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Added Stiffness Relating to Small Motions of Two Concentric Cylinders Submitted to Axial Annular Incompressible Flow

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Abstract

Axial incompressible, viscous flow is considered in an annular gap between two rigid cylinders. If the cylinders are displaced from their concentric positions in a certain manner, the displacement will cause reaction forces exerted by the fluid pressure. In this investigation the pressure fluctuations caused by the displacement of the structures are approximated by analytical means. Pressure fluctuations in phase with acceleration and velocity of the structure are not calculated here. Stationary flow is considered, because structural displacements are assumed to be small.

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List of Symbols

A	Outer cylinder.
B	Inner cylinder.
C_{Fi}	Spring constant relating to the structural displacement q_i.
F_i	Force or torque relating to the structural displacement q_i.
$J_{A/B}$	Jacket of the cylinders A, B, respectively.
L	Length of the cylinders.
$\underline{N}$	Column matrix (8×1) of interpolation functions N_i.
N_i	The i-th interpolation function of spatial variables in the matrix $\underline{N}$.
R	Mean value of the radii of the two cylinders.
R_A	Radius of the outer cylinder.
R_B	Radius of the inner cylinder.
Re	Reynolds number.
S	Width of the gap between the cylinders if both cylinders are at rest.
U	Mean velocity of the basic stream, averaged over the cross-section of the annular gap.
U_0	Reference velocity.
Z_A	z-Coordinate of the hinge of cylinder A, $Z_A \leq L/2$.
Z_B	z-Coordinate of the hinge of cylinder B, $Z_B \leq L/2$.

a_{i0}, a_{il}	Constant numbers or lengths for a given system.
b_{i0}, b_{il}	Constant numbers or lengths for a given system.
$\underline{c}_p$	Coeffizient matrix (8×1) in the Fourier expansion of the pressure field.
c_{pi}	Element i of the column matrix $\underline{c}_p$.
c_{pi}^{hom}	Term of a sum of the coefficient c_{pi}. This term is derived from a homogeneous solution of a certain differential equation.

c_{pi}^{part}	Term of a sum of the coefficient c_{pi}. This term is derived from a particular solution of a certain differential equation.
$\underline{c}_r$, $\underline{c}_\varphi$, $\underline{c}_z$	Coefficient matrices (8×1) in the Fourier expansions of the velocity components u_r, u_φ, u_z, respectively.
c_{ri}, $c_{\varphi i}$, c_{zi}	The i[th] elements in the column matrices $\underline{c}_r$, $\underline{c}_\varphi$, $\underline{c}_z$, respectively.
$\vec{e}_z$	Unit vector in axial (z-)direction.
g	Gravitational acceleration.
g_i	Shape function, relating to the structural displacement q_i.
h_i	Interpolation function, relating to the structural displacement q_i.
l_S	Distance from the hinge of a test pendulum to the centre of gravity of the test pendulum.
p	Static pressure.
p_1	Pressure averaged over the circumference at the top of the annular gap.
p_2	Pressure at the bottom of the annular gap.
m_B	Mass of the cylinder B.
m_B^*	Mass of the cylinder B reduced by the displaced mass of the fluid.
$\underline{q}$	Column matrix (8×1) of the structural displacements of both cylinders.
$\underline{q}_A$	Column matrix (4×1) of the structural displacements of cylinder A.
$\underline{q}_B$	Column matrix (4×1) of the structural displacements of both cylinders.
q_i	Element i of column matrix $\underline{q}$.
r	Radial coordinate of the laboratory-fixed cylindrical coordinates (r, φ, z).
$\underline{s}_p$	Coefficient matrix (8×1) in the Fourier expansion of the pressure field.
s_{pi}	Element i of the column matrix $\underline{s}_p$.
$\underline{s}_r$, $\underline{s}_\varphi$, $\underline{s}_z$	Coefficient matrices (8×1) in the Fourier expansions of the velocity components u_r, u_φ, u_z, respectively.
s_{rj}, $s_{\varphi j}$, s_{zj}	The j[th] elements in the column matrices $\underline{s}_r$, $\underline{s}_\varphi$, $\underline{s}_z$, respectively.

$\vec{u}$	Velocity field of the fluid.
$\bar{u}_z$	Axial fluid velocity component, averaged over the circumference of the gap.
x_A , x_B	Parallel displacements of the cylinders A, B, respectively, in x-direction. See fig. 2.
y_A , y_B	Parallel displacements of the cylinders A, B, respectively, in y-direction. See fig.2.
z	Axial coordinate of the laboratory-fixed cylindrical coordinates (r, φ, z).

