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A massive quasi-particle model of the SU(3) gluon plasma

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Abstract

Recent SU(3) gauge field lattice data for the equation of state are interpreted by a quasi-particle model with effective thermal gluon masses. The model is motivated by lowest-order perturbative QCD and describes very well the data. The proposed quasi-particle approach can be applied to study color excitations in the non-perturbative regime. As an example we estimate the temperature dependence of the Debye screening mass and find that it declines sharply when approaching the confinement temperature from above, while the thermal mass continuously rises.

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I. INTRODUCTION

The recent progress in calculating properties of quarks and gluons by numerical methods on space-time lattices also provides new information on the equation of state. It is generally believed that at sufficiently high temperature the strongly interacting matter appears as plasma of quarks and gluons, while at low temperatures the matter constituents are represented by hadrons (we consider here charge symmetric matter). Due to subtleties in incorporating fermions on a lattice the most accurate information is available for the pure gluon plasma. There are extensive studies of the SU(2) [1,2] and SU(3) [3,4] gluon plasma. An intriguing question concerns finite-size effects and the extrapolation to the continuum limit, which has been investigated recently [2,4].

Even if precision data for the equation of state are available, one must ask whether they allow for an interpretation in terms of physical quantities. Indeed, there are various attempts to find suggestive interpretations of the lattice numerology. In an early attempt the SU(2) data [1] are described by a low-momentum cut-off model. In ref. [5] a finite gluon mass and a vacuum pressure are fitted to previous SU(3) data. More accurate SU(3) data [3] are described in ref. [6] by a modified cut-off model with perturbative corrections and a bag constant, while in refs. [7,8] a thermal mass alone is found to be sufficient for describing the data. The latter approach has also proven to be successful for the SU(2) data [9]. Despite the accuracy of the SU(3) data on a $16^3 \times 4$ lattice [3], by now there are data on larger lattices available [4]. These new data seem to permit a safe extrapolation to the continuum limit and are worth to be interpreted.

The aim of our note is to present an interpretation of the recent SU(3) data [4] in terms of an ideal gas model of quasi-particles with thermal masses $m(T)$. This model can be applied for studying various physical quantities (such as Debye or screening mass of heavy quark potential, transport coefficients, dilepton and photon rates) at physically relevant, low temperatures near the confinement temperature T_c, where perturbative QCD can not be utilized directly. The particular point we adopt is that the high temperature limit of our thermal mass follows essentially from perturbative QCD. Such a functional dependence of $m(T)$ turns out to reproduce quite well the newest SU(3) lattice data. We extend our model here to estimate the Debye mass in SU(3) gauge theory near T_c.

II. IDEAL QUASI-PARTICLE GAS MODEL

Our goal is a quasi-particle model for the equation of state of a gluon plasma which is compatible with both the continuum extrapolation of lattice data, currently available up to $4\,T_c$, and the perturbative region, i.e., QCD at $T \to \infty$. We utilize the dispersion relation

$$\omega^2 = k^2 + m^2(T) \tag{1}$$

(ω and k are the quasi-particle energy and momentum). The effective Hamiltonian for such a system of quasi-particle excitations may be written as [10] $H_{eff} = \sum_{i=1}^{d} \sum_{\vec{k}} \omega(k,T)\, a^{\dagger}_{\vec{k},i}\, a_{\vec{k},i} + E_0(T)$, with $a^{\dagger}$ and a as usual creation and destruction operators for bosons, and E_0 denoting the ground state energy; d is the degree of degeneracy of energy eigen states. With the distribution function $f(k) = [\exp\{\sqrt{k^2 + m^2(T)}\} - 1]^{-1}$ the entropy density takes then the ideal gas form

$$s(T) = \frac{d}{2\pi^2 T} \int_0^\infty dk\, f(k)\, k^2 \, \frac{\frac{4}{3}k^2 + m^2(T)}{\sqrt{k^2 + m^2(T)}}, \tag{2}$$

while the primary thermodynamical potential pressure p and the energy density e read

$$p(T) = \frac{d}{6\pi^2} \int_0^\infty dk\, f(k)\, \frac{k^4}{\sqrt{k^2 + m^2(T)}} - B(T), \tag{3}$$

$$e(T) = \frac{d}{2\pi^2} \int_0^\infty dk\, f(k)\, k^2 \sqrt{k^2 + m^2(T)} + B(T). \tag{4}$$

