} - 2i\pi\langle\chi_E^{c'(-)}|V|\xi_E^{c(+)}\rangle - i\sum_R \frac{\tilde{\gamma}_{Rc'}^{1/2*}\;\tilde{\gamma}_{Rc}^{1/2}}{E - (\tilde{E}_R - \frac{i}{2}\tilde{\Gamma}_R)} \end{aligned} \tag{33}$$

where eqs. (25), (31) and (32) are used. In (33), the $\tilde{\gamma}_{Rc}^{1/2}$, $\tilde{\Gamma}_R$ as well as $\tilde{E}_R$ are energy dependent functions.

The representation of Ψ_E^c in eqs. (22) and (25) is the solution of eq. (1). In (22), it is represented in terms of three sets of wavefunctions $\{\Phi_R^{SM}\}$, $\{\xi_E^c\}$ and $\{\omega_R\}$. All three sets are obtained by solving standard equations: The $\{\Phi_R^{SM}\}$ are solutions of the shell model equations (5) while the $\{\xi_E^c\}$ and $\{\omega_R\}$ follow from the solution of coupled channel equations without and with source term, eqs. (6) and (21), respectively. The coupled channel equations are solved in the channel representation. The corresponding equations read

$$\sum_{t'}(t|H_{PP} - E|t')(t'|\xi_E^c\rangle = 0 \tag{34}$$

and

$$\sum_{t'}(t|H_{PP} - E|t')(t'|\omega_R\rangle = -\sum_{c'}\int dE'\;(t|\xi_{E'}^{c'}\rangle\;\langle\xi_{E'}^{c'}|H|\Phi_R^{SM}\rangle\;. \tag{35}$$

Here, $|t)$ is the wavefunction of the target (residual) nucleus, $(t|\xi_E^c\rangle$ is the scattering wavefunction in coupled channel representation ($target{+}nucleon$), and $(t|H_{PP}|t')$ is the corresponding one-particle Hamilton operator in the P subspace. Eqs. (34) and (35) are solved by using the relation $P = 1 - Q$. To that purpose, the channel representation of the shell model wavefunctions Φ_R^{SM} in the Q-subspace is needed,

$$\sum_{t'}(t|H_{QQ} - E|t')(t'|\Phi_R^{SM}\rangle = 0\;. \tag{36}$$

Here and in the following, the (+) and (−) at the wavefunctions are omitted.

Details of the numerical solution of the equations given above can be found in [16]. Here, the approximations underlying the solution method ("continuum shell model") are summarized.

1. The existence of a central potential in which the nucleons move.
   The shape of the potential is *not* calculated in a selfconsistent manner but assumed to be of Woods-Saxon type. It is assumed to be independent of the excitation energy of the nucleus. The parameters of the potential are fitted to experimental data. They are assumed to be the same for the compound nucleus consisting of A nucleons and for the target (residual) nucleus consisting of $A-1$ nucleons.

2. $\Psi = P\Psi + Q\Psi$ is assumed to be the full function space.
   That means, no other decay channels as those described by $P\Psi$ with one nucleon in relative motion to the residual nucleus consisting of $A-1$ nucleons are considered. The states of the residual nucleus are assumed to be stable. Due to the assumption $P+Q=1$, the unitarity condition of the decay process is fulfilled.

3. The residual interaction V between two nucleons is of the same type for bound and unbound nucleons.
   In the numerical calculations of the continuum shell model, V is spin and isospin dependent and $V \propto \delta(r-r')$.

4. Truncation of the configuration space.
   In the numerical calculations, the configuration space is truncated in the same manner as in shell model calculations. In every calculation, the number N of resonances of the many-particle system and the number Λ of open and closed decay channels is fixed. N and Λ can be varied independently of each other.

In the continuum shell model, the equations (5), (6) and (21) are solved numerically without further approximations. Some specific features of the calculations are the following.

1. The energy dependence of the coupling matrix elements $\langle \Omega_R | V | \chi_E^c \rangle = \langle \Phi_R^{SM} | V | \xi_E^c \rangle$ (amplitudes of the partial widths of isolated states) is taken into account.

2. The energy dependence of the eigenvalues $\tilde{\mathcal{E}}_R = \tilde{E}_R - \frac{i}{2}\tilde{\Gamma}_R$ of H_{QQ}^{eff} (energies and widths of the resonances) is taken into acount. Therefore, deviations from the Breit-Wigner line shape appear.

3. The calculations are performed with non-vanishing energy dependent channel-channel coupling $\langle \chi^c_E | V | \chi^{c'}_{E'} \rangle$ where V is the residual interaction.

4. Due to the cut-off technique used for the single-particle resonances, the number of shell model states is exactly equal to the number of resonance states. The eigenvalues $\tilde{\mathcal{E}}_R = \tilde{E}_R - \frac{i}{2}\tilde{\Gamma}_R$ of H^{eff}_{QQ} at the energy $E = E_R$ of the system can be interpreted as the energies $E_R = \tilde{E}_R(E = E_R)$ and widths $\Gamma_R = \tilde{\Gamma}_R(E = E_R)$ of the resonance states R [16].
The cut-off technique for single-particle resonances leads to QBSEC ("quasibound states embedded in the continuum") instead of the BSEC ("bound states embedded in the continuum") [16]. The cut-off technique for single-particle resonances has no influence on the calculated cross section. The only aim of doing the cut-off technique is to make possible nuclear spectroscopic investigations.

5. The level distribution follows from a shell model calculation. The energy shifts $\Delta E = E^{SM}_R - \tilde{E}_R(E = E_R)$ due to the coupling of the system to the continuum of decay channels are of the order of magnitude of the widths, as a rule.

3 Nonlinearities

Nonlinearities occur in the equations of the continuum shell model at different places.

