

Archiv-Ex.:

FZR-132

April 1996

Preprint

M. Schleif and R. Wunsch

Inertial Mass
of the Chiral Quark-Loop Soliton
in the Nambu & Jona-Lasinio Model
at Finite Temperature and Density

Forschungszentrum Rossendorf e.V.

Postfach 51 01 19 · D-01314 Dresden

Bundesrepublik Deutschland

Telefon (0351) 260 2209

Telefax (0351) 260 3700

E-Mail schleif@fz-rossendorf.de

Inertial Mass of the Chiral Quark–Loop Soliton in the Nambu & Jona-Lasinio Model at Finite Temperature and Density

M. Schleif and R. Wünsch

Institut für Kern- und Hadronenphysik, Forschungszentrum Rossendorf e.V.,
Postfach 51 01 19, D-01314 Dresden, Germany

Abstract

We consider the mass of the one-loop hedgehog soliton of the bosonized SU(2) Nambu & Jona-Lasinio model embedded in hot nuclear matter mimiced by a gas of constituent quarks. We prove that the proper-time regularized and self-consistently determined soliton in a heat bath obeys Poincare's invariance up order V^2 . At finite temperature and chemical potential, we show that the inertial mass obtained in the perturbative pushing approach coincides with the total internal energy of the soliton.

PACS numbers: 12.39.Fe, 11.10.Lm

1 Introduction

Chiral soliton models have proved to be a fruitful approach to the description of nucleon structure. Starting from isolated nucleons one has investigated the influence of a strongly interacting medium on the structure of the nucleon. Parameters of the nucleon characterizing its static properties and behavior in nuclear reactions have been calculated in dependence on temperature (T) and density (ρ) of the medium.

We consider a non-topological soliton which is defined by the Euclidean effective action of the bosonized Nambu & Jona-Lasinio (NJL) model restricted to the two lightest quarks with time-independent meson fields treated in mean-field approximation (MFA). The polarization of the quarks (fermion loop) is fully taken into account. The mesonic fields are restricted to hedgehog configurations and to the chiral circle. They are self-consistently determined by minimizing the corresponding effective action (Hartree approximation). For a detailed review of the model at $T=0$ and $g=0$ cf. refs. [1, 2].

Within an approach where the quarks are the fundamental degrees of freedom the simplest realization of a strongly interacting medium is a non-interacting gas of constituent quarks. On the first view such an approach seems to be reasonable only in the deconfined phase above the critical values of temperature and density. Nevertheless there are arguments in favor of such an approach for a soliton embedded in hadronic matter below the critical point. This becomes obvious if one considers the way how the medium influences the quarks in a soliton in detail.

The one-loop NJL-soliton is made out of valence quarks and an infinite number of sea quarks. The soliton is bound by meson fields which are generated by the quarks themselves. Meson and quark fields have to be consistent with each other. This is realized by the mesonic equation of motion which contains a source term produced by the quarks. The attractive part of the mean field in the center of the soliton stems from the valence quarks while the asymptotic value, which determines the constituent quark mass, is a result of the sea quarks as a whole. On the other hand the self-consistent meson field is solely able to bind N_c valence quarks. The sea quarks move almost freely. They are mainly influenced by the asymptotic value of the meson fields, which determines the constituent quark mass. Considering the medium as consisting of solitons the valence quarks in the various solitons are quite isolated from each other while the sea quarks can be found at any place approximately with the same probability. So it seems to be reasonable to assume that the thermodynamic equilibrium is established by sea quarks with constituent quark mass.

The rest mass is one of the most important parameters of the nucleon. Its variation in dependence on temperature and density of a surrounding medium is of fundamental interest. As well known the mean-field hedgehog configuration breaks translational as well as iso-rotational symmetry of the Lagrangian. These symmetries can approximately be restored *e. g.* within the semi-classical pushing and cranking approaches [3, 4]. Spurious motions of the soliton contribute to the soliton rest mass. They have to be removed. The size of the spurious contributions to the soliton mass is controlled by the inertial parameters. For an elementary particle the rest mass M_0 , which is defined as the total energy in the rest frame, is identical with the inertial mass M^* which describes its kinetic energy. Relativistic invariance states that the total energy $E(\mathbf{V})$ of a particle depends on its velocity \mathbf{V} according to

$$E(\mathbf{V}) = M_0/\sqrt{1-V^2} = M_0 + \frac{M^*}{2}V^2 + \mathcal{O}(V^4) \quad \text{with} \quad M_0 = M^*. \quad (1.1)$$

For a composite, extended particle this relation is by no means a matter of course, in particular if it is in a heat bath. In addition, the non-local nature of the quark determinant and its inevitable regularization intricate this problem.

The equivalence of rest mass and inertial mass for the one-loop NJL hedgehog soliton at $T=0$ and $\rho=0$ has been shown in refs. [5, 6]. To examine eq. (1.1) for finite values of temperature and density we expand the contribution of a single moving soliton to the internal energy with respect to its velocity \mathbf{V} up to second order. The energy at $\mathbf{V}=0$ is the internal energy of the soliton at rest and coincides with the rest mass ($E(0)=M_0$). The second-order term will determine the inertial mass M^* of the soliton and will be compared with the rest mass M_0 .

In sect. 2, we shortly review the formula defining the regularized NJL soliton at finite temperature and density and calculate its rest mass. In sect. 3, we consider a boosted soliton moving adiabatically through a medium of constituent quarks and expand its energy with respect to the boost velocity. The second order term defines the inertial soliton mass. Using the mesonic equation of motion and performing algebraic manipulations we show that the inertial mass coincides with the internal energy of the static, self-consistent hedgehog soliton. Conclusions are drawn in sect. 4.

