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Asymmetry of the Dielectron Emission Rate in an Isospin-Asymmetric Pion Medium

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Abstract

The dielectron emission by pion annihilation in an isospin-asymmetric pion gas at finite temperature is considered. Due to the difference between the longitudinal and transverse parts of the in-medium ρ meson self-energy a specific asymmetry of the rates is caused for electron pairs with relative momenta perpendicular or parallel to the total pair momentum. This asymmetry may be considered as a sensitive signal of in-medium modifications of the ρ properties.

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1. Introduction: In-medium modifications of hadrons attract presently much theoretical and experimental interest. In particular, dileptons can be considered as a probe of the vector meson dynamics in an excited nuclear environment. According to the vector dominance model [1], the pion electromagnetic form factor is almost completely dominated by the ρ meson below an invariant mass of about 1 GeV [2], and this gives rise to the hope to explore in-medium properties of the ρ meson via the lepton pair production in the $\pi^+\pi^-$ annihilation process [3] in the course of heavy-ion collisions. However, the up to now calculated in-medium modification of the ρ mass and decay width within different models gives too a modest affection of the dielectron distribution as function of the dielectron invariant mass M [4] to be seen, at least at the present level of the available accuracy of experimental data. The other problem here is the large number of competing dielectron sources in nucleus-nucleus collisions [5]. In the nuclear medium the properties of the sources might be modified and it is difficult to extract the contribution of a separate channel (e.g., $\pi^+\pi^-$ annihilation) from the data. In Ref. [6] the lepton decay anisotropy (or more general, the lepton pair angular distribution) has been proposed for disentangling different sources.

In the present letter we focus on another possible observable in the dielectron spectra, namely the dielectron asymmetry. The asymmetry is a measure of the difference between the dielectron production rates for pairs with the relative pair momentum $\mathbf{t} = \mathbf{p}_+ - \mathbf{p}_-$ perpendicular and parallel to the total pair momentum $\mathbf{p} = \mathbf{p}_+ + \mathbf{p}_-$ ($\mathbf{p}_\pm$ are the momenta of the electrons and positrons). This quantity can provide information on the delicate in-medium properties of the ρ meson. It is caused by a difference between longitudinal and transverse parts of the ρ self-energy. In vacuum this difference is exactly zero because of the spatial isotropy. Generally, in matter the asymmetry vanishes only for those ρ mesons with the spatial momentum $\mathbf{p} = 0$. In an isospin-symmetric pion medium at finite temperature T the asymmetry has been considered in Ref. [7] and is found to be comparatively small. Here we analyze the case of an isospin-asymmetric pion medium with large values of the charge chemical potential μ_Q and show that the difference between longitudinal and transverse parts of the ρ self-energy is larger, which in turn leads to the above mentioned distinctive asymmetry of the dielectron production rates.

Our prediction has still a qualitative character. For a more detailed quantitative estimate one needs to include yet the baryonic degrees of freedom and other mesons in order to describe realistically the fireball in intermediate energy heavy-ion collisions, or the fragmentation region or the midrapidity region in relativistic heavy-ion collisions. Nevertheless, the model of a pure pionic isospin-asymmetric medium at finite temperature and chemical potential μ_Q has its own interest as simplest model of strongly interacting hadrons and represents a necessary step towards the more complete theory. An example of this interest is the recent work of Weldon [8], where the dilepton production by $\pi^+\pi^-$ annihilation via the intermediate scalar σ meson is considered. This process can occur only in a CP-violating medium with $\mu_Q \neq 0$. We consider here another generic non-trivial in-medium effect which can be, in principle, verified experimentally with sufficient accuracy.

