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Effects of nuclear Coulomb field on two-meson correlations

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Abstract:

The influence of the nuclear Coulomb field on two-pion and two-kaon correlations is investigated for sources with charge number of Z = 160. The source radii extracted from the correlation function determined in sidewards and outwards direction are remarkably affected for meson pairs with average momenta below 200 MeV/c.

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I. Motivation

Measurements of correlations of pions and kaons as a function of their relative momenta are used to determine the source radius in heavy ion reactions at intermediate and relativistic energies. On the basis of the Hanbury-Brown and Twiss effect(see [1, 2]) the assumption is made that the pions propagate freely as soon as the leave the source region. While this might be correct for the strong interaction the long range Coulomb force still affects the motion of the pions outside the source. This could become important if heaviest nuclei are used as target and projectile forming a source with a common charge of about Z=160. We are aiming to investigate this effect of the Coulomb field on the correlation for positively and negatively charged pions and kaons.

II. Basic equations

The basic assumptions for applying interferometry to nuclei is the chaoticity of the source and the absence of initial correlations in the source of the observed pions. This allows one to express the two-particle density-matrix by the one-particle density matrices $\rho(p, p')$, where p denotes the four-momentum $p = (\omega, \boldsymbol{p})$. Following the derivation of Pratt [3, 4] one can write the probability to observe two mesons with momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ as proportional to

$$F(\boldsymbol{K}, \boldsymbol{q}) = \int\int dp dp' \phi^*_{\boldsymbol{q}}(\boldsymbol{p})\phi_{\boldsymbol{q}}(\boldsymbol{p}') \Big[\rho(K - \frac{p}{2}, K - \frac{p'}{2}) \times \rho(K + \frac{p}{2}, K + \frac{p'}{2}) + \rho(K - \frac{p}{2}, K + \frac{p'}{2})\rho(K + \frac{p}{2}, K - \frac{p'}{2})\Big], \quad (1)$$

where $\boldsymbol{K} = \frac{1}{2}(\boldsymbol{k}_1 + \boldsymbol{k}_2)$ is half the total momentum of the pair and $\boldsymbol{q} = \boldsymbol{k}_1 - \boldsymbol{k}_2$ is its relative momentum. The function $\phi_{\boldsymbol{q}}(\boldsymbol{p})$ is the Fourier transformed wave function describing the relative motion of the two mesons with relative momentum $\boldsymbol{q}$. It contains the final state interaction of the pair and depends only on the relative momentum.

Eq. (1) is not calculable in the general case without introducing further approximations. For charged pions the strong interaction in the isospin 2 channel is weak and the same holds for the charged kaons in the $|S| = 2$ channel, since no deep lying resonance states are known. So, one replaces the two-particle wave functions by plane waves $g_{\boldsymbol{q}}(\boldsymbol{p}) = \sqrt{P}\delta(\boldsymbol{q}-\boldsymbol{p})$, where P is the Coulomb penetrability $P = 2\pi\eta/(exp(2\pi\eta)-1), \eta = e^2m/(2q)$. The symbols m and e denote the particle mass and charge, respectively, and the convention $\hbar = c = 1$ is used. For an improved evaluation of the penetrability we refer to [5].

The one-particle density-matrix is connected with the the source function which describes the probability that a meson with momentum P is emitted from the position $\boldsymbol{r}$ at time t:

$$\rho(p,p') = \int d^4x g(x,P)\psi_p^*(x)\psi_{p'}(x), \tag{2}$$

where $x = (t,\boldsymbol{r})$. Here, $p = (\omega,\boldsymbol{p})$ is taken on shell, $\omega = \sqrt{m^2+\boldsymbol{p}^2}$, while $P = (p+p')/2 = ((\omega+\omega')/2, (\boldsymbol{p}+\boldsymbol{p}')/2)$ is slightly off-shell. The wave function ψ_p describes the motion of the particle with respect to the source. We assume that the source expands slowly so that the emitted particles are essentially outside the source. This allows us to use stationary solutions for $\psi_p = exp(-i\omega t)\,\psi_{\boldsymbol{p}}$. Now, using a standard source with a Gaussian distribution $g = \exp\left(-t^2/2\tau^2\right)S(\boldsymbol{r})$ of the emission points which is separable in time and position space one obtains