2 NJL model at finite temperature

Starting point is the two-flavor NJL Lagrangian [7] with a chirally invariant non-linear quark-antiquark interaction part

$$\mathcal{L}_{NJL} = \bar{q} (i\gamma^\mu \partial_\mu - m_0) q + \frac{G}{2} [(\bar{q}q)^2 + (\bar{q}i\gamma_5 \hat{\tau} q)^2] \quad (2.1)$$

where q represents a Dirac quark field with two flavors (u,d) and $N_c=3$ colors. Here, the isospin operator $\hat{\tau}$ is given by the 2×2 Pauli matrices, and m_0 is the average current quark mass of the light quarks $m_0 = (m_u + m_d)/2$.

To study the effects of a surrounding medium we consider a grand canonical ensemble of u- and d-quarks with the chemical potential $\mu_u = \mu_d = \mu$ and the temperature $T=1/\beta$. The grand canonical partition function in imaginary-time path integral formulation is given by [8, 9]

$$\mathcal{Z}(T, \mu, \mathcal{V}) = \int_{q(\mathbf{x}, \tau=0)=-q(\mathbf{x}, \tau=\beta)} \mathcal{D}q \mathcal{D}q^\dagger \exp \left[-\mathcal{A}[q, q^\dagger](T, \mathcal{V}) + \mu \int_0^\beta d\tau \int_{\mathcal{V}} d^3\mathbf{x} q^\dagger q \right] \quad (2.2)$$

where the quark fields are anti-commuting complex Grassmann variables and satisfy anti-periodic boundary conditions at the imaginary times $\tau=0$ and $\tau=\beta$. The quantum statistical Euclidean action \mathcal{A} of the interacting quarks is determined by the Wick rotated ($t \rightarrow -i\tau$) Lagrangian (2.1) according to

$$\mathcal{A}[q, q^\dagger](T, \mathcal{V}) = - \int_0^\beta d\tau \int_{\mathcal{V}} d^3\mathbf{x} \mathcal{L}_{NJL}(\bar{q}(\mathbf{x}, \tau), q(\mathbf{x}, \tau)). \quad (2.3)$$

The grand canonical potential can be determined from the partition function

$$\Omega(T, \mu, \mathcal{V}) = -T \ln \mathcal{Z}(T, \mu, \mathcal{V}), \quad (2.4)$$

and the particle number N and the entropy S are given by

$$N(T, \mu, \mathcal{V}) = -\frac{\partial}{\partial \mu} \Omega(T, \mu, \mathcal{V}) \Big|_{T, \mathcal{V}} \quad S(T, \mu, \mathcal{V}) = -\frac{\partial}{\partial T} \Omega(T, \mu, \mathcal{V}) \Big|_{\mu, \mathcal{V}}. \quad (2.5)$$

The internal energy E is obtained from the relation

$$E(S, N, \mathcal{V}) = \Omega + TS + \mu N = \left[1 - T \frac{\partial}{\partial T} - \mu \frac{\partial}{\partial \mu} \right] \Omega = T \left[T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} \right] \ln \mathcal{Z}, \quad (2.6)$$

where the independent variables have been changed from T and μ to S and N by means of eqs. (2.5).

Because of the non-linear interaction terms in the NJL Lagrangian (2.1) the integration over the quark fields in eq. (2.2) can not be carried out. By means of scalar-isoscalar (σ) and pseudoscalar-isovector (π) mesonic auxiliary fields one can eliminate the quartic terms. Restricting the auxiliary fields onto their classical values (mean-field or stationary-phase approximation) the system of interacting quarks can be described by an effective action, which differs from the grand canonical potential only by a factor T . The resulting grand canonical potential is a functional of the classical meson fields and consists of a quark part Ω^q , where the mesons fields contribute only via the quark fields, and a purely mesonic part Ω^m

$$\Omega_{\text{MFA}}[\sigma_{\text{cl}}, \pi_{\text{cl}}] = \Omega^q[\sigma_{\text{cl}}, \pi_{\text{cl}}] + \Omega^m[\sigma_{\text{cl}}, \pi_{\text{cl}}]. \quad (2.7)$$

The classical fields σ_{cl} and π_{cl} have to fulfill the equations of motion

$$\frac{\delta \Omega_{\text{MFA}}}{\delta \sigma} \Big|_{\sigma=\sigma_{\text{cl}}, \pi=\pi_{\text{cl}}} = \frac{\delta \Omega_{\text{MFA}}}{\delta \pi} \Big|_{\sigma=\sigma_{\text{cl}}, \pi=\pi_{\text{cl}}} = 0. \quad (2.8)$$

From now on, we shall drop the indices cl and MFA and assume Ω , σ and π as to be determined by eqs. (2.7) and (2.8). The solitonic configuration we are interested in constitutes a localized deviation from the homogeneous background field $\sigma = \sigma_0 = \text{const}$, $\pi = \mathbf{0}$. To get the canonical potential characterizing the soliton we have to subtract the potential of the background field

$$\Omega[\sigma, \pi] \longrightarrow \Omega[\sigma, \pi] - \Omega[\sigma_0, \mathbf{0}] = \Omega[\sigma, \pi] - \{h \rightarrow h_0\}. \quad (2.9)$$