2. Dilepton rate and asymmetry: Recall that the thermal lepton pair production rate within the vector dominance model is related to the imaginary part of the ρ propagator as follows [7, 9] (we utilize units with $\hbar = c = 1$)

$$E_+E_-\frac{dR}{d^3\mathbf{p}_+d^3\mathbf{p}_-} = \frac{1}{(2\pi)^6}\frac{e^4}{g_\rho^2}\frac{m_\rho^4}{M^2}\left[(1-\xi)W_L + (1+\xi)W_T\right]\left[\exp\{\frac{E}{T}\} - 1\right]^{-1}, \quad (1)$$

with

$$\xi = 1 - (\mathbf{t}^2 - \frac{(\mathbf{t}\cdot\mathbf{p})^2}{\mathbf{p}^2})M^{-2} = \frac{M^2\cos^2\Theta_{\mathbf{tp}}}{M^2 + \mathbf{p}^2\sin^2\Theta_{\mathbf{tp}}}, \quad (2)$$

$$W_L = \frac{-\mathrm{Im}\,F}{\left(M^2 - m_\rho^2 - \mathrm{Re}\,F\right)^2 + (\mathrm{Im}\,F)^2}, \quad W_T = \frac{-\mathrm{Im}\,G}{\left(M^2 - m_\rho^2 - \mathrm{Re}\,G\right)^2 + (\mathrm{Im}\,G)^2}, \quad (3)$$

where $p = (E, \mathbf{p})$, $p^2 = M^2$, and F and G are the complex valued longitudinal (L) and transverse (T) parts of the in-medium ρ propagator

$$D^{\mu\nu} = -\frac{P_L^{\mu\nu}}{p^2 - m_\rho^2 - F} - \frac{P_T^{\mu\nu}}{p^2 - m_\rho^2 - G} \quad (4)$$

($P_{L,T}$ denote the corresponding projectors [7]). In Eq. (1) the quantity ξ depends on the angle $\Theta_{\mathbf{tp}}$ between the vectors $\mathbf{t}$ and $\mathbf{p}$ (to measured in the medium's rest frame) and varies from 0 at $\Theta_{\mathbf{tp}} = \pi/2$ up to 1 at $\Theta_{\mathbf{tp}} = 0$.

The effect of the difference of the longitudinal and transverse polarization contributions manifests itself most clearly in the angular dependence of the asymmetry of the differential distributions, which we define as

$$A(\Theta_{\mathbf{tp}}) = \frac{dR(\Theta_{\mathbf{tp}}) - dR(\Theta_{\mathbf{tp}} = 0)}{dR(\Theta_{\mathbf{tp}} = \pi/2)} = (1-\xi)\frac{W_L - W_T}{W_L + W_T}. \quad (5)$$

In vacuum the transverse and longitudinal parts of the ρ self-energy are equal to each other and, as consequence, $A(\theta_{\mathbf{tp}})$ vanishes. Gale and Kapusta [7] found that at $\mu_Q = 0$ and finite temperature the difference between F and G is rather small even at T=150 MeV which leads to a hardly measurable value of $A(\theta_{\mathbf{tp}})$. The situation changes at finite μ_Q.

3. Self-energies: Calculating the ρ self-energy as starting point we use an effective Lagrangian which describes a system of charged pions and ρ mesons

$$\mathcal{L} = \frac{1}{2}(D^\nu\phi)^* D_\nu\phi - \frac{1}{2}m_\pi^2\phi\phi^* - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho^\mu\rho_\mu, \tag{6}$$

where ϕ is the complex pion field, ρ^μ stands for the neutral vector field with the strength $\rho_{\mu\nu} = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu$, and $D_\nu = \partial_\nu - ig_\rho\rho_\nu$ denotes the covariant derivative; μ, ν are Lorentz indices. The Hamiltonian of the system is related to the Lagrangian (6) in the usual way by $\mathcal{H} = \frac{\partial\mathcal{L}}{\partial(\partial_0\varphi)}\partial_0\varphi - \mathcal{L}$ with $\varphi = (\phi,\, \phi^*,\, \rho)$.