These relations are thermodynamically self consistent, i.e., they fulfill $e + p = sT$ and $s = \partial p / \partial T$. The function $B(T) = \lim_{V \to \infty} \frac{E_0(T)}{V}$ (with V as volume of the system) is a necessary quantity when allowing for a temperature dependent quasi-particle energy $\omega(k, T)$ [10]. $B(T)$ is not a second independent function, but related to the thermal mass, due to self consistency, via

$$B(T) = B_0 - \frac{d}{4\pi^2} \int_{T_0}^{T} d\tau \, \frac{dm^2(\tau)}{d\tau} \int_0^\infty \frac{dk\, k^2\, f(k)}{\sqrt{k^2 + m^2(\tau)}}. \tag{5}$$

The integration constant B_0 resembles somewhat the bag constant. Note that the previous approaches [8,9] used $p(T)$ with $B(T) \equiv 0$, and that consequently neither the entropy density nor the energy density take the structure of an ideal gas. Eqs. (3, 4) are used in ref. [5].

To determine the functional dependence of $m(T)$ on the temperature let us first consider the perturbative regime. The thermodynamical properties of the gluon plasma depend predominantly on the transverse part of the gluon self-energy [11,12]. In the weak coupling regime the transversal gluon self-energy in a gluon plasma with N_c colors results in a dispersion relation which can be approximated [11,12] by $\omega^2 = \alpha k^2 + \beta \omega_0^2$, with $\alpha = 1$ ($\frac{6}{5}$) and $\beta = \frac{3}{2}$ (1) at large (small) momenta and gauge invariant plasma frequency $\omega_0^2 = \frac{N_c}{9} g^2 T^2$ (here g^2 denotes the perturbative QCD coupling constant). Numerically, the large momentum approximation to the full transverse one-loop dispersion relation [13] holds at $k/T > 2\sqrt{N_c/9}\, g$; longitudinal excitations are there overdamped. Otherwise, the large momentum region dominates the statistical integrals in eqs. (2 - 4), e.g., more than 96.5% of the contribution to the energy density come from $k/T \geq 1$. Therefore, eq. (1) represents an excellent approximation of QCD properties, relevant for evaluating eqs. (2 - 4), and $m^2(T) = \frac{1}{2}\beta\omega_0^2$ with $\beta = \frac{3}{2}$ is supported within this approximation. We have here included the factor $\frac{1}{2}$ which, according to Goloviznin, Satz and Shuryak [9], accounts for partitioning the self-energy in lowest order between two interaction partners. Hence, $m^2(T) = \frac{1}{\Gamma} g^2(T)\, T^2$, with $\Gamma = \frac{12}{N_c}$, emerges approximately from perturbative QCD.

Let us now compare the obtained pressure potential (3) at high temperature with the corresponding pressure obtained within first-order QCD. The high-temperature expansion (i.e., $m/T \ll 1$) of eq. (3) reads

$$p = p_{SB}\left[1 - \frac{15}{4\pi^2}\left(\frac{m(T)}{T}\right)^2 + \cdots\right] - B(T), \tag{6}$$

with $p_{SB} = \frac{d\pi^2}{90}T^4$. From QCD it is known [11] that the perturbative contribution to the pressure is

$$p_{pQCD} = \frac{2(N_c^2-1)\pi^2}{90}T^4\left[1 - \frac{5N_c}{16\pi^2}g^2 + \cdots\right]. \tag{7}$$