The internal mixing of two basic states with the wavefunctions $\Phi^{(0)}_R$ and $\Phi^{(0)}_{R'}$ (Slater determinants) is *linear* in the residual interaction V,

$$W^{int}_{R'R} = \langle \Phi^{(0)}_{R'} | V | \Phi^{(0)}_R \rangle \, . \tag{37}$$

It is contained in the Φ^{SM}_R according to (5). The external mixing is *nonlinear* in V,

$$\begin{aligned} W^{ext}_{R'R} &= \langle \Phi^{SM}_{R'} | H_{QP} G^{(+)}_P H_{PQ} | \Phi^{SM}_R \rangle \\ &= \sum_c \int_{\epsilon_c}^{\infty} dE' \langle \Phi^{SM}_{R'} | V | \xi^c_{E'} \rangle (E^{(+)} - E')^{-1} \langle \xi^c_{E'} | V | \Phi^{SM}_R \rangle \, . \end{aligned} \tag{38}$$

The difference between the Φ^{SM}_R and $\tilde{\Phi}_R$ is caused by W^{ext} according to (5), (14) and (24). The value of W^{ext} is a measure for the degree $\bar{\Gamma}/\bar{D}$ of overlapping of the resonances where $\bar{\Gamma}$ is the average value of the $\tilde{\Gamma}_R$ and $\bar{D}$ the

average distance of two neighboured resonances. The larger $\bar{\Gamma}/\bar{D}$, the larger W^{ext}, i.e. the larger the overlap region of the resonances where the nucleons do not clearly belong to exactly one of the resonance states.

The external mixing is most important for the redistribution processes and the trapping effect occuring in nuclei at high level density [5]. A part of it can, of course, be simulated by an additional part to the residual interaction in a typical nuclear structure calculation which is performed in a closed system with only bound states. In such a case, the energy shift according to

$$\Delta E = \sum_c \mathcal{P} \int_{\epsilon_c}^{\infty} dE' \, \langle \Phi_R^{SM} | V | \xi_E^c \rangle \, (E^{(+)} - E')^{-1} \, \langle \xi_E^c | V | \Phi_R^{SM} \rangle \qquad (39)$$

is effectively taken into account by a phenomenological residual interaction ($\mathcal{P}$ is the principal value of the integral [17]).

The behaviour of the nuclear system is investigated in several papers as a function of the strength of the mixing matrix elements W. In [1, 5], the distance D between the resonance states is variied by hand while all the other values are fixed. By this, the average degree of mixing $\bar{\Gamma}/\bar{D}$ of the resonances and therefore W^{ext} is changed. The same is achieved in [6, 7, 11, 13, 14], where the mixing W^{ext} via the residual interaction V between an unbound and a bound particle as well as between two unbound particles is varied while the mixing W^{int} via V between two bound particles is fixed to its realistic value. The external mixing W^{ext}, eq. (38), of two states R and R' contains the residual interaction V in a *nonlinear* manner.

In another paper [10], W^{ext} is fixed while W^{int} is varied. The internal mixing W^{int} depends *linearly* on V, eq. (37). The external mixing W^{ext} is, however, *nonlinearly* in W^{int} because both matrix elements in eq. (38) depend on W^{int} via the wavefunctions Φ_R^{SM} and $\Phi_{R'}^{SM}$. Thus, the behaviour of the system described as a function of W^{ext} or as a function of W^{int} is expected to show the same general behaviour. In both types of calculations, the trapping effect is observed, indeed.

Further, the Schrödinger equation $(H-E)\Psi_E^c$ is solved in Section 2. This is, of course, a *linear* equation for Ψ_E^c. Investigating the properties of an open quantum mechanical system, we are, however, interested not in the Ψ_E^c but in the wavefunctions of the resonance states. These wavefunctions are given by the $\tilde{\Omega}_R(E)$ at the energy $E = E_R$ of the resonances. For isolated resonances, $\tilde{\Omega}_R = \Omega_R$.

The $\tilde{\Omega}_R$ and Ω_R are proportional to the $\tilde{\Phi}_R$ and Φ_R^{SM}, respectively. The Φ_R^{SM} are solutions of a Schrödinger equation in the Q subspace ($Q < 1$), eq. (5), while the $\tilde{\Phi}_R$ are solutions of an equation the Hamilton operator of which is non-Hermitean, eqs. (14) and (24). The eigenfunctions $\tilde{\Phi}_R$ are complex and orthogonal,

$$\langle \tilde{\Phi}_R^* | \tilde{\Phi}_R \rangle = 1 \,. \tag{40}$$

Since the Φ_R^{SM} in (38) are real, it follows $W_{RR'}^{ext} = W_{R'R}^{ext}$ and

$$H_{QQ}^{eff} \tilde{\Phi}_R = \tilde{\mathcal{E}}_R \tilde{\Phi}_R \,, \tag{41}$$

$$H_{QQ}^{eff\,+} \tilde{\Phi}_R^+ = \tilde{\mathcal{E}}_R \tilde{\Phi}_R^+ \,. \tag{42}$$

From (40), one obtains

$$\langle \tilde{\Phi}_R | \tilde{\Phi}_R \rangle \geq 1 \,. \tag{43}$$

Eqs. (24) and (43) lead to

$$\langle \tilde{\Phi}_R | H_{QQ}^{eff} | \tilde{\Phi}_R \rangle = (\tilde{E}_R - \frac{i}{2} \tilde{\Gamma}_R) \langle \tilde{\Phi}_R | \tilde{\Phi}_R \rangle \,. \tag{44}$$

Therefore, the superposition principle does *not* hold for the $\tilde{\Phi}_R$, generally. The eq. (24) can be rewritten in the following form

$$(H - \tilde{\mathcal{E}}_R)\, \tilde{\Phi}_R = X\, \tilde{\Phi}_R \tag{45}$$

where

$$X = \{1 - H_{QP}\, G_P^{(+)}\}\, H_{PQ} \,. \tag{46}$$

Eq. (45) is a Schrödinger equation with source term. The $\tilde{\Phi}_R$ are functions of the Q subspace while the source term has components in both subspaces with *nonlinear* coupling. If $Q = 1$ or $H_{PQ} = H_{QP} = 0$, eq. (45) turns into a usual Schrödinger equation without any source term and with real eigenvalues.