A finite value of the chemical potential μ_Q leads to a transformation of the Hamiltonian, which we use for the calculation of the partition function, $\mathcal{H} \to \mathcal{H} - \mu_Q J_0$, where J_0 is the time component of Noether's current $J_\nu \equiv i\frac{1}{2}(\phi^*(D_\nu\phi) - \phi(D_\nu\phi)^*)$. Then using the standard functional integral methods (e.g., Refs. [10, 11]) we calculate the partition function up to second order of the coupling constant g_ρ and express the ρ polarization operator via loop integrals. The divergent part of the self-energy is regularized with counterterms by using the dimensional regularization. For details of the lengthy calculation we refer the interested reader to Ref. [12]. The final expressions of the in-medium (*mat*) parts of the self-energies F and G read

$$\begin{aligned}
\operatorname{Re} G_{mat} &= \frac{1}{2}\frac{g_\rho^2}{4\pi^2}\int_0^\infty \frac{k^2dk}{\omega}N(\omega)\Big[\frac{2(E^2+\mathbf{p}^2)}{\mathbf{p}^2} - \frac{M^2E\omega}{k\mathbf{p}^3}\ln|b| - \\
&\quad \frac{E^2(4\omega^2+E^2) - \mathbf{p}^2(4k^2-\mathbf{p}^2+2E^2)}{4k\mathbf{p}^3}\ln|a|\Big],
\end{aligned} \tag{7}$$

$$\operatorname{Re} F_{mat} = \frac{M^2}{\mathbf{p}^2}\frac{1}{2}\frac{g_\rho^2}{4\pi^2}\int_0^\infty \frac{k^2dk}{\omega}N(\omega)\Big[\frac{4\omega^2+E^2}{2k|\mathbf{p}|}\ln|a| + \frac{2E\omega}{k|\mathbf{p}|}\ln|b| - 4\Big], \tag{8}$$

$$\operatorname{Im} G_{mat} = \frac{1}{2}\frac{g_\rho^2}{4\pi}\int_0^\infty \frac{k^2dk}{\omega}N(\omega)\frac{E^2(4\omega^2+E^2) - \mathbf{p}^2(4k^2-\mathbf{p}^2+2E^2) - 4M^2E\omega}{4k|\mathbf{p}|^3}\zeta, \tag{9}$$

$$\operatorname{Im} F_{mat} = -\frac{1}{2}\frac{M^2}{\mathbf{p}^2}\frac{g_\rho^2}{2\pi}\int_0^\infty \frac{k^2dk}{\omega}N(\omega)\frac{(2\omega-E)^2}{4k|\mathbf{p}|}\zeta, \tag{10}$$

$$a = \frac{(E^2 - \mathbf{p}^2 + 2k|\mathbf{p}|)^2 - 4E^2\omega^2}{(E^2 - \mathbf{p}^2 - 2k|\mathbf{p}|)^2 - 4E^2\omega^2}, \quad b = \frac{(E^2 - \mathbf{p}^2)^2 - 4(E\omega + 2k|\mathbf{p}|)^2}{(E^2 - \mathbf{p}^2)^2 - 4(E\omega - 2k|\mathbf{p}|)^2},$$

where $\zeta = \theta(k - k_-) \cdot \theta(k_+ - k)$ with $k_\pm = \left| E(1 - 4m_\pi^2/M^2)^{1/2} \pm |\mathbf{p}| \right|$. These quantities are needed for $F = F_{mat} + F_{vac}$ and $G = G_{mat} + G_{vac}$, where the vacuum (vac) contributions $F_{vac} = G_{vac}$ are given in Eq. (19) of Ref. [7]. The expressions (7) - (10) look very similar to the results of Gale and Kapusta [7], but the pion distribution function becomes now $N(\omega) = [\exp\{(\omega - \mu_Q)/T\} - 1]^{-1} + [\exp\{(\omega + \mu_Q)/T\} - 1]^{-1}$ with $\omega^2 = m_\pi^2 + k^2$. Please notice that the F's and G's depend on the invariant mass M and the total pair momentum $|\mathbf{p}|$. All dependence on the angle $\Theta_{\mathbf{tp}}$ is described by the quantity ξ which enters Eq. (1).