Since the meson fields enters the canonical potential via the quark Hamiltonian

$$h = h(\sigma, \pi) = \boldsymbol{\alpha} \cdot \mathbf{p} + \gamma^0 [\sigma(\mathbf{x}) + i\gamma_5 \hat{\tau} \cdot \boldsymbol{\pi}(\mathbf{x})] \quad (2.10)$$

we have introduced the short notation $-\{h \rightarrow h_0\}$ for the subtraction of the corresponding background value calculated with

$$h_0 \equiv h(\sigma_0, \mathbf{0}) = \boldsymbol{\alpha} \cdot \mathbf{p} + \gamma^0 \sigma_0. \quad (2.11)$$

Furthermore we restrict ourselves to time-independent spherical hedgehog meson fields with

$$\sigma(\mathbf{x}, \tau) = \sigma(r) \quad \boldsymbol{\pi}(\mathbf{x}, \tau) = \pi(r) \mathbf{x}/|\mathbf{x}| \quad (2.12)$$

and to the chiral circle

$$\sigma^2 + \pi^2 = \sigma_0^2 = \text{const}. \quad (2.13)$$

The quark contribution to the grand canonical potential (2.7) of the soliton can be written

$$\Omega^q[\sigma, \pi] = -T \ln \text{Det} D(\mu) - \{h \rightarrow h_0\} = -T \text{Tr} \ln D(\mu) - \{h \rightarrow h_0\} \quad (2.14)$$

with

$$D(\mu) = \partial_\tau + h - \mu. \quad (2.15)$$

The mesonic part of the potential (2.7) is given by

$$\Omega^m[\sigma] = \frac{m_0}{G} \int d^3 \mathbf{x} (\sigma_0 - \sigma(\mathbf{x})). \quad (2.16)$$

While the mesonic part (2.16) is local and has the familiar appearance of an action the quark part (2.14) is non-local. (The trace Tr includes both functional and matrix (Dirac, flavor, color) trace.) For time-independent meson fields the determinant in eq. (2.14) is real and one gets

$$\Omega^q = -\frac{T}{2} \text{Tr} \ln A(\mu) - \{h \rightarrow h_0\} = -\frac{T}{2} \ln \text{Det} [-\partial_\tau^2 + (h - \mu)^2] - \{h \rightarrow h_0\}, \quad (2.17)$$

where we have introduced the operator

$$A(\mu) \equiv D(\mu)^\dagger D(\mu) = -\partial_\tau^2 + (h - \mu)^2. \quad (2.18)$$

The fermion determinant can be expressed by odd Matsubara frequencies $\omega_n = (2n + 1)\pi T$ [10] and by the eigenvalues ε_α and ε_α^0 of the Hamiltonian h (2.10) and h_0 (2.11), respectively

$$\Omega^q = -\frac{T}{2} N_c \ln \prod_{n=-\infty}^{\infty} \prod_{\alpha} [\omega_n^2 + (\varepsilon_\alpha - \mu)^2] - \{\varepsilon_\alpha \rightarrow \varepsilon_\alpha^0\}. \quad (2.19)$$

Lagrangian (2.1) and Hamiltonian (2.10) are independent of the color degree of freedom giving rise to a general factor N_c . We shall treat this factor explicitly and products (\prod_α) or sums (\sum_α) will not include the color degree of freedom.

The product in eq. (2.19) can be evaluated and written as the sum of two components

$$\Omega^q = \Omega^{q,\text{sea}} + \Omega^{q,\text{med}}. \quad (2.20)$$

The first term is independent of the actual occupation numbers of the various quark levels. It is the only quark contribution which survives in the limit $T \rightarrow 0, \mu \rightarrow 0$. It describes the contribution of the Dirac sea and consists of the difference between the sum of single-particle quark energies with and without the soliton (Casimir energy)

$$\Omega^{q,\text{sea}} \equiv \lim_{\substack{\mu=0 \\ \beta \rightarrow \infty}} \Omega^q = -\frac{T}{2} \text{Tr}_0 \ln A(0) - \{h \rightarrow h_0\} = -\frac{N_c}{2} \sum_{\alpha} [|\varepsilon_\alpha| - |\varepsilon_\alpha^0|]. \quad (2.21)$$

The trace Tr_0 denotes the trace at $T=0$ where a sum over the Matsubara frequencies ω_n can be replaced by an integral ($\omega_n \rightarrow \omega, T \sum_{n=-\infty}^{\infty} \rightarrow \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}$). It is ultraviolet divergent and has to be regularized introducing a cut-off parameter Λ . We use Schwinger's proper-time scheme [11] and get [12]

$$\begin{aligned} \Omega^{q,\text{sea}} \longrightarrow \Omega_{\Lambda}^{q,\text{sea}} &= \frac{T}{2} \text{Tr}_0 \int_{1/\Lambda^2}^{\infty} ds s^{-1} e^{-sA(0)} - \{h \rightarrow h_0\} \\ &= -\frac{N_c}{2} \sum_{\alpha} [R(\varepsilon_\alpha, \Lambda) |\varepsilon_\alpha| - R(\varepsilon_\alpha^0, \Lambda) |\varepsilon_\alpha^0|], \end{aligned} \quad (2.22)$$

where $R(\varepsilon, \Lambda)$ is the regularization function

$$R(\varepsilon; \Lambda) = \frac{-1}{\sqrt{4\pi}|\varepsilon|} \int_{1/\Lambda^2}^{\infty} dt t^{-3/2} e^{-\varepsilon^2 t} = \frac{-1}{\sqrt{4\pi}} \Gamma\left(-\frac{1}{2}, \frac{\varepsilon^2}{\Lambda^2}\right) \quad (2.23)$$

with the incomplete Gamma-function on the right-hand side. It cuts off contributions with $|\varepsilon| \gg \Lambda$. The term $\Omega_{\Lambda}^{\text{q,sea}}$ does not explicitly depend on temperature and chemical potential. An implicit dependence on T and μ is caused by the restriction of the meson fields to their classical values via the equation of motion (2.8), which evidently depends on T and μ .