$$\rho(p,p') = e^{-(\omega+\omega')/2T}\, e^{-(\omega-\omega')^2\tau^2/2} \int d\boldsymbol{r}\, S(\boldsymbol{r})\psi_{\boldsymbol{p}}^*(\boldsymbol{r})\psi_{\boldsymbol{p}'}(\boldsymbol{r}) \tag{3}$$

with τ being the emission time. The Boltzmann factor comes from the assumption of a canonical ensemble characterized by a temperature T, a recent foundation can be found in Ref. [6]. The ratio of Eq.(1) to the uncorrelated part $\rho(k_1,k_1)\rho(k_2,k_2)$ defines the correlation function C_2 which reads with the above approximations

$$C_2(K,q) = P\left(1 + \frac{|\rho(K-\frac{q}{2},K+\frac{q}{2})|^2}{\rho(K-\frac{q}{2},K-\frac{q}{2})\,\rho(K+\frac{q}{2},K+\frac{q}{2})}\right). \tag{4}$$

If the emitted particles move freely as plane waves the correlation is given by the expression $C_2 = P\,(1 + exp(-\boldsymbol{q}^2R_0^2 - (\omega-\omega')^2\tau^2)$ for the Gaussian source distribution $S = exp(-\boldsymbol{r}^2/(2R_0^2))$.

However, we aim to investigate the influence of the Coulomb field of the source on the correlation function C_2. In this case we expect that the correlation depends also on momentum K even for vanishing emission time τ. Thus, we investigate the correlation as a function of the momentum $\boldsymbol{q}_{out}$ pointing in the direction of $\boldsymbol{K}$ and momentum $\boldsymbol{q}_{side}$ being orthogonal to $\boldsymbol{K}$, respectively, and the obtained function is fitted to the common expression [7]

$$C_2 = 1 + exp(-\boldsymbol{q}_{side}^2 R_{side}^2 - \boldsymbol{q}_{out}^2 R_{out}^2). \tag{5}$$

To incorporate the Coulomb field we solve the Klein-Gordon equation

$$\left(-\frac{\partial^2}{\partial \boldsymbol{r}^2} - (\sqrt{m^2 + \boldsymbol{p}^2} - U)^2 + m^2\right)\psi_{\boldsymbol{p}}(\boldsymbol{r}) = 0, \tag{6}$$

where U means the Coulomb potential of the source with charge number Z. For the Gaussian source we take the potential $U = \pm Ze^2\,\Phi(r/\sqrt{2}R_0)/r$, with Φ being the error function. The solution is obtained numerically by using partial-wave expansion technique. The boundary conditions are chosen such that ψ behaves asymptotically like an outgoing Coulomb wave.

III. Results

The correlation Eq. (4) has been calculated for various vectors $\boldsymbol{K}$ as a function of the relative momentum $\boldsymbol{q}$. The obtained correlation function has nearly a Gaussian shape and is fitted to Eq. (5) in the region of $C_2 = 1.5$. The extracted radii do not depend on the penetrability factor and the temperature entering the Boltzmann factor in Eq.(3). The results are demonstrated for a charge number of $Z = 160$ and the source radius R_0 has been chosen to be 4.5 fm. Such a value is compatible with recent analyses of one-pion spectra in collisions of gold and gold at an energy of one GeV per nucleon [8]. There, hard sphere radii R_h were obtained ranging from 7 fm to 13 fm for decreasing pion energies. Those radii relate to corresponding values R_0 via $R_h/\sqrt{5}$.

In Fig. 1 the fitted source radii R_{side} are shown in relation to the actual source radius R_0 as a function of half the total momentum $|\boldsymbol{K}|$. Compared to

R_0 the observed radii are increased for negatively charged pions and diminished for positively charged ones. However, these changes are significant only for pion momenta $|\boldsymbol{K}| < 200$ MeV/c. The effect is stronger for kaons than for pions at the same $\boldsymbol{K}$. As the energy reaches the threshold for positively charged particles the extracted radii start to increase.

The extracted outward radii are displayed in Fig. 2. Here, the effect of the Coulomb force is much weaker for large momenta $\boldsymbol{K}$ and opposite to that of the sideward correlation. Again, in the threshold region the effect becomes strongest and changes its direction. While the sideward correlation does not depend on the emission time the radius extracted for the outward correlation depends on the emission time in good approximation via $R_{out} = \sqrt{R_0^2 + (\tau \boldsymbol{K}/\omega(\boldsymbol{K}))^2}$. As an example the extracted radii for K^+ and K^- are included into Fig. 2 by the dotted and dash-dotted lines using τ= 7 fm/c . The effect of the emission time is small in the low momentum region.