4. The asymmetry: Eqs. (1), (7) - (10) allow to calculate several in-medium corrections to the dielectron production rate. In the present letter we limit ourselves onto the discussion of the asymmetry (5). For this goal we compare first the difference between longitudinal and transverse parts of the self-energy. In our calculations we utilize the numerical values $m_\pi = 139.6$ MeV, $m_\rho = 770$ MeV, and $g_\rho^2/4\pi = 2.93$. Fig. 1 displays the ratios $\mathrm{Re}\,F/\mathrm{Re}\,G$ and $\mathrm{Im}\,F/\mathrm{Im}\,G$ as function of the invariant dielectron mass M for different values of $|\mathbf{p}|$. To catch a realistic temperature of the hadron gas, where our effect is largest, we choose $T = 150$ MeV. For the charge chemical potential we employ $\mu_Q = 120$ MeV, which represents an upper limit of values to be expected in heavy-ion collisions [12]. For sufficiently small values of μ_Q we recover the results of Gale and Kapusta [7]. From Fig. 1 follows that the real part of the longitudinal contribution is smaller than that of the transverse contribution, i.e., $\mathrm{Re}\,F < \mathrm{Re}\,G$, while the imaginary parts have the opposite behavior. At $|\mathbf{p}| = 0$ we get $F = G$, but at finite values of $|\mathbf{p}|$ we see a significant difference between F and G. The largest difference appears in the region of $M \sim 0.4 - 0.6$ GeV, and its absolute value increases with $|\mathbf{p}|$. The averaged ρ momentum at given temperature of the hadron gas is $< |\mathbf{p}| >= 0.66$ GeV.

The angular dependence of the asymmetry $A(\Theta_{\mathbf{tp}})$, defined in Eq. (5), at $M = 0.5$ GeV and for different values of $|\mathbf{p}|$ is displayed in Fig. 2. At small values of $|\mathbf{p}|$ one gets $W_L \approx W_T$, and the asymmetry vanishes. At finite $|\mathbf{p}|$ the asymmetry increases smoothly with $\Theta_{\mathbf{tp}}$ and reaches its maximum value at $\Theta_{\mathbf{tp}} = \frac{\pi}{2}$.

Fig. 3 shows the maximum asymmetry at $\Theta_{\mathbf{tp}} = \frac{\pi}{2}$ as function of the invariant mass at different values of $|\mathbf{p}|$. At $\mathbf{p} > 0$, and when $M \to 2m_\pi$, $\mathrm{Im}\,F \to \mathrm{Im}\,G$ holds and we find sharply decreasing values of $A(\frac{\pi}{2})$. On the other hand, one can see that $A(\frac{\pi}{2})$ has a zero at

$M \simeq m_\rho + \delta(\mathbf{p})$, where $\delta(\mathbf{p})$ is a smoothly decreasing function of $\mathbf{p}^2$ with $\delta(\mathbf{p})/m_\rho < 10^{-2}$. So, the asymmetry reaches a maximum between $M = 2m_\pi$ and this zero, i.e., within the interval $2m_\pi < M < m_\rho$, because of the inequality $\mathrm{Im}\, F > \mathrm{Im}\, G$ at $M < m_\rho$. We find that the maximum asymmetry increases again with $|\mathbf{p}|$ and it may be as much as 0.24 for $|\mathbf{p}| = 0.8$ GeV. For the isospin-symmetric medium with $\mu_Q = 0$ it is about four times smaller, while for $\mu_Q = 60$ MeV it becomes 0.13 (at $|\mathbf{p}| = 0.8$ GeV).

5. Summary: In summary, we predict a specific asymmetry in the differential electron pair distribution in an isospin-asymmetric pion medium: The rates for pairs with relative momentum parallel and perpendicular to the total pair momentum differ due to in-medium effects. We find that the asymmetry increases with the charged pion chemical potential μ_Q and is largest for large values of the total pair momentum. However, the presence of the other mesons and baryons needs to be regarded for more realistic quantitative predictions. Nevertheless, relying on the present schematic model we are able to figure out hints for an interesting generic phenomenon which is sensitive to in-medium effects and which is worth investigating in more details. Potentially, this medium effect can give us a new insight into the dilepton production as a probe of hadron properties under extreme conditions.