Comparing the leading terms in eqs. (6,7) one reveals that, despite of massive quasi-particles, one needs to include only the two transverse degrees of freedom, i.e., $d = 2(N_c^2-1)$. The next-to-leading order terms in the parenthesis confirm our above ansatz for $m^2(T)$. In this way the perturbative part of QCD pressure (7) is related to the quasi-particle excitations with energy $\omega(k,T)$, while the ground state contribution E_0 is related to $B(T)$ and can be estimated in such an expansion from eq. (5) as $B(T) = -p_{SB}\frac{15}{8\pi^2}\left(\frac{m(T)}{T}\right)^2$. This interpretation points to the non-trivial rôle of the ground state contribution even at high temperatures. This particular aspect is discussed in ref. [11] and exemplified in ref. [14].

Finally we specify the coupling constant in accordance with perturbative QCD as

$$G^2(T) = \frac{48\pi^2}{11\,N_c\,\ln(\lambda T/T_c + T_s/T_c)^2}, \tag{8}$$

with T_s/T_c as phenomenological regularization as in ref. [8] and $\lim_{T\to\infty} G^2(T) \to g^2(T)$; T_c/λ represents the usual regularization scale parameter Λ. In the following we utilize in eqs. (2, 3, 4) the thermal mass

$$m^2(T) = \frac{1}{\Gamma}G^2(T)\,T^2, \quad \Gamma = \frac{12}{N_c}. \tag{9}$$

III. ANALYSIS OF LATTICE DATA

We apply our model now to the SU(3) lattice data [4]. In fig. 1 we demonstrate that our model, defined by eqs. (1, 2 - 5, 8, 9), describes very well the continuum-extrapolated data. As fit parameters we obtain $\lambda = 1.66$, $T_s/T_c = -0.53$. $B_0 = 0.25\,T_c^4$ turns out as an optimum choice for the present data. As seen in fig. 1 the function $B(T)$, which becomes small at $T > 1.5T_c$, changes its sign at $2T_c$ (a similar observation is made in ref. [10] for the older data [3]). This might be considered as a hint on a complicated non-perturbative vacuum structure.

Fig. 2 displays the interaction measure $(e - 3p)\,T^{-4}$, which is a sensitive quantity. One observes that for $T > 1.2\,T_c$ the $32^3 \times 6$ and $32^3 \times 8$ lattice data are nicely reproduced. In the region T_c - $1.2\,T_c$ the scaled energy density is a rapidly varying function. It might turn out that our quasi-particle model does not cover perfectly the very details of forthcoming high-precision lattice data in this region. However, it seems that the gross features of the equation of state in the physically relevant region are fairly well described. This gives some confidence in our quasi-particle interpretation.

As matter of fact we mention three obvious aspects of our phenomenological approach. (i) The flexibility, introduced by the definition of $G(T)$ in eq. (8), allows to some extent for the description of the data. (ii) Otherwise, this flexibility is insufficient to describe the data on the basis of a different model, wherein eqs. (6) and (7) would be completely identified. (iii) Higher order agreement of the corresponding parts of eqs. (6,7) is not achievable due to a mismatch of numerical prefactors, therefore, eq. (3) can not be considered as resumed expression.

IV. SCREENING MASS

Our model can be applied to study various collective properties of a colored quark-gluon system. Since even at comparatively low temperatures the gas of quasi-particles still remains weakly interacting, we have a chance to treat this gas in a perturbative way down to T_c. Here we estimate as an example the temperature dependence of the Debye screening mass. The Debye mass reflects the property of a plasma medium to screen the static chromoelectric interactions. Following the standard definition [11,15] the Debye mass m_D for an electromagnetic plasma is given by the small momentum limit of the static longitudinal photon self-energy function $\Pi_{00}(\omega, k)$,