The second, explicitly T and μ dependent term in eq. (2.20)

$$\begin{aligned} \Omega^{\text{q,med}} &\equiv \Omega^{\text{q}} - \Omega^{\text{q,sea}} = -\frac{T}{2} [\text{Tr} \ln A(\mu) - \text{Tr}_0 \ln A(0)] - \{h \rightarrow h_0\} \\ &= -TN_c \sum_{\alpha} \ln \frac{1 + \exp\{-\beta [|\varepsilon_{\alpha}| - \mu \text{sign}(\varepsilon_{\alpha})]\}}{1 + \exp\{-\beta [|\varepsilon_{\alpha}^0| - \mu \text{sign}(\varepsilon_{\alpha}^0)]\}} - \mu N_c B^{\text{sea}} \end{aligned} \quad (2.24)$$

is finite and depends on the occupation probability of the various levels α . It describes the polarization of the medium due to the solitonic meson fields. Levels which are not shifted by the solitonic field ($\varepsilon_{\alpha} \approx \varepsilon_{\alpha}^0$), *i. e.* levels highly and deeply in the continuum do not contribute to the sum (2.24). The largest contributions result from the valence level which is bound for the soliton and unbound for the pure medium. Quark levels in the neighborhood of the gap in the spectrum do also remarkably contribute. The term $-\mu N_c B^{\text{sea}}$ differs from zero only in the particular case that the Dirac sea has a finite baryon number

$$B^{\text{sea}} = \frac{1}{2} \left[\sum_{\varepsilon_{\alpha} < 0} - \sum_{\varepsilon_{\alpha} > 0} \right]. \quad (2.25)$$

This happens if the solitonic mean field is strong enough to change the sign of the energy of one or several quark levels, *i. e.* $\exists \alpha$ with $\text{sign}(\varepsilon_{\alpha}) \neq \text{sign}(\varepsilon_{\alpha}^0)$. Otherwise the number of quark levels with negative energy equals the number with positive energy and B^{sea} vanishes.

Using eqs. (2.6-2.8, 2.14, 2.16) and (2.20-2.24) one gets the following internal energy of the soliton

$$E \equiv M_0 = E_{\Lambda}^{\text{sea}} + E^{\text{med}} + \Omega^{\text{m}}, \quad (2.26)$$

which is equivalent to the soliton rest mass M_0 . It consists of the (regularized) Casimir energy

$$E_{\Lambda}^{\text{sea}} = \Omega_{\Lambda}^{\text{q,sea}} \quad (2.27)$$

the medium-polarization energy

$$E^{\text{med}} = N_c \sum_{\alpha} \tilde{n}(\varepsilon_{\alpha}; T, \mu) |\varepsilon_{\alpha}| - \{\varepsilon_{\alpha} \rightarrow \varepsilon_{\alpha}^0\} \quad (2.28)$$

with the typical fermionic occupation numbers

$$\tilde{n}(\varepsilon; T, \mu) = \frac{1}{1 + \exp\{\beta [|\varepsilon| - \mu \text{sign}(\varepsilon)]\}} = \frac{1}{1 + \exp\{\beta [\varepsilon - \mu]\}} - \Theta(-\varepsilon) \quad (2.29)$$

for quarks ($\varepsilon > 0$) and antiquarks ($\varepsilon < 0$), and of the purely mesonic energy Ω^{m} (2.16).

3 Pushed soliton and inertial mass

Now we consider the adiabatic motion of a soliton with fixed particle number N through a medium with a constant velocity \mathbf{V} . For non-relativistic velocities \mathbf{V} , the grand canonical potential $\Omega(\mathbf{V})$ of the moving soliton is given by same expressions (2.7), which was obtained for a soliton at rest, with the Hamiltonian h (2.10) replaced by the shifted Hamiltonian

$$h(\mathbf{V}) = h - \mathbf{V} \cdot \mathbf{p}. \quad (3.1)$$

Working in Euclidean space the shift velocity has to be anti-hermitian. Eq. (3.1) is the pushing analog to the cranking procedure considered in ref. [12]. The term $\mathbf{V} \cdot \mathbf{p}$ acts on the quark fields in the co-moving system like an induced external field. The velocity \mathbf{V} can be regarded as a Lagrange multiplier fixing the expectation value of the total soliton momentum \mathbf{P} , which can be represented as

$$\langle P^i \rangle(\mathbf{V}) = -\frac{\partial \Omega(\mathbf{V})}{\partial V^i}. \quad (3.2)$$

Now we expand the energy of the moving soliton, which is a function of S, N, \mathcal{V} and of the velocity \mathbf{V} at fixed values of S, N and \mathcal{V} , up to second order in the velocity \mathbf{V}

$$E(\mathbf{V}) = E(0) + \left. \frac{\partial E(\mathbf{V})}{\partial V^i} \right|_{\mathbf{V}=0} V^i + \frac{1}{2} \left. \frac{\partial^2 E(\mathbf{V})}{\partial V^i \partial V^k} \right|_{\mathbf{V}=0} V^i V^k + \dots \quad (3.3)$$