As follows from (22) together with (14), the relation between Ψ_E^c and a special Φ_R^{SM} contains the coupling $\langle \Phi_R^{SM} | V | \xi_E^c \rangle = \langle \Omega_R | V | \chi_E^c \rangle$ between the two subspaces via H_{QQ}^{eff} in a *nonlinear* manner, in general,

$$\langle \Phi_R^{SM} | \Psi_E^c \rangle = \sum_{R'} \langle \Phi_R^{SM} | (E - H_{QQ}^{eff})^{-1} | \Phi_{R'}^{SM} \rangle \langle \Phi_{R'}^{SM} | V | \xi_E^c \rangle \,. \tag{47}$$

If $\langle \Phi_R^{SM}|V|\xi_E^c\rangle \propto \delta_{Rc}$ for K states and $\langle \Phi_R^{SM}|V|\xi_E^c\rangle \approx 0$ for the remaining states, then the relation is (almost) linear in the coupling matrix elements for K states.

The relation between $\tilde{\Phi}_R$ and Ψ_E^c is given by

$$\langle \tilde{\Phi}_R|\Psi_E^c\rangle = (E - \tilde{E}_R + \frac{i}{2}\tilde{\Gamma}_R)^{-1}\langle \tilde{\Phi}_R|V|\xi_E^c\rangle \ \langle \tilde{\Phi}_R|\tilde{\Phi}_R\rangle \tag{48}$$

according to (25). If $\langle \tilde{\Phi}_R|\tilde{\Phi}_R\rangle \approx 1$ then the relation between $\tilde{\Phi}_R$ and Ψ_E^c is (almost) linear. This happens for the K broad states in the strong coupling regime with K open decay channels [13].

According to (44), the widths $\tilde{\Gamma}_R$ follow from the relation

$$Im\{\langle \tilde{\Phi}_R|H_{QQ}^{eff}|\tilde{\Phi}_R\rangle\} = -\frac{1}{2}\,\tilde{\Gamma}_R\,\langle \tilde{\Phi}_R|\tilde{\Phi}_R\rangle\,. \tag{49}$$

It holds

$$Im\{\langle \tilde{\Phi}_R|H_{QQ}^{eff}|\tilde{\Phi}_R\rangle\} = \sum_{i,j} a_{Ri}^* a_{jR} Im\{\langle \Phi_i^{SM}|H_{QQ}^{eff}|\Phi_j^{SM}\rangle\} \tag{50}$$

and

$$Im\{\langle \Phi_i^{SM}|H_{QQ}^{eff}|\Phi_j^{SM}\rangle\} = -\pi \sum_c \langle \Phi_i^{SM}|V|\xi_E^c\rangle\ \langle \xi_E^c|V|\Phi_j^{SM}\rangle\,. \tag{51}$$

Therefore,

$$\begin{aligned} Im\{\langle \tilde{\Phi}_R|H_{QQ}^{eff}|\tilde{\Phi}_R\rangle\} &= -\pi \sum_c \langle \tilde{\Phi}_R|V|\xi_E^c\rangle\ \langle \xi_E^c|V|\tilde{\Phi}_R\rangle \\ &= -\frac{1}{2}\sum_c |\tilde{\gamma}_{Rc}| \end{aligned} \tag{52}$$

by using the definition (32). From (49) and (52), it follows

$$\tilde{\Gamma}_R\,\langle \tilde{\Phi}_R|\tilde{\Phi}_R\rangle = \sum_c |\tilde{\gamma}_{Rc}|\,. \tag{53}$$

That means, only if $\langle \tilde{\Phi}_R|\tilde{\Phi}_R\rangle \approx 1$, the width $\tilde{\Gamma}_R$ of the state R is (nearly) equal to the sum of the partial widths $\tilde{\gamma}_{Rc}$. Such an assumption is basic to the method of calculation of widths in the frame of the shell model + R matrix formalism. It is justified for non-overlapping resonance states as well as for the short-lived states formed at high level density as numerical calculations have shown [13] (see also section 7).

4 Coupling between system and environment

The three basic equations (5), (6) and (21) can be rewritten in the following manner. Eq. (5) reads

$$(H - E_R^{SM})\, \Phi_R^{SM} = H_{PQ} \Phi_R^{SM} \; . \tag{54}$$

The Φ_R^{SM} are functions of the Q-subspace while the source term satisfies the relation $Q \cdot H_{PQ} \Phi_R^{SM} = 0$.

From (21) it follows

$$(H - E)\, \omega_R = -H_{PQ} \Phi_R^{SM} + H_{QP}\; G_P^{(+)}\; H_{PQ}\; \Phi_R^{SM} \; . \tag{55}$$

It holds $Q \cdot \omega_R = 0$ while the source term has components in both subspaces. Eqs. (54) and (55) together lead to

$$(H - E)\, \Omega_R = H_{QP} G_P^{(+)} H_{PQ}\; \Phi_R^{SM} - H_{PQ} \Phi_R^{SM} (1 - \delta_{E, E_R^{SM}}) \; . \tag{56}$$

In an analogous manner, one gets from (6)

$$(H - E)\; \xi_E^c = H_{QP}\; \xi_E^c \; . \tag{57}$$

The ξ_E^c are functions of the P subspace while the source term satisfies $P \cdot H_{QP} \xi_E^c = 0$.