$$m_D^2 = \lim_{k\to 0} \Pi_{00}(\omega = 0, k). \tag{10}$$

It is connected with the longitudinal part of the plasma dielectric tensor $\epsilon_L(\omega, k)$ via $k^2 + \Pi_{00}(0, k) = k^2 \epsilon_L(0, k)$ [16]. At leading order in α_s the above definition is valid also for the QCD plasma [17]. In our model α_s is the coupling constant of the color interaction between quasi-particles. The chromoelectrical tensor $\epsilon_L(\omega, k)$ can be calculated in lowest order in α_s within the kinetic theory of collective color excitations [18] with the corresponding corrections related to the non-zero effective mass $m(T)$ of our quasi-particles. (The analogous approach has been employed for calculations within the cut-off model [19].)

For the gluon plasma the chromoelectrical tensor is

$$\epsilon_L = 1 + \frac{g^2 N_c \gamma}{\omega k^2} \int \frac{d^3p}{(2\pi)^3} \frac{\vec{k}\vec{p}}{E\omega - \vec{k}\vec{p} + i\varepsilon} \left(\vec{k} \frac{\partial}{\partial \vec{p}} \right) f(p), \tag{11}$$

where $k^\mu = (\omega, \vec{k})$ is the wave four-vector, and $f(p)$ denotes the above distribution function of quasi-particles with four momentum $p^\mu = (E, \vec{p})$ and the dispersion relation (1). The factor $\gamma = 2$ accounts for the spin degrees of freedom of the quasi-particles with respect to the asymptotic limit above and to ref. [5]. Solving eq. (11) in the limit (10) yields

$$m_D^2 = \frac{N_c}{\pi^2} g^2 T^2 J_g(T), \tag{12}$$

$$J_g(T) = T^{-3} \int_0^\infty dp\, p^2 f^2(p) \exp\left\{ \frac{\sqrt{p^2 + m^2(T)}}{T} \right\}.$$

In the limit $m(T)/T \to 0$ one recovers the well known perturbative QCD limit $m_D^2 = \frac{1}{3} N_c g^2 T^2$. The perturbative QCD coupling constant can be expressed by

$$\alpha_s = \frac{g^2}{4\pi} = \frac{12\pi}{11 N_c \log \frac{M^2}{\Lambda^2}}, \tag{13}$$

where the quantity M^2 is determined by averaging over the squared quasi-particle momenta [11], i.e.,

$$M^2(T) = \frac{4}{3} \frac{\int_0^\infty dp\, f(p)\, p^4}{\int_0^\infty dp\, f(p)\, p^2}. \tag{14}$$

We choose the scale parameter Λ in accordance with the high-temperature limit of eqs. (8, 13). Since in this limit $M \approx 3.7\,T$, we find $\Lambda/T_c = 3.7\,\lambda^{-1}$. For the temperature dependence of $m(T)$, extracted above from the lattice data, the coupling constant (13) remains as small as 0.68 at T_c. So based on the perturbative ansatz for m_D^2 we get in our quasi-particle model the non-perturbative behavior of the Debye mass displayed in fig. 3. As seen in fig. 3 the thermal mass increases when approaching T_c from above, while the screening mass m_D^2 drops; above $1.3T_c$ m_D/T stays roughly constant. The obtained behavior of the Debye screening mass is confirmed by lattice calculations [20]. Such a sharp dropping of m_D near T_c might have quite interesting consequences for several deconfinement probes in ultrarelativistic heavy-ion collisions, e.g., the mono-jets considered in ref. [21]. At the same time we stress that the direct comparison of the screening mass in our model with the lattice data requires a more improved analysis of the static quark potential due to contributions from the vacuum state to the free energy, where the potential is extracted from. This quantitative comparison is envisaged, as also an extension to SU(2) [22].