The first term on the right-hand side is the internal energy (2.26) of the soliton and the third term determines the inertial mass tensor. Derivatives of $E(S, N, \mathcal{V}; \mathbf{V})$ with respect to \mathbf{V} at fixed S and N can be replaced by derivatives of the canonical potential $\Omega(T, \mu, \mathcal{V}; \mathbf{V})$ of the moving soliton at fixed T and μ

$$\frac{\partial E(S, N, \mathcal{V}; \mathbf{V})}{\partial V^i} = \frac{\partial \Omega(T, \mu, \mathcal{V}; \mathbf{V})}{\partial V^i} \quad (3.4)$$

and we get

$$M_{ik}^* \equiv - \left. \frac{\partial^2 E(\mathbf{V})}{\partial V^i \partial V^k} \right|_{\mathbf{V}=0} = - \left. \frac{\partial^2 \Omega(\mathbf{V})}{\partial V^i \partial V^k} \right|_{\mathbf{V}=0} = \frac{\partial}{\partial V^i} \langle P^k \rangle(\mathbf{V}) \Big|_{\mathbf{V}=0}. \quad (3.5)$$

The minus sign results from the anti-hermitian property of the boost velocity with $\mathbf{V}^2 < 0$. The linear term in eq. (3.3) vanishes since the expectation value (3.2) equals zero at $\mathbf{V} = 0$, in accordance with Ehrenfest's theorem.

Considering rotationally symmetric solitons the tensor M_{ik}^* is diagonal with identical diagonal elements M^*

$$M_{ik}^* = M^* \delta_{ik}. \quad (3.6)$$

Evaluating M_{ik}^* according to eq. (3.5) we have to consider only those contributions to the grand canonical potential $\Omega(\mathbf{V})$ which depend via $h(\mathbf{V})$ on the velocity \mathbf{V} . Only sea-quark (2.22) and medium part (2.24) contribute to the mass tensor (3.5)

$$M_{ik}^* = (M_{\text{sea}}^*)_{ik} + (M_{\text{med}}^*)_{ik} \quad (3.7)$$

with

$$(M_{\text{sea}}^*)_{ik} = - \left. \frac{\partial^2 \Omega_{\Lambda}^{\text{q,sea}}(\mathbf{V})}{\partial V^i \partial V^k} \right|_{\mathbf{V}=0} = - \frac{T}{2} \text{Tr}_0 \int_{1/\Lambda^2}^{\infty} ds s^{-1} \left. \frac{\partial^2}{\partial V^i \partial V^k} e^{-sA(\mathbf{V})} \right|_{\mathbf{V}=0} - \{h \rightarrow h_0\} \quad (3.8)$$

and

$$(M_{\text{med}}^*)_{ik} = - \left. \frac{\partial^2 \Omega_{\text{q,med}}(\mathbf{V})}{\partial V^i \partial V^k} \right|_{\mathbf{V}=0} \quad (3.9)$$

$$= \frac{T}{2} \text{Tr} \left. \frac{\partial^2}{\partial V^i \partial V^k} \ln A^{\mathbf{V}}(\mu) \right|_{\mathbf{V}=0} - \{h \rightarrow h_0\} - \frac{T}{2} \text{Tr}_0 \left. \frac{\partial^2}{\partial V^i \partial V^k} \ln A^{\mathbf{V}}(0) \right|_{\mathbf{V}=0} - \{h \rightarrow h_0\},$$

where we have introduced the shifted operators

$$A^{\mathbf{V}}(\mu) \equiv D^{\mathbf{V}}(\mu)^\dagger D^{\mathbf{V}}(\mu) = A(\mu) + B^i V^i - (\mathbf{V} \cdot \mathbf{p})^2 \quad (3.10)$$

with

$$D^{\mathbf{V}}(\mu) = D(\mu) - \mathbf{V} \cdot \mathbf{p} = \partial_\tau + h - \mu - \mathbf{V} \cdot \mathbf{p}. \quad (3.11)$$

The quantity $A(\mu)$ is defined in eq. (2.18) and

$$B^i = \left. \frac{\partial}{\partial V^i} A^{\mathbf{V}}(\mu) \right|_{\mathbf{V}=0} = p^i D(\mu) - D(\mu)^\dagger p^i = 2p^i \partial_\tau - [h, p^i] \quad (3.12)$$

is independent of the chemical potential μ . The commutator $[h, p^i]$ is determined by the derivative of the mean field

$$[h, p^i] = i\gamma^0 \partial_i [\sigma(\mathbf{x}) + i\gamma_5 \hat{\tau} \cdot \boldsymbol{\pi}(\mathbf{x})]. \quad (3.13)$$

Using the commutator representation

$$B^i = [C^i, A(0)] = [C^i, A(\mu) + 2\mu h] \quad (3.14)$$

with

$$C^i = \frac{\alpha^i}{2} - ix^i \partial_\tau \quad (3.15)$$

the masses (3.8) and (3.9) can be written

$$(M_{\text{sea}}^*)_{ik} = -T \text{Tr}_0 \int_{1/\Lambda^2}^{\infty} ds e^{-sA(0)} \left(p^i p^k + \frac{1}{2} [C^i, B^k] \right) - \{h \rightarrow h_0\} \quad (3.16)$$

and

$$(M_{\text{med}}^*)_{ik} = -T \text{Tr} \left[A(\mu)^{-1} \left(p^i p^k + \frac{1}{2} [C^i, B^k] \right) \right] - \{h \rightarrow h_0\}$$

$$+ T \text{Tr}_0 \left[A(0)^{-1} \left(p^i p^k + \frac{1}{2} [C^i, B^k] \right) \right] - \{h \rightarrow h_0\} \quad (3.17)$$

$$- T \mu \text{Tr} \left[A(\mu)^{-1} \left((h - \mu) \delta^{ik} + ix^i [h, p^k] \right) \right] - \{h \rightarrow h_0\}$$

with

$$[C^i, B^k] = 2\delta^{ik} \partial_\tau^2 + i\gamma^i \partial_k (\sigma + i\gamma_5 \hat{\tau} \cdot \boldsymbol{\pi}). \quad (3.18)$$