Using the expression (22) for the solution of (1), we obtain

$$\begin{aligned} (H - E)\Psi_E^c &= 0 \\ &= (H - E)\{\xi_E^c + \sum_{R,R'} (\Phi_R^{SM} + \omega_R) \times \\ &\quad \langle \Phi_R^{SM} | (E^{(+)} - H_{QQ}^{eff})^{-1} | \Phi_{R'}^{SM} \rangle \; \langle \Phi_{R'}^{SM} | V | \xi_E^c \rangle \} \; . \end{aligned} \tag{58}$$

By using (56) and (57), it follows

$$\begin{aligned} 0 \;=\; & H_{QP} \xi_E^c + \sum_{R,R'} \{ H_{QP} G_P^{(+)} H_{PQ}\; \Phi_R^{SM} - H_{PQ} \Phi_R^{SM} (1 - \delta_{E, E_R^{SM}}) \} \; \times \\ & \langle \Phi_R^{SM} | (E^{(+)} - H_{QQ}^{eff})^{-1} | \Phi_{R'}^{SM} \rangle \; \langle \Phi_{R'}^{SM} | V | \xi_E^c \rangle \; . \end{aligned} \tag{59}$$

Multiplication (59) by $\Phi_{R''}^{SM}$ from left leads to

$$\begin{aligned} \langle \Phi_{R''}^{SM} | V | \xi_E^c \rangle \;=\; & -\sum_{R,R'} \langle \Phi_{R''}^{SM} | H_{QP}\; G_P^{(+)}\; H_{PQ} | \Phi_R^{SM} \rangle \; \times \\ & \langle \Phi_R^{SM} | (E^{(+)} - H_{QQ} - H_{QP}\; G_P^{(+)}\; H_{PQ})^{-1} | \Phi_{R'}^{SM} \rangle \; \times \\ & \langle \Phi_{R'}^{SM} | V | \xi_E^c \rangle \; . \end{aligned} \tag{60}$$

Due to the relation (30), $\langle \Phi_{R"}^{SM} | V | \xi_E^c \rangle$ describes the direct coupling of a discrete state $R"$ to the channel c. The r.h.s. of eq. (60) describes the coupling between $R"$ and c via other states and other channels. The sum of both parts vanishes in accordance with the fact that the Φ_R^{SM} are discrete non-decaying states: The direct coupling between the discrete state $R"$ and the channel c is compensated by the coupling of this state via other states and other channels to the channel c. The square of eq. (60) plays therefore the role of a rate equation.

Eq. (60) follows from our basic assumption according to which we restrict ourselves to the solution of the equation $(H - E)\Psi_E^c = 0$ without any source term and with $\Psi_E^c = Q\Psi_E^c + P\Psi_E^c$. The environment ($P$ subspace) of the system (Q subspace) is well defined. It holds $P = 1 - Q$.

The definition of $P\Psi_E^c$ as an environment differs from the usual definition of an environment which is supposed to be a thermal bath in which the system is assumed to be embedded. The thermal bath is infinite, but it holds $P = 1 - Q$.

Due to the uncertainty relation in quantum mechanics, the short-time behaviour of the system is determined by the states which are strongly coupled to the continuum. In other words, if the coupling matrix element $\langle \Phi_{R"}^{SM} | V | \xi_E^c \rangle$ at $E = E_{R"}$ is large, then the decay is determined by this matrix element. If it is small, the time scale may be determined by a few fast transitions in the chain $R" \Rightarrow c' \Rightarrow R \Rightarrow c" \Rightarrow R' \Rightarrow c$ in the r.h.s. of (60). Deviations from the exponential decay law holding for an isolated resonance state may occur (see section 7). The corrections to the Breit-Wigner line shape can be written down analytically due to the relations (31) and (33) [18]. Threshold effects are considered numerically in [19] for one (isolated) resonance and two channels.

Further, the different long-lived states are coupled to the continuum by more or less the same strength due to the r.h.s. of (60). As a consequence, the conditions of equilibrium are fulfilled in the long-time scale (formation of a "compound nucleus").

As follows from eq. (31), the coupling matrix element $\langle \tilde{\Phi}_R | V | \xi_E^c \rangle = \langle \tilde{\Omega}_R | V | \chi_E^c \rangle$ is the amplitude of the partial width for the decay of the state R into the channel c. In the $\langle \tilde{\Phi}_R | V | \xi_E^c \rangle$, all couplings of the r.h.s. of (60) are involved. Fast and slow transitions are separated. As a consequence, the partial widths observed in a realistic system may be very different from the

square of the original matrix elements $\langle \Phi_R^{SM}|V|\xi_E^c\rangle$.

The evolution of the nuclear system at high level density proceeds via the short-lived resonance states and is determined by the coupling matrix elements $\langle \tilde{\Omega}_R|V|\chi_E^c\rangle$ of the K short-lived states to the K open decay channels. That means, it proceeds mainly via break-up (pre-equilibrium) processes.

5 The equation for the motion of one particle

As mentioned at the end of Sect. 2, the numerical solution of $(H-E)\Psi_E^c$ requires the knowledge of the one-particle or channel wavefunctions. These wavefunctions may be very different from the single-particle wavefunctions which describe the motion of single particles independently of each other in the central potential. The single-particle wavefunctions form the basis for the construction of the Slater determinants $\Phi_R^{(0)}$ used in the ansatz (2) for solving eq. (1). In contrast to this, the channel wavefunctions contain all the correlations between the particles which are caused by the residual interaction V.