V. SUMMARY

In summary we present an interpretation of new SU(3) gluon lattice data within a model of an ideal gas of quasi-particles with effective thermal masses, which is motivated by perturbative QCD. Such a functional dependence of the effective mass is found to reproduce rather perfectly the recent SU(3) lattice data of thermodynamical parameters. We utilize our model to deduce the behavior of the Debye screening mass near the confinement temperature and find a sharply dropping Debye mass, when approaching close to T_c from above, while the thermal mass continuously rises.

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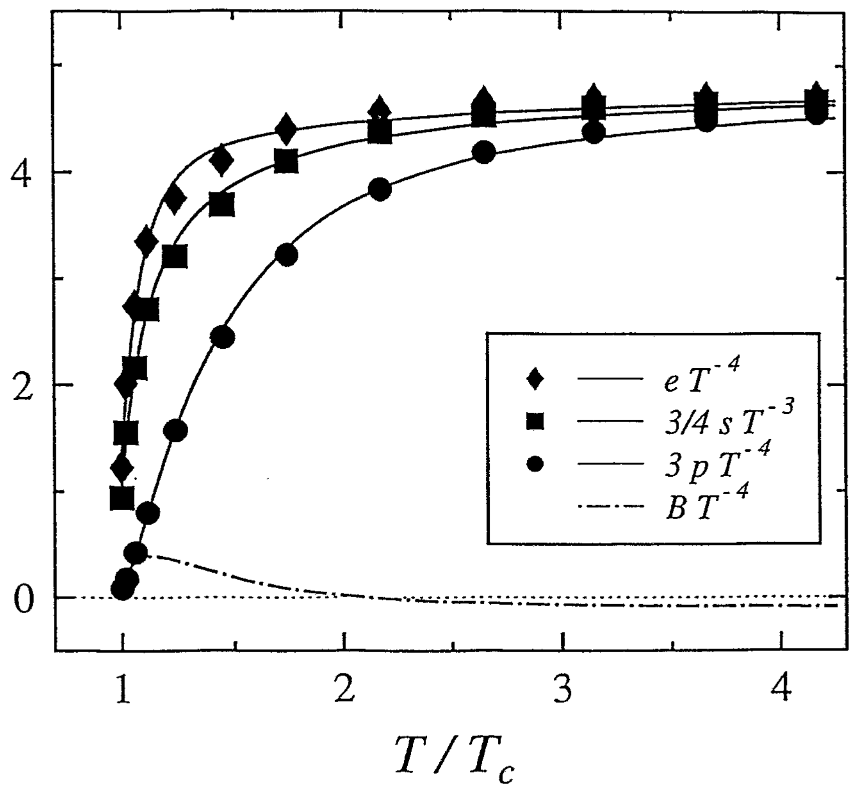

FIG. 1. Comparison of our model (thin lines) with continuum-extrapolated lattice data (symbols, from [4]) of scaled energy density, pressure and entropy density. The dash-dotted curve depicts the function $B(T)/T^4$.