Details of the calculation can be found in the appendix. The diagonal elements (3.6) of the tensors (3.16, 3.17) are given by

$$M_{\text{sea}}^* = -\frac{T}{3} \text{Tr}_0 \int_{1/\Lambda^2}^{\infty} ds e^{-sA(0)} \left(\mathbf{p}^2 + 3\partial_\tau^2 + \frac{i}{2} \boldsymbol{\gamma} \cdot \nabla (\sigma + i\gamma_5 \hat{\boldsymbol{\tau}} \cdot \boldsymbol{\pi}) \right) - \{h \rightarrow h_0\} \quad (3.19)$$

$$\begin{aligned} M_{\text{med}}^* &= -\frac{T}{3} \text{Tr} \left[A(\mu)^{-1} \left(\mathbf{p}^2 + 3\partial_\tau^2 + \frac{i}{2} \boldsymbol{\gamma} \cdot \nabla (\sigma + i\gamma_5 \hat{\boldsymbol{\tau}} \cdot \boldsymbol{\pi}) \right) \right] - \{h \rightarrow h_0\} \\ &+ \frac{T}{3} \text{Tr}_0 \left[A(0)^{-1} \left(\mathbf{p}^2 + 3\partial_\tau^2 + \frac{i}{2} \boldsymbol{\gamma} \cdot \nabla (\sigma + i\gamma_5 \hat{\boldsymbol{\tau}} \cdot \boldsymbol{\pi}) \right) \right] - \{h \rightarrow h_0\} \\ &- \frac{T}{3} \mu \text{Tr} \left[A(\mu)^{-1} \left(3(h - \mu) + i\mathbf{x}^k [h, p^k] \right) \right] - \{h \rightarrow h_0\}. \end{aligned} \quad (3.20)$$

Now we make use of the fact that any variation of the canonical potential Ω (2.7) with respect to the meson fields has to vanish around the stationary point

$$\delta\Omega = \delta\Omega_\Lambda^{\text{q,sea}} + \delta\Omega_\Lambda^{\text{q,med}} + \delta\Omega^{\text{m}} = 0. \quad (3.21)$$

A variation which is in accordance with the restrictions to spherically symmetric hedgehog fields and to the chiral circle (2.13) respecting the boundary conditions $\delta\sigma=0, \delta\boldsymbol{\pi}=0$ for $|\mathbf{x}| \rightarrow 0, \infty$ is given by

$$\delta\sigma = \epsilon x^k \partial_k \sigma \quad \text{and} \quad \delta\boldsymbol{\pi} = \epsilon x^k \partial_k \boldsymbol{\pi} \quad (3.22)$$

with an infinitesimal variation parameter ϵ . The corresponding variation of the meson contribution (2.16) to Ω is given by

$$\delta\Omega^{\text{m}}/\epsilon = -\frac{m_0}{G} \int_{\mathcal{V}} d^3\mathbf{x} \delta\sigma(\mathbf{x})/\epsilon = -\frac{m_0}{G} \int_{\mathcal{V}} d^3\mathbf{x} x^k \partial_k \sigma = 3\frac{m_0}{G} \int_{\mathcal{V}} d^3\mathbf{x} (\sigma - \sigma_0) = -3\Omega^{\text{m}}. \quad (3.23)$$

The quark contribution depends on the meson fields via Hamiltonian (2.10) and the variation is given by

$$\delta\Omega_\Lambda^{\text{q,sea}} = -\frac{T}{2} \text{Tr}_0 \int_{1/\Lambda^2}^{\infty} ds e^{-sA(0)} \delta h^2 \quad (3.24)$$

and

$$\delta\Omega^{\text{q,med}} = -\frac{T}{2} \text{Tr} \left[A(\mu)^{-1} \delta(h - \mu)^2 \right] + \frac{T}{2} \text{Tr}_0 \left[A(0)^{-1} \delta h^2 \right] \quad (3.25)$$

with

$$\begin{aligned} \delta h/\epsilon &= \gamma^0 (\delta\sigma + i\gamma_5 \hat{\boldsymbol{\tau}} \cdot \delta\boldsymbol{\pi})/\epsilon = \gamma^0 \mathbf{x} \cdot \nabla (\sigma + i\gamma_5 \hat{\boldsymbol{\tau}} \cdot \boldsymbol{\pi}) = -i\mathbf{x}^k [h, p^k] \\ &= \boldsymbol{\alpha} \cdot \mathbf{p} - i[h, \mathbf{x} \cdot \mathbf{p}], \end{aligned} \quad (3.26)$$

$$\delta h^2/\epsilon = \{h, \delta h\}/\epsilon = 2\mathbf{p}^2 + i\boldsymbol{\gamma} \cdot \nabla (\sigma + i\gamma_5 \hat{\boldsymbol{\tau}} \cdot \boldsymbol{\pi}) - i[h^2, \mathbf{x} \cdot \mathbf{p}], \quad (3.27)$$

$$\delta(h - \mu)^2 = \delta h^2 - 2\mu \delta h. \quad (3.28)$$

Introducing δh and δh^2 into eqs. (3.19) and (3.20) we get with the help of eq. (A.12)

$$M_{\text{sea}}^* = -T \text{Tr}_0 \int_{1/\Lambda^2}^{\infty} ds e^{-sA(0)} \left(\partial_\tau^2 + \delta h^2/6\epsilon \right) - \{h \rightarrow h_0\} \quad (3.29)$$

and

$$M_{\text{med}}^* = -T\text{Tr} \left[A(\mu)^{-1} \left(\partial_\tau^2 + \mu(h - \mu) + \delta(h - \mu)^2/6\epsilon \right) \right] - \{h \rightarrow h_0\} \\ + T\text{Tr}_0 \left[A(0)^{-1} \left(\partial_\tau^2 + \delta h^2/6\epsilon \right) \right] - \{h \rightarrow h_0\}. \quad (3.30)$$