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References

[1] J.J. Sakurai, *Currents and mesons*, Chicago, University of Chicago Press, 1969

[2] G.J. Gounaris, J.J. Sakurai, Phys. Rev. Lett. **21** (1968) 244

[3] R.D. Pisarski, Phys. Lett. **B11** (1982) 157

[4] Gy. Wolf, W. Cassing, W. Ehehalt, U. Mosel, Progr. Part. Nucl. Phys. **30** (1993) 273

[5] Gy. Wolf, G. Batko, W. Cassing, U. Mosel, K. Niita, M. Schäfer, Nucl. Phys. **A517** (1990) 615
L. Xiong, Z.G. Wu, C.M. Ko, J.Q. Wu, Nucl. Phys. **A512** (1990) 772
L.A. Winkelmann, H. Sorge, H. Stöcker, W. Greiner, Phys. Lett. **B298** (1993) 22
K. Haglin, C. Gale, Phys. Rev. **C49** (1994) 401
M. Schäfer, H.C. Dönges, A. Engel, U. Mosel, Nucl. Phys. **A575** (1994) 429
A.I. Titov, B. Kämpfer, E.L. Bratkovskaya, Phys. Rev. **C51** (1995) 227

[6] E.L. Bratkovskaya, O.V. Teryaev, V.D. Toneev, Phys. Lett. **B348** (1995) 283
E.L. Bratkovskaya, M. Schäfer, W. Cassing, U. Mosel, O.V. Teryaev, V.D. Toneev, Phys. Lett. **B348** (1995) 325

[7] J.L. Kapusta, C. Gale, Nucl. Phys. **B357** (1991) 65

[8] H.A. Weldon, Phys. Lett. **B274** (1992) 133

[9] H.A. Weldon, Phys. Rev. **D42** (1990) 2384

[10] C.W. Bernard, Phys. Rev. **D9** (1974) 3312

[11] J.I. Kapusta, *Finite temperature field theory*, Cambridge, University Press, 1989

[12] T.I. Gumalov, A.I. Titov, B. Kämpfer, preprints FZR-66 (1994), FZR-90 (1995)