The basic equations of the channel wavefunctions are given by eqs. (34), (35) and (36). The Hamilton operator is $h = (t|H|t')$ with the projections $h_{PP} = (t|H_{PP}|t')$, $h_{QQ} = (t|H_{QQ}|t')$ and so on. It follows (see eqs. (54) to (57))

$$\sum_{t'}(t|H-E_R^{SM}|t')(t'|\Phi_R^{SM}\rangle = \sum_{t'}(t|H_{PQ}|t')\,(t'|\Phi_R^{SM}\rangle \tag{61}$$

$$\sum_{t'}(t|H-E|t')(t'|\omega_R\rangle = \sum_{t'}\{(t|H_{QP}\,G_P^{(+)}\,H_{PQ}|t') - (t|H_{PQ}|t')\}\,(t'|\Phi_R^{SM}\rangle \tag{62}$$

$$\sum_{t'}(t|H-E|t')(t'|\{\Phi_R^{SM}+\omega_R\}\rangle = \sum_{t'}\{(t|H_{QP}\,G_P^{(+)}\,H_{PQ}|t') - (t|H_{PQ}|t')\times(1-\delta_{E,E_R^{SM}})\}\,(t'|\Phi_R^{SM}\rangle \tag{63}$$

$$\sum_{t'}(t|H-E|t')(t'|\xi_E^c\rangle \;=\; \sum_{t'}(t|H_{QP}|t')\,(t'|\xi_E^c\rangle\,. \tag{64}$$

Eqs. (61) to (64) are Schrödinger equations with source term arising from the (nonlinear) coupling between the two subspaces. Using these equations, the one-particle Schrödinger equation in $P+Q$ without source term can be written down,

$$\sum_{t'}(t|H-E|t')(t'|\Psi_E^c\rangle \;=0 \tag{65}$$

with

$$(t'|\Psi_E^c\rangle \;=(t'|\xi_E^c\rangle+\sum_R(t'|\Phi_R^{SM}+\omega_R\rangle\cdot A \tag{66}$$

where

$$A=\sum_{R'}\langle\Phi_R^{SM}|(E-H_{QQ}^{eff})^{-1}|\Phi_{R'}^{SM}\rangle\langle\Phi_{R'}^{SM}|V|\xi_E\rangle \tag{67}$$

according to (22).

Using (25), it follows

$$(t'|\Psi_E^c\rangle \;=(t'|\xi_E^c\rangle+\sum_R(t'|\tilde{\Phi}_R+\tilde{\omega}_R\rangle\cdot\tilde{A} \tag{68}$$

where

$$\tilde{A}=\frac{\langle\tilde{\Phi}_R|V|\xi_E\rangle\;\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle}{(E-\tilde{E}_R+\frac{i}{2}\tilde{\Gamma}_R)}\,. \tag{69}$$

Eq. (65) is a linear equation. The relation between $(t|\Phi_R^{SM}\rangle$ (or $(t|\tilde{\Phi}_R\rangle)$ and $(t|\Psi_E^c\rangle$ contains the nonlinear coupling between the two subspaces. As a consequence, the superposition principle holding for the $(t|\Psi_E^c\rangle$ in the whole function space $Q+P$ is not valid for the $(t|\Phi_R^{SM}\rangle$ (or $(t|\tilde{\Phi}_R\rangle)$ in $Q+P$, in general.

In eqs. (61) to (65), *all correlations between the nucleons* caused by the two-particle residual interaction V are taken into account including those via the continuum. The last ones are contained in the non-diagonal matrix elements of $H_{QP}\;G_P^{(+)}\;H_{PQ}$. The factors A and $\tilde{A}$ are calculated in the framework of the continuum shell model by using the channel wavefunctions $(t|\Phi_R^{SM}\rangle$, $(t|\omega_R\rangle$ and $(t|\xi_E^c\rangle$.

6 The number of degrees of freedom in the open quantum system

The number of degrees of freedom of a system is given by the number of states, in which the system can exist. This number is equal to the number N of resonance states (except for spurious states arising from the centre of mass motion) if the internal mixing V_{QQ} of the residual interaction

$$H^{eff}_{QQ} - (H_0)_{QQ} = V_{QQ} + V_{QP} G^{(+)}_P V_{PQ} \tag{70}$$

(see eq. (14)) is the dominant part. In this case, the properties of the system are determined by the average potential and the two-body residual interaction V_{QQ} (nuclear structure calculations at low level density). If, however, $V_{QP} G^{(+)}_P V_{PQ}$ dominates in the residual interaction (70), then the number of relevant degrees of freedom is given by the number K of open decay channels. In such a case, the properties of the system are determined mainly by the K coupled channels (Feshbach unified theory of nuclear reactions [20]).

The states of a system are mixed usually in the basic wavefunctions. As long as the N states Φ_i are mapped to exactly N other states according to the relation

$$\Phi_i = \sum_{j=1}^{N} a_{ij} \Phi_j \quad i = 1, ..., N \tag{71}$$

the number of degrees of freedom does not change as a consequence of the mixing. The transformation (71) is linear.

If, however, N states Φ_i are transformed to K other states with $K \neq N$, then the number of degrees of freedom of the system will be changed. For $K < N$, it is reduced [22]. Such a map

$$\chi_c = \sum_{j=1}^{N} a_{cj} \Phi_j \quad c = 1, ..., K \tag{72}$$

is nonlinear, as a rule, due to the additional conditions for the coefficients a_{cj} following from

$$\chi_c = 0 \qquad c = K+1, ..., N \,. \tag{73}$$

In the nuclear system considered in this paper, the non-diagonal matrix elements $\langle \Phi_{R'}^{SM} | H_{QQ}^{eff} | \Phi_R^{SM} \rangle$ contain the internal mixing W^{int} (in the Φ_R^{SM}) as well as the external mixing W^{ext} of the resonance states (eqs. (37) and (38)).