Finally we exploit the equation of motion (3.21) with the variations (3.23-3.25) and get

$$M^* = M_{\text{sea}}^* + M_{\text{med}}^* = -T\text{Tr}_0 \int_{1/\Lambda^2}^{\infty} ds e^{-sA(0)} \partial_\tau^2 - \{h \rightarrow h_0\} \quad (3.31) \\ + T\text{Tr}_0 \left[A(0)^{-1} \partial_\tau^2 \right] - \{h \rightarrow h_0\} - T\text{Tr} \left[A(\mu)^{-1} \left(\partial_\tau^2 + \mu(h - \mu) \right) \right] - \{h \rightarrow h_0\} + \Omega^m.$$

The trace with the anti-periodic boundary conditions can be expressed by an integral (sum) over Matsubara frequencies ω (ω_n) and by a sum over the eigenvalues ε_α and ε_α^0 of the Hamiltonians h and h_0

$$M^* = N_c \sum_\alpha \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{1/\Lambda^2}^{\infty} ds e^{-s(\omega^2 + \varepsilon_\alpha^2)} \omega^2 - \{\varepsilon_\alpha \rightarrow \varepsilon_\alpha^0\} - N_c \sum_\alpha \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\omega^2}{\omega^2 + \varepsilon_\alpha^2} - \{\varepsilon_\alpha \rightarrow \varepsilon_\alpha^0\} \\ + N_c T \sum_\alpha \sum_{n=-\infty}^{\infty} \frac{\omega_n^2 - \mu(\varepsilon_\alpha - \mu)}{\omega_n^2 + (\varepsilon_\alpha - \mu)^2} - \{\varepsilon_\alpha \rightarrow \varepsilon_\alpha^0\} + \Omega^m. \quad (3.32)$$

Performing integration (summation) over $d\omega$ (ω_n) the first term gives the sea energy (2.27) while second and third term add up to the medium-polarization energy (2.28). Altogether we can conclude that the inertial mass M^* equals to its total internal energy E (2.26)

$$M^* = E = M_0. \quad (3.33)$$

4 Conclusions

We have investigated the adiabatic motion of the non-topological hedgehog soliton defined by the NJL Lagrangian in self-consistent mean-field approximation through a medium of constituent quarks. Defining the soliton by the difference between fields with and without valence quarks and regularizing only that part of the grand canonical potential which survives at $T, \mu \rightarrow 0$ we could show, that the soliton behaves like an elementary particle with respect to the identity of inertial and rest mass independently of the thermodynamic parameters of the medium.

The paper was supported by the Bundesministerium für Forschung und Technologie (contract 06 DR 666). M. S. thanks for the warm hospitality of the University of Rostock, in particular D. Blaschke as well as E. E. Kolomeitsev, A. Pfitzner and F. Creutzburg for helpful discussions.

A Appendix

Evaluating expressions (3.8) and (3.9) we follow partially the way indicated in ref.[6]. First we treat the sea contribution and notice that the first derivative of the exponential

function is given by

$$\frac{\partial}{\partial V^k} e^{-sA^{\mathbf{V}(0)}} = -s \int_0^1 dt e^{-(1-t)sA^{\mathbf{V}(0)}} [B^k - 2p^k p^l V^l] e^{-tsA^{\mathbf{V}(0)}}. \quad (\text{A.1})$$

At $\mathbf{V}=\mathbf{0}$ only B^k survives in the inner bracket which can be replaced by the commutator (3.14). The integral is just the commutator between C^k and $e^{-sA(0)}$ (see *e.g.* [13])

$$\left. \frac{\partial}{\partial V^k} e^{-sA^{\mathbf{V}(0)}} \right|_{\mathbf{V}=\mathbf{0}} = \int_0^1 dt e^{-(1-t)sA(0)} [C^k, -sA(0)] e^{-tsA(0)} = [C^k, e^{-sA(0)}]. \quad (\text{A.2})$$

The second derivative is obtained by differentiating eq. (A.1) once more. At $\mathbf{V}=\mathbf{0}$ we can apply eq. (A.2) and get

$$\begin{aligned} \left. \frac{\partial^2}{\partial V^i \partial V^k} e^{-sA^{\mathbf{V}(0)}} \right|_{\mathbf{V}=\mathbf{0}} &= -s \int_0^1 dt [C^i, e^{-(1-t)sA(0)}] B^k e^{-tsA(0)} \\ &+ s \int_0^1 dt e^{-(1-t)sA(0)} 2p^i p^k e^{-tsA(0)} - s \int_0^1 dt e^{-(1-t)sA(0)} B^k [C^i, e^{-tsA(0)}]. \end{aligned} \quad (\text{A.3})$$

After taking the trace expression (A.3) can be rearranged and simplified. The integration becomes trivial

$$\text{Tr}_0 \left. \frac{\partial^2}{\partial V^i \partial V^k} e^{-sA^{\mathbf{V}(0)}} \right|_{\mathbf{V}=\mathbf{0}} = \text{Tr}_0 [s e^{-sA(0)} (2p^i p^k + [C^i, B^k])]. \quad (\text{A.4})$$