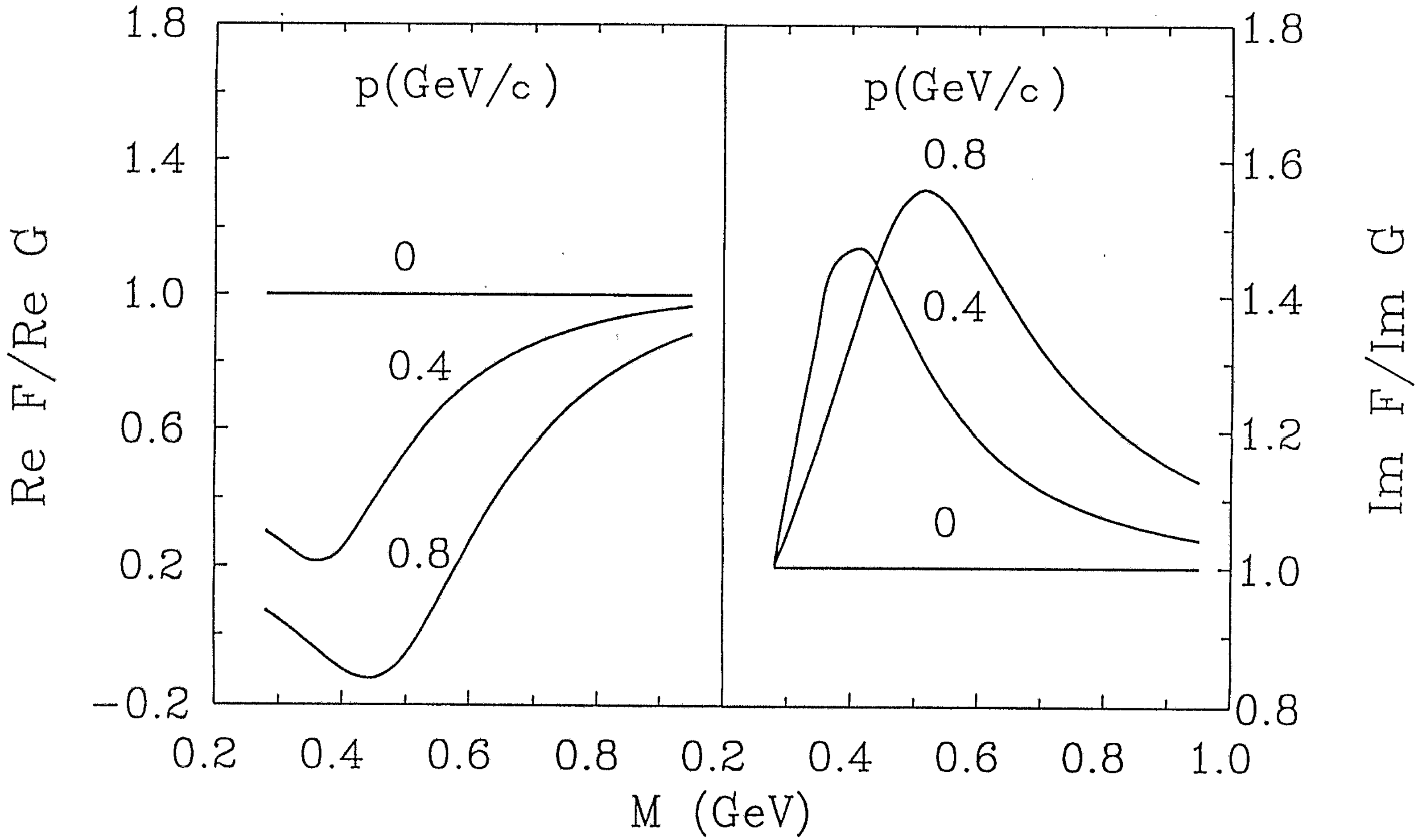

FIG. 1. The ratios $\mathrm{Re}\,F/\mathrm{Re}\,G$ (left panel) and $\mathrm{Im}\,F/\mathrm{Im}\,G$ (right panel) as function of the dielectron invariant mass M for different values of $|\mathrm{p}|$ and for $\mu_Q = 120$ MeV and $T = 150$ MeV.