The calculated change of the radii is roughly proportional to the charge number of the source. For a smaller source radius these changes become somewhat larger since the Coulomb field increases near the source. We have also investigated the effect of a source function g which is homogeneously distributed within a sphere with the same rms radius. It turns out that the result does not depend very sensitively on the shape as long as the charge and the source distribution match one another.

The qualitative behavior of the apparent radii can be understood in the spirit of the eikonal approach to the mesonic orbits. The interference is determined by the particle momenta $\tilde{\mathrm{p}}$ near the source. Thus the overlap integral in Eq. (3) is approximately a function of $R_0^2(\tilde{\mathrm{p}} - \tilde{\mathrm{p}}')^2 \approx R_{obs}^2(\mathrm{p} - \mathrm{p}')^2$. E. g. in the case of sideward correlation of two positively charged mesons, the observed relative momentum q is given by the opening angle and the asymptotic momentum p which is larger than $\tilde{\mathrm{p}}$. Thus, the observed relative momentum q is larger than $\tilde{q}$, and the deduced R_{obs} radius is therefore smaller than R_0. In addition, the deflection of the particles in the Coulomb field increases the opening angle and enhances the deviation of the deduced radius from R_0.

For the outward correlation **p** and **p'** are parallel to one another, therefore the ratio $R_{obs}/R_0 = \tilde{q}/q$ can be approximated for small differences q by $d\tilde{p}/dp = (1-\bar{U}/\sqrt{m^2+p^2})p/|\tilde{p}|$, where $\bar{U}$ stands for the average potential around the source. This ratio is larger than unity above the threshold. It becomes singular just at the threshold $\tilde{p} = 0$, and $\tilde{p}$ gets imaginary below. The more accurate numerical calculation produces a smooth maximum instead, as can be seen in Fig. 2 for positively charged pairs. The threshold effect in the sideward correlation originates essentially from the change of the direction if the damped waves penetrate into the region of high potential. Such a phenomenon is known in optics when light waves in an optically dense medium are totally reflected at the surface of a medium with a smaller index of refraction.

The behavior below the threshold energy of about 30 MeV (i.e. momenta of $|\mathbf{K}| = 100$ MeV/c for pions and 170 MeV/c for kaons) may not be accessible in experiment, since those mesons are slow and may stay for a long time within the expanding source. At the moment of their final release the Coulomb field has already become weaker.

In summary, we have found that the Coulomb field reduces (increases) the radii extracted from the sideward correlation for positively (negatively) charged pairs with small momenta. The effect is opposite and weaker for the outward correlation. Differences between correlations of positively and negatively charged mesons may also arise from the strong interaction within the source which was not considered here.

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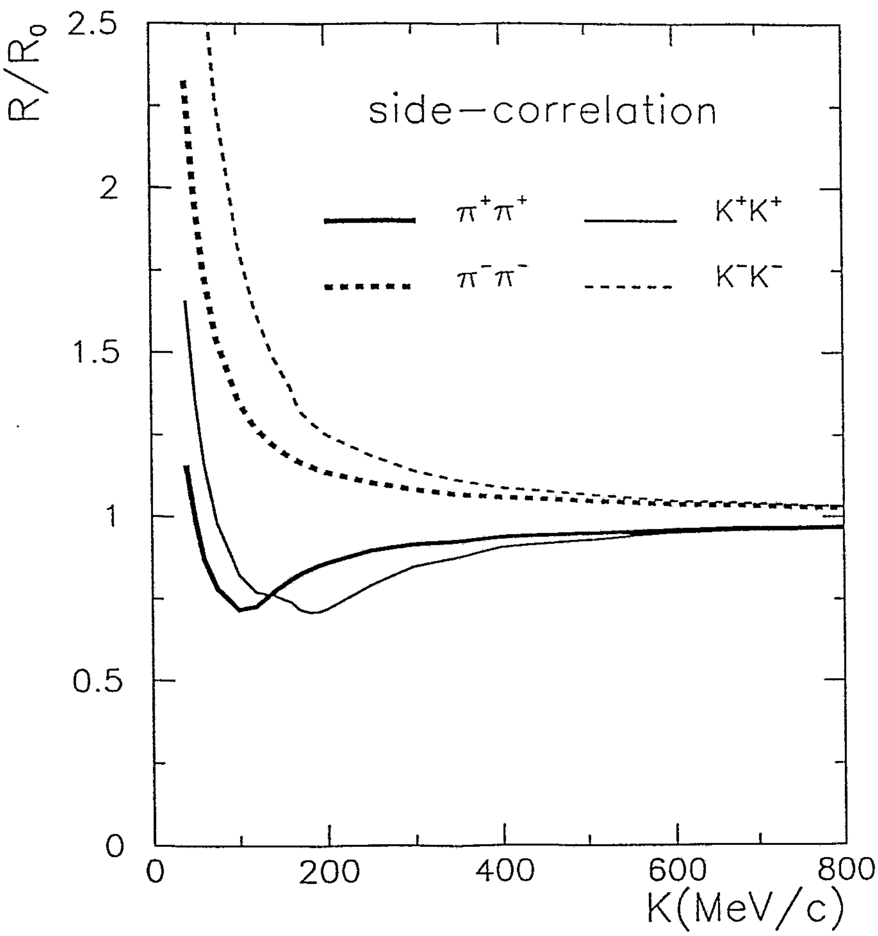