Now we consider the term $\partial^2/\partial V^i \partial V^k \ln A^{\mathbf{V}(\mu)} \Big|_{\mathbf{V}=\mathbf{0}}$ in the medium contribution (3.9) to the inertial mass

$$\left. \frac{\partial^2}{\partial V^i \partial V^k} \text{Tr} \ln A^{\mathbf{V}(\mu)} \right|_{\mathbf{V}=\mathbf{0}} = -\text{Tr} [2A(\mu)^{-1} p^i p^k + A(\mu)^{-1} B^i A(\mu)^{-1} B^k]. \quad (\text{A.5})$$

The second term can be rewritten using the commutator representation (3.14) of B^i

$$\begin{aligned} \text{Tr} [A(\mu)^{-1} B^i A(\mu)^{-1} B^k] &= \text{Tr} [A(\mu)^{-1} [C^i, A(\mu) + 2\mu h] A(\mu)^{-1} B^k] \\ &= \text{Tr} [A(\mu)^{-1} [C^i, A(\mu)] A(\mu)^{-1} B^k] + 2\mu \text{Tr} [A(\mu)^{-1} [C^i, h] A(\mu)^{-1} B^k]. \end{aligned} \quad (\text{A.6})$$

The first term can be treated as in ref. [6] and we get $\text{Tr} [A(\mu)^{-1} [C^i, B^k]]$. To reformulate the second one we rewrite the commutator

$$[C^k, h] = -\frac{i}{2} \{x^k, A(\mu)\} + iD(\mu)^\dagger x^k D(\mu) \quad (\text{A.7})$$

and get

$$\begin{aligned} \text{Tr} [A(\mu)^{-1} [C^i, h] A(\mu)^{-1} B^k] &= -i \text{Tr} [A(\mu)^{-1} B^k A(\mu)^{-1} \left(\frac{1}{2} \{x^i, A(\mu)\} - D(\mu)^\dagger x^i D(\mu) \right)] \\ &= -i \text{Tr} [A(\mu)^{-1} \frac{1}{2} \{B^k, x^i\}] + i \text{Tr} \left[(D(\mu)^\dagger)^{-1} B^k D(\mu)^{-1} x^i \right]. \end{aligned} \quad (\text{A.8})$$

Using eqs. (2.18, 3.12) one obtains

$$\frac{1}{2} \{x^i, B^k\} = (2x^i p^k - i\delta^{ik}) \partial_\tau - x^i [h, p^k] \quad (\text{A.9})$$

and

$$\begin{aligned} \text{Tr} \left[(D(\mu)^\dagger)^{-1} B^k D(\mu)^{-1} x^i \right] &= \text{Tr} \left[A(\mu)^{-1} (p^k x^i D(\mu) - D(\mu)^\dagger x^i p^k) \right] \\ &= \text{Tr} \left[A(\mu)^{-1} (2x^i p^k \partial_\tau - i\delta^{ik} D(\mu) + [x^i p^k, h]) \right]. \end{aligned} \quad (\text{A.10})$$

The last term does not contribute to the trace since h commutes with $A(\mu)^{-1}$. Altogether we have

$$\frac{\partial^2}{\partial V^i \partial V^k} \text{Tr} \ln A^V(\mu) \Big|_{V=0} = -\text{Tr} \left[A(\mu)^{-1} (2p^i p^k + [C^i, B^k] + 2\mu [(h - \mu)\delta^{ik} + ix^i [h, p^k]]) \right] \quad (\text{A.11})$$

Considerations (A.1-A.11) are independent of h, T and μ . So eqs. (A.4, A.11) are valid for $h \rightarrow h_0, T \rightarrow 0$ ($\text{Tr} \rightarrow \text{Tr}_0$) and $\mu \rightarrow 0$.

A useful rule, which we have applied several times, is

$$\text{Tr} [A [B, C]] = 0 \quad \text{if } [A, B] = 0 \quad \text{or} \quad [A, C] = 0. \quad (\text{A.12})$$

References

- [1] Christov, Chr. V., A. Blotz, H.-C. Kim, P. V. Pobylitsa, T. Watabe, Th. Meissner, E. Ruiz Arriola and K. Goeke, *Prog. Part. and Nucl. Phys.* **37** (1996) (Pergamon Press, Oxford, 1996)
- [2] R. Alkofer, H. Reinhardt and H. Weigel, UNITU 25/1994; hep-ph 9501213; *Phys. Rep.* (in print)
- [3] P. Ring, P. Schuck, *The nuclear many-body problem* (Springer, New York, 1980)
- [4] M. C. Birse, *Prog. Part. and Nucl. Phys.* **26** (1990) 1
- [5] M. Betz and R. Goldflam, *Phys. Rev. D* **28** (1983) 2848
- [6] P. V. Pobylitsa, E. Ruiz Arriola, Th. Meißner, F. Grümmer, K. Goeke, W. Broniowski, *J. Phys. G* **18** (1992) 1455
- [7] J. Nambu, G. Jona-Lasinio, *Phys. Rev.* **122** (1961) 345; **124** (1961) 246
- [8] J. I. Kapusta *Finite Temperature Field Theory* (Cambridge, University Press, 1989)
- [9] J. W. Negele, H. Orland *Quantum Many-Particle Systems* (Addison-Wesley, Amsterdam, 1988)
- [10] T. Matsubara, *Prog. Theor. Phys.* **14** (1957) 570
- [11] J. Schwinger, *Phys. Rev.* **82** (1951) 664
- [12] H. Reinhardt, *Nucl. Phys. A* **503** (1989) 825
- [13] M. Veltman, *Diagrammatica: The Path to Feynman Diagrams*. Cambridge Lecture Notes in Physics 4 (University Press, Cambridge, 1994)