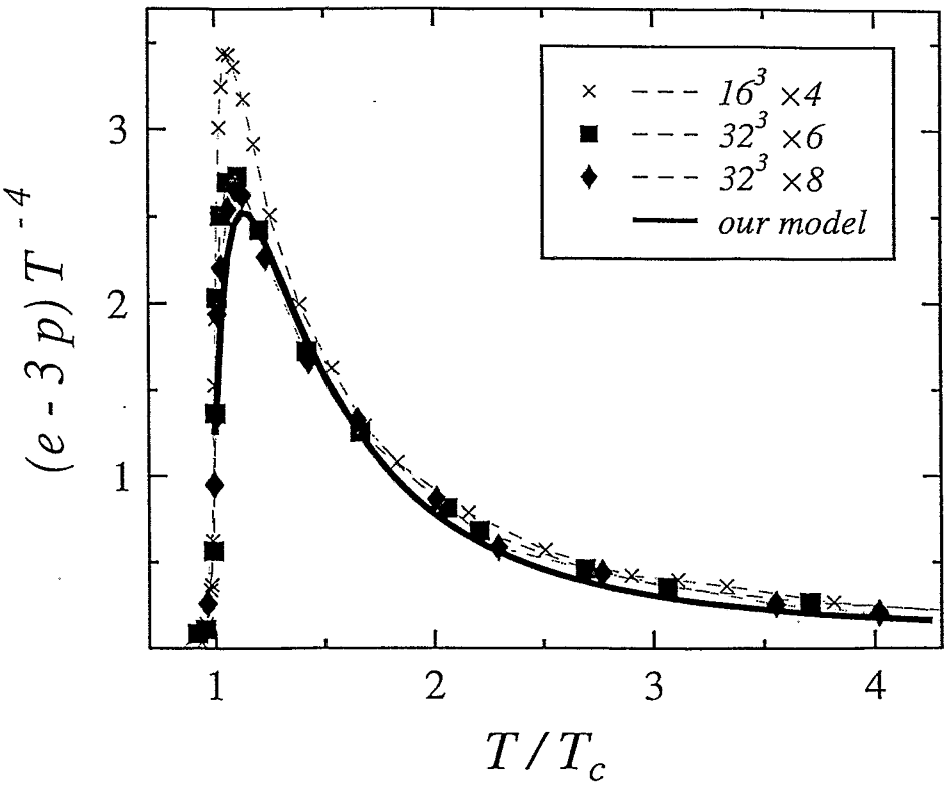

FIG. 2. The interaction measure as function of temperature (heavy full line: our model; symbols: lattice data [4].

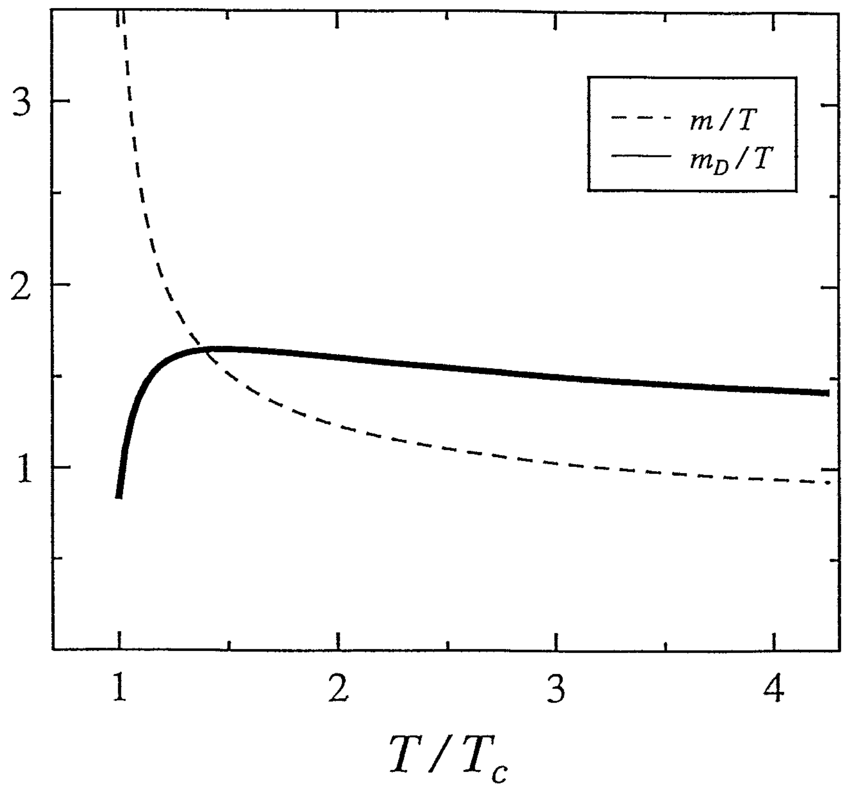

FIG. 3. The thermal mass (dashed line) and the estimated Debye screening mass (heavy full line) in our model.