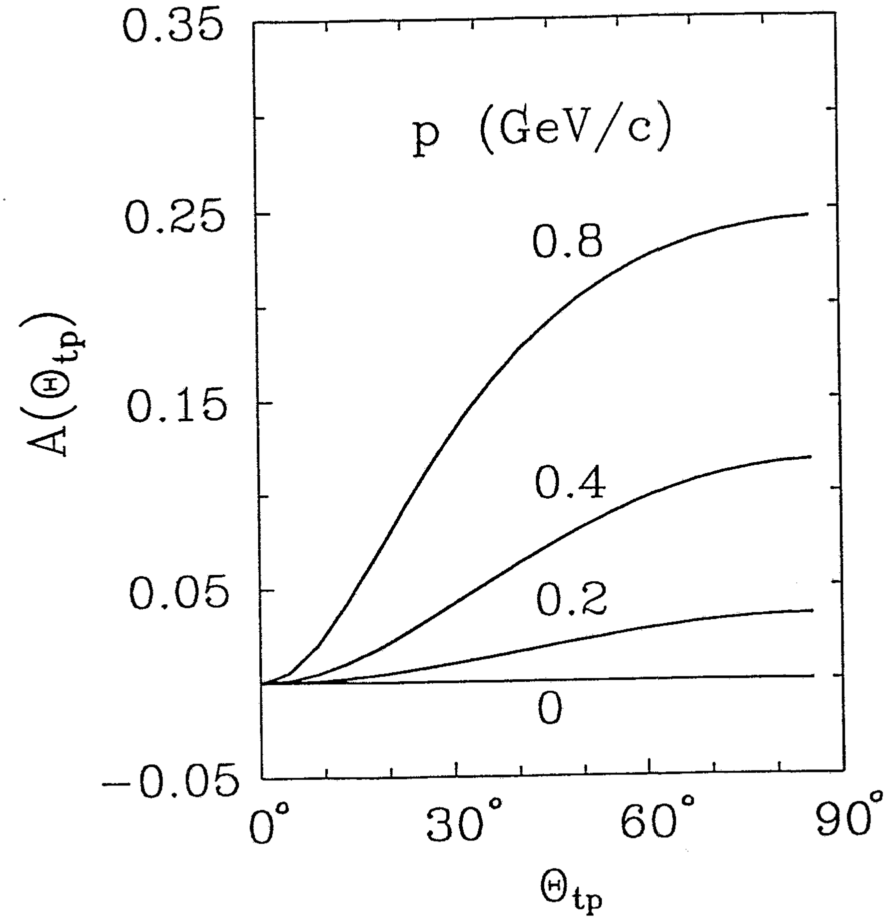

FIG. 2. The angular dependence of the asymmetry $A(\Theta_{\mathrm{tp}})$ for $M = 0.5$ GeV and for different values of $|\mathbf{p}|$ ($\mu_Q = 120$ MeV, $T = 150$ MeV).

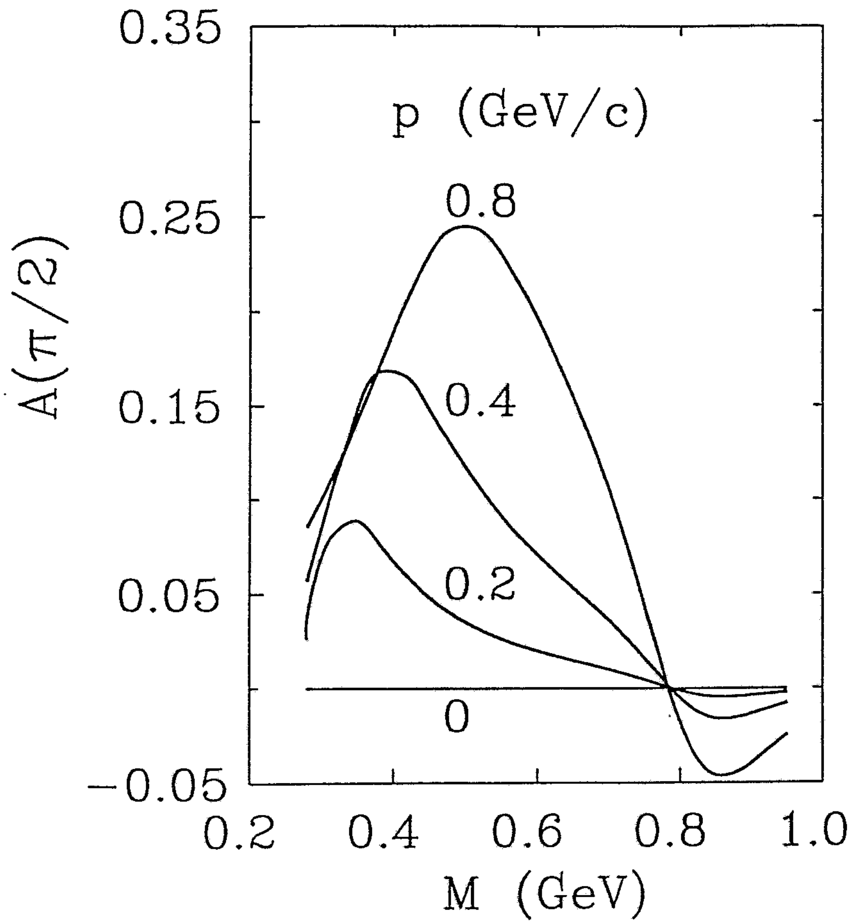

FIG. 3. The asymmetry $A(\frac{\pi}{2})$ as function of the invariant mass M for different values of $|\mathbf{p}|$ ($\mu_Q = 120$ MeV, $T = 150$ MeV).