The internal mixing W^{int} is linear in the residual interaction V. Therefore, it cannot lead to a reduction of the number of degrees of freedom. It leads to a "spreading" of the information over all N states if the states were originally unmixed. The information entropy increases with increasing internal mixing W^{int},

$$\Delta i = lnN - ln1 > 0 \, . \tag{74}$$

Here i denotes the maximum information entropy

$$i = lnN \tag{75}$$

of the system with N states. The system tends to thermal equilibrium.

The external mixing W^{ext} is nonlinear in V. In this case, a reduction in the number of degrees of freedom is possible. An information which was originally distributed over N states is concentrated finally in $K < N$ states. The information entropy of the system is reduced according to

$$\Delta i = lnK - lnN < 0 \tag{76}$$

if $K < N$. One observes formation of structures.

The results of numerical calculations for nuclei have shown that external mixing can, indeed, lead to a "formation of structures in space and time" [13, 14]. At sufficiently strong external mixing, a system with N states and $K < N$ open decay channels forms K states with short lifetime and small spatial extension. The remaining $N-K$ states have a lifetime which is longer by at least one order of magnitude. Furthermore, the nucleons emitted from these states originate from the surface of the nucleus mainly in contrast to those from the short-lived states. The K short-lived states behave like "normal" resonances with $\langle \tilde{\Phi}_R | \tilde{\Phi}_R \rangle \approx 1$ (eqs. (43) and (53), see section 7) and

$$\begin{aligned} \tilde{\Gamma}_R \; &\approx \; 2\pi \sum_c |\langle \tilde{\Phi}_R | V | \xi_E^c \rangle)|^2 \\ &= \sum_c |\tilde{\gamma}_{Rc}| \, . \end{aligned} \tag{77}$$

They are characteristic of the system and relevant for its evolution [4].

The remaining $N-K$ states are "slaved" by the K short-lived states. They decay according to

$$\begin{aligned} \tilde{\Gamma}_R &= (\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle)^{-1} \sum_c |\tilde{\gamma}_{Rc}| \\ &< \sum_c |\tilde{\gamma}_{Rc}| \end{aligned} \tag{78}$$

due to $\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle > 1$ [13]. They retain not only the nonlinearity but carry also a large information entropy due to their large number [13] so that there is no contradiction of (76) with the second law. Further, these states have generic (stochastic) properties [6]. They determine the long-time behaviour of the system and *not* the evolution which proceeds at the short-time scale. The differences between the short-lived and long-lived states are the stronger the larger $N-K$ is.

Accordingly, the closed channels play only a minor role for the evolution of the system. Their contribution consists mainly in fluctuations caused by the virtual emission of nucleons.

Thus, the number of (relevant) degrees of freedom of the system will be reduced if the resonance states are able to mix via the continuum of decay channels. This happens at high level density, starting at a certain critical value of the degree of overlapping of the resonances, when the external mixing of the resonance states cannot be neglected.

7 The nucleus at low excitation energy

It is very well known that the properties of the nucleus are described well by restricting to bound states as long as the level density is low. The relevant part of the function space corresponds to the Q subspace defined in section 2. The lifetime against particle decay can be calculated by combining the nuclear structure calculation with the R-matrix theory. That means, the P subspace is irrelevant for nuclear structure studies (nuclear spectroscopic investigations).

At higher level density, the function space of open decay channels is relevant while the discrete states (compound nucleus states) and the closed decay channels are irrelevant. This statement is basic to the unified theory of nuclear reactions formulated by Feshbach [20]. Let us call the two corresponding subspaces the P' and Q' subspaces. At low excitation energy of

the nucleus, the number of open decay channels is much smaller than the number of resonances.

Thus, the properties of a nucleus at low and at high level density are described by models which differ essentially from each other. The transition from one to the other description can be traced only if the Schrödinger equation is solved with an ansatz which contains both the relevant and the irrelevant parts of the function space since at the transition a great deal of the relevant part becomes irrelevant and vice versa. The model described in section 2 (continuum shell model) fulfills this condition.

In [5], the transition is traced numerically in the continuum shell model. It takes place at $\bar{\Gamma}/\bar{D} \approx 1$, where $\bar{\Gamma}$ is the average width of the resonance states and $\bar{D}$ is the average distance of two neighboured states. The transition takes place in accordance with the known properties of the nucleus at low and at high level density. It is $P + Q = P' + Q' = 1$.

Analytically, the residual interaction between two nucleons consists of two parts

$$V_{QQ}^{eff} = V_{QQ} + V_{QP} G_P^{(+)} V_{PQ} \tag{79}$$

(see eq. (14)). The corresponding mixing matrix elements are

$$\begin{aligned} W_{R'R} &= W_{R'R}^{int} + W_{R'R}^{ext} \\ &= \langle \Phi_{R'}^{(0)} | V | \Phi_R^{(0)} \rangle + \langle \Phi_{R'}^{SM} | V G_P^{(+)} V | \Phi_R^{SM} \rangle \end{aligned} \tag{80}$$

according to (37), (38). As long as $\bar{\Gamma}/\bar{D} \ll 1$, it follows

$$\begin{aligned} W_{R'R}^{<} &\approx W_{R'R}^{int} \\ &= \langle \Phi_{R'}^{(0)} | V | \Phi_R^{(0)} \rangle \end{aligned} \tag{81}$$

to a good approximation while for $\bar{\Gamma}/\bar{D} \gg 1$,

$$\begin{aligned} W_{R'R}^{>} &\approx W_{R'R}^{ext} \\ &\approx \langle \Phi_{R'} | V G_P^{(+)} V | \Phi_R \rangle \end{aligned} \tag{82}$$

since V_{QQ} can be neglected relatively to $Re\{V_{QP} G_P^{(+)} V_{PQ}\}$ in (79). In this case, the Φ_R can be obtained, to a good approximation, from a statistical distribution of the eigenvalues and contain, in this manner, $Re\{V_{QP} G_P^{(+)} V_{PQ}\}$ (and V_{QQ}). Usually, the distribution of the Gauss Orthogonal Ensemble (GOE) is used [17]. Then

$$W_{R'R}^{>} \approx Im\{\langle \Phi_{R'} | V G_P^{(+)} V | \Phi_R \rangle\} \,. \tag{83}$$

It is clear that the symmetry properties of the system under the conditions (81) and (83) are different. In the first case, the symmetry is determined by H_0 while this symmetry is broken in the second case. It is replaced by a symmetry against formation and decay of the compound nucleus ("time reversal symmetry").