Fig. 1. Ratios of sidewards radii to the radius $R_0 = 4.5$ fm of a Gaussian shaped source extracted from sidewards correlations of a source with charge number 160 as a function of the average momentum K of the pair. Thick lines refer to charged pions while thin lines refer to kaons.

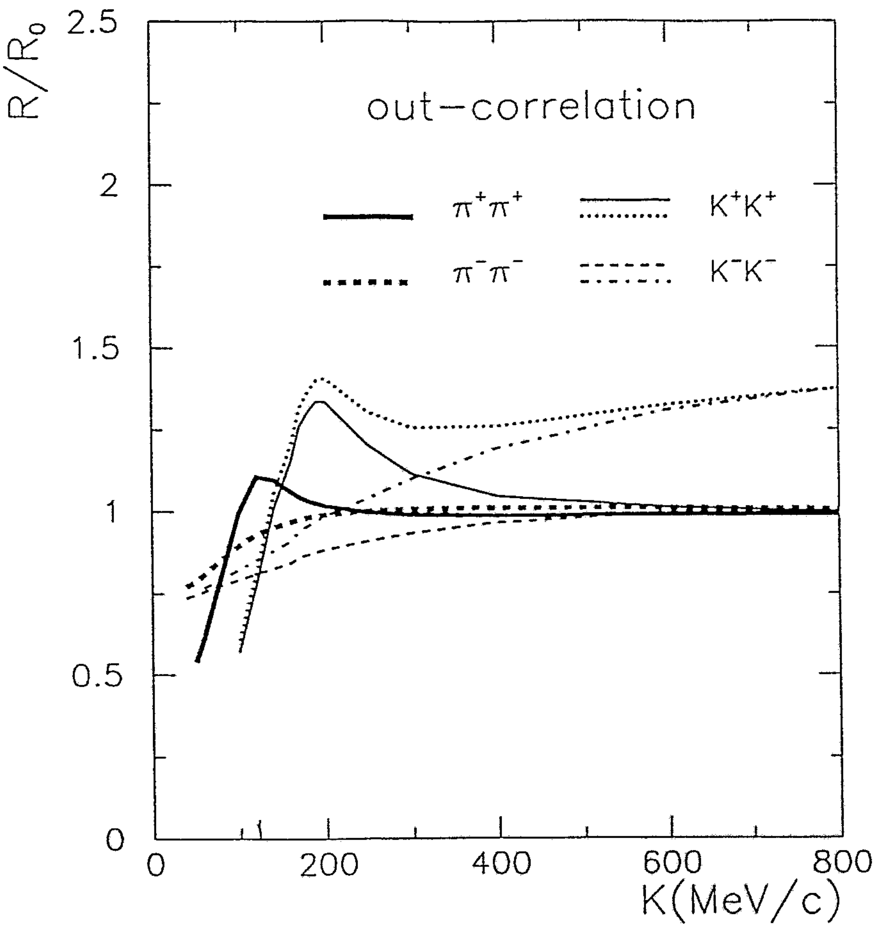

Fig. 2. Ratios of radii to the radius $R_0 = 4.5$ fm extracted from outwards correlations as a function of the average momentum K of the pair for pions (thick lines) and kaons (thin lines). The dotted and dash-dotted lines show the ratios for kaons emitted during a time of 7 fm/c while the other curves are calculated for a sudden break-up.