In heavy nuclei, $Re\{V_{QP}G_P^{(+)}V_{PQ}\}$ may be large and create strongly mixed states Φ_R even if $Im\{V_{QP}G_P^{(+)}V_{PQ}\}$ vanishes or is very small as it is the case for discrete states below the particle decay thresholds and for resonance states just above the first threshold for particle decay (e.g. neutron resonances), respectively. This can be seen from eq. (39) to which the matrix elements $\langle\Phi_R^{SM}|V|\xi_E^c\rangle$ at all energies contribute due to the integral and from eq. (51) or (52) where the corresponding matrix elements $\langle\Phi_R^{SM}|V|\xi_E^c\rangle$ or $\langle\tilde{\Phi}_R|V|\xi_E^c\rangle$ are taken at the energy of the system. The resonances *in the very neighbourhood of the elastic threshold* have small matrix elements (83) *and* small widths, therefore, *in any case*, i.e. also in that case in which the level density is high and the conditions for the trapping effect, eq. (82), are fulfilled.

Let us multiply the residual interaction V in the matrix elements $W_{R'R}$ by α in the same manner as it is done in the numerical calculations [5, 6, 11, 13, 14]. Then, the properties of the system can be considered as a function of the parameter α. For small α, the resonances do not overlap and $W_{R'R}^{>}$ is small. Therefore, $W_{R'R} \approx W_{R'R}^{<}$. At some critical value α_{cr}, the resonances start to overlap and $W_{R'R}^{<} \approx W_{R'R}^{>}$. For $\alpha \gg \alpha_{cr}$, $W_{R'R} \approx W_{R'R}^{>}$.

In the first case ($W_{R'R} \approx W_{R'R}^{<}$), the relevant part of the function space is the Q subspace. The coupling to the continuum does (almost) not influence the spectroscopic properties of the nuclear states. The symmetry of the system is determined by H_0. It is only weakly disturbed by the residual interaction V.

In the third case ($W_{R'R} \approx W_{R'R}^{>}$), the relevant part is the P' subspace of open decay channels since the number of short-lived states is equal to the number of open decay channels. This equality [8] is a consequence of the unitarity of the S-matrix as can be seen from the continuum shell model equations: The resonance part of the S-matrix, eq. (33), reads

$$S_{cc'}^{(2)} = i \sum_R \frac{\tilde{\gamma}_{Rc'}^{1/2*} \; \tilde{\gamma}_{Rc}^{1/2}}{E - \tilde{E}_R + \frac{i}{2}\tilde{\Gamma}_R} \,. \tag{84}$$

Let us consider an ensemble of N resonances which lie densely in an energy region ΔE comparable to the uncertainty of energy of the system ($E \approx E_1 \approx$

$E_2 ... \approx E_N$). Due to external mixing, they interfere strongly and one gets for estimation

$$S_{cc'}^{(2)} \approx 2 \sum_R \frac{\tilde{\gamma}_{Rc'}^{1/2*} \tilde{\gamma}_{Rc}^{1/2}}{\tilde{\Gamma}_R} . \tag{85}$$

It is

$$S_{cc}^{(2)} \approx 2 \sum_{R_f} \frac{|\tilde{\gamma}_{R_f c}|}{\sum_c |\tilde{\gamma}_{R_f c}|} \langle \tilde{\Phi}_{R_f} | \tilde{\Phi}_{R_f} \rangle \tag{86}$$

where the R_f denote the relevant fast modes. According to the unitarity of the S-matrix, $|S_{cc}^{(2)}| \leq 2$ or

$$\sum_{R_f} \frac{|\tilde{\gamma}_{R_f c}|}{\sum_c |\tilde{\gamma}_{R_f c}|} \leq \langle \tilde{\Phi}_{R_f} | \tilde{\Phi}_{R_f} \rangle^{-1} . \tag{87}$$

Therefore, the number R_f of relevant fast modes cannot be larger than 1 in the case with one open decay channel (and $\langle \tilde{\Phi}_{R_f} | \tilde{\Phi}_{R_f} \rangle \approx 1$). An analogous conclusion can be drawn in the many-channel case: The number of fast relevant modes is exactly equal to the number K of open decay channels. The symmetry of the system is determined, therefore, by its coupling to the P' subspace of open decay channels.

Thus, by means of varying the parameter α the transition from nuclear structure calculations at low level density to coupled channels calculations at high level density can be traced. This is in accordance with the results of numerical calculations [5, 13, 14]. Expressions of the type (81) are used, indeed, in the nuclear structure calculations at low level density while an expression of the type (83) is basic of the shell model approach to nuclear reactions [17] at high level density. Here, the open decay channels are relevant (unified theory of nuclear reactions [20]).

Further, the parameter α appears *linearly* in $W_{R'R}^{\leq}$, eq. (81). At low level density, the *superposition principle* holds, $\langle \tilde{\Phi}_R | \tilde{\Phi}_R \rangle \approx 1$, since the resonance states do not overlap and the external mixing is small. The decay takes place according to an *exponential law* as follows from the solution of the time dependent Schrödinger equation:

$$\begin{aligned} i\hbar \frac{\partial \tilde{\Psi}_R}{\partial t} &= H \tilde{\Psi}_R \\ &= \tilde{\mathcal{E}}_R \tilde{\Psi}_R \end{aligned} \tag{88}$$

where

$$\tilde{\Psi}_R = e^{-\frac{i}{\hbar}\tilde{\mathcal{E}}_R t}\tilde{\Phi}_R \,. \tag{89}$$

Here, $\tilde{\Phi}_R$ is the solution of the time independent Schrödinger equation with the eigenvalue $\tilde{\mathcal{E}}_R = \tilde{E}_R - \frac{i}{2}\,\tilde{\Gamma}_R$ (at the energy $E_R = \tilde{E}_R(E = E_R)$ of the system) and $\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle = 1$ (threshold effects are considered in [19]). It exists therefore a *natural scale* $\tilde{\mathcal{E}}$ what is very well known from the numerous nuclear structure calculations at low level density. The *symmetry* properties of the system are determined by H_0 as stated above.

In $W^{>}_{R'R}$, eqs. (82) and (83), the parameter α appears *nonlinearly*. Here, the *superposition principle* does *not* hold for the trapped (long-lived) states, $\langle\Phi_R|\Phi_R\rangle > 1$ in general, and the decay of the long-lived states takes place according to a *power law* [21]. A *natural scale* does *not* exist for the long-lived states as it is very well known from, e.g., the neutron resonances in heavy nuclei. In the eigenvalue equation, $\tilde{\mathcal{E}}_R\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle$ appears instead of $\tilde{\mathcal{E}}_R$, see eq. (44). The *symmetry* arising from H_0 is *broken*.

For the short-lived states at high level density holds $\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle \approx 1$. This follows from the unitarity of the S-matrix according to (86) since the widths of these states are near to the Wigner limit (spectroscopic factor 1). Moreover, it is proven numerically [13]. Thus, the few *short-lived states appearing at high level density behave very much like the isolated states at low level density*. They determine the evolution of the system but cannot be seen in high-resolution experiments, where they appear as a background.

In table 1, the properties of nuclear states at low and high level density are summarized. The words "regular" and "chaotic" motion are used in an analogous manner as in classical systems [15]. The short-lived states at high level density are not considered in table 1 since they are not investigated in high-resolution experiments (although they are relevant for the evolution of the system [20]).

Thus, the redistribution happening in an open quantum mechanical system at a critical value α_{cr} of a "control parameter" α takes place in accordance with our knowledge on the properties of nuclei at low excitation energy. The nuclear structure and coupled channels approaches describe the different behaviour of the nucleus below and beyond the critical value of the control parameter. Further, this redistribution observed in the open nuclear system at low excitation energy and described by the continuum shell model, satifies

Table 1: Nuclear states in high resolution experiments

⋆ **Low level density**

— Standard nuclear structure theory:
$H = H_0 + V$
closed system $\Longleftrightarrow Q = 1$
symmetry of $H_0 \Longleftrightarrow$ spectroscopy of discrete states

— Open quantum system:
$W_{RR'} \approx W^{int}_{RR'} = \langle \Phi^{(0)}_{R'} | V | \Phi^{(0)}_{R} \rangle$
regular motion of the nucleons:
– $\langle \tilde{\Phi}_R | \tilde{\Phi}_R \rangle = 1$
– exponential decay
– natural scale

⋆ **High level density**

— Standard nuclear reaction theory:
$H^{eff}_{QQ} = H_{QQ} + V_{QP} G^{(+)}_P V_{PQ}$
open system $\Longleftrightarrow Q < 1$
symmetry breaking $\Longleftrightarrow$ statistical distribution of discrete states

— Open quantum system:
$W_{RR'} \approx W^{ext}_{RR'} \approx \langle \Phi_{R'} | V G^{+}_P V | \Phi_R \rangle$
chaotic motion of the nucleons:
– $\langle \tilde{\Phi}_R | \tilde{\Phi}_R \rangle > 1$
– power law decay
– no natural scale

the same rules which are known from other selforganizing systems.

8 Summary

In this paper, the formalism of the continuum shell model is reformulated by emphazing the origin of the interferences which lead to the trapping effect at high level density. The interferences are caused by *nonlinear* couplings between the system (Q subspace) and the environment (P subspace) in which the system is embedded. The wavefunctions, energies and widths of the (decaying) resonance states follow from the diagonalization of an effective Hamilton operator which is non-Hermitean due to the (non-linear) coupling between the system and its environment.

The relation between the wavefunctions Ω_R of the discrete states (or the wavefunctions $\tilde{\Omega}_R$ of the resonance states) and the solution Ψ_E^c of $(H - E)\Psi_E^c = 0$ in the full function space is nonlinear, in general. The superposition principle in the whole function space $Q + P$ holds only for the Ψ_E^c and - approximately - for the short-lived states, but *not* for the trapped ones.

As a result, selforganization in a quantum system is caused by similar conditions as in a classical system although the mathematical formalism used is completely different. At a critical value of the degree of overlapping of the resonances, a redistribution of the spectroscopic properties takes place as a consequence of which states with very different lifetimes appear. The number of degrees of freedom which are relevant for the evolution of the system is reduced. "Structures in space and time" are formed. The corresponding short-lived states are described well by (almost) linear equations. They decay according to an exponential law and it exists a natural scale. The long-lived (slaved) states decay according to a power law and do not have a natural scale due to the non-negligible nonlinearities in the equations. The properties of the open system below and beyond the critical value of the degree of resonance overlapping are very well known in nuclei at low excitation energy and are described well by phenomenological models.

Acknowledgment: Valuable discussions with F.M. Dittes, W. Iskra, M. Müller and A. Pelster are gratefully acknowledged. The present investigations are supported by the Deutsche Forschungsgemeinschaft (Ro 922/1).

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