Γ_1, Γ_2	Undetermined functions, dependent on the radial coordinate ξ.
Θ	Moment of inertia of a rigid body.
Θ_F	Moment of inertia caused by the fluid pressure.

$\alpha_0, \alpha_1, \alpha_2, \ldots$	Undetermined real numbers.
γ_1, γ_2	Undetermined functions, dependent on the radial coordinate ξ.
$\delta_H, \delta_L, \delta_R$	Different damping coefficients.
ζ	Axial coordinate of the coordinates (ξ, ϕ, ζ).
η	Radial coordinate, linear in ξ.
μ	Dynamic, molecular viscosity of the fluid.
ξ	Radial coordinate of the coordinates (ξ, ϕ, ζ).
ρ	Density of the fluid.
ϕ	Circumferential coordinate of the coordinates (ξ, ϕ, ζ).
φ	Circumferential coordinate of the laboratory-fixed coordinates (r, φ, z).
ψ_{xA} , ψ_{xB}	Rotational displacements of the cylinders A, B, respectively, around the x-axis.
ψ_{yA} , ψ_{yB}	Rotational displacements of the cylinders A, B, respectively, around the y-axis.

$_{,r}$	Partial derivation with respect to r with φ and z being constant.
$_{,\varphi}$	Partial derivation with respect to φ with r and z being constant.
$_{,z}$	Partial derivation with respect to z with r and φ being constant.
∂_ξ	Partial derivation with respect to ξ with ϕ and ζ being constant.
∂_ϕ	Partial derivation with respect to ϕ with ξ and ζ being constant.
∂_ζ	Partial derivation with respect to ζ with ξ and ϕ being constant.

$[f]_{\xi \equiv R_B}$, $[f]_{\zeta \equiv 0}$	Restriction of the function f on the domain with $\xi \equiv R_B$ or $\zeta \equiv 0$, repectively.

1 Introduction

Modelling the vibrations of a reactor pressure vessel and a core barrel, the forces of the fluid onto the solid structure have to be taken into account [1]. Grunwald and Altstadt [2,3] regard two rigid cylinders which are concentric at rest. The cylinders can perform arbitrary small motions except axial displacements and rotations. In between the cylinders an axial fluid stream is driven by a pressure difference from the top to the bottom of the annular gap. Acceleration and velocity of the cylindrical structures are causing reaction forces of the fluid. In [2,3] a pressure field is derived which describes the added mass and the added damping effects. In this report a pressure field is derived which approximates the reaction force of the streaming fluid caused by the displacement of the structure.

It is assumed that the displacements of the cylinders are small in comparison with the fluid filled gap between them. This assumption is required to linerarize the governing equations with repect to displacement, velocity and acceleration of the structure. Presuming these conditions, it suffices to consider displaced but fixed cylinders and stationary flow.

A number of papers on similar or related problems has been published in [4]. Other work is presented in [5]-[8].

In the second section of this report the general problem is stated shortly. In section three the geometry of the system is exploited in order to simplify the mathematical problem. In section four and in section five the problem is restricted: The outer cylinder is fixed at rest and the inner cylinder can principally move in one degree of freedom, but it is assumed that the inner cylinder is fixed at any (non-vanishing) constant displacement. Other displacements can be treated by analogy with section four. A displacement of the structure in one special degree of freedom is causing a specific disturbance of the basic stream. These disturbances can be superimposed because they are very small. The superposition of the pressure disturbances is carried out in section six. In section seven theoretical predictions following from the result are compared with experimental data. Some remarks concerning the influence of the boundary conditions are given at the end of this report.

2 Presentation of the Problem

In this investigation the same system is considered as in the work of Grunwald and Altstadt [2, 3]. The geometry of the system is shown in fig.1 The length of the cylinders is L. The outer cylinder is denoted by A, the inner one by B. At rest the two axes of the cylinders coincide with the z-axis. The external pressure gradient $(p_1 - p_2)/L\ (-\vec{e}_z)$ drives the basic stream $\bar{u}_z$.

The eight degrees of freedom of the motion of the cylinders are shown in fig. 2. The positions of the cylinders are denoted by

$$\underline{q}^T = [x_A, y_A, \psi_{xA}, \psi_{yA}, x_B, y_B, \psi_{xB}, \psi_{yB}] = [\underline{q_A}^T, \underline{q_B}^T] , \qquad (1)$$

with ψ_{xA} and ψ_{yA} being the rotation angles of the axis of cylinder A in the y-z-plane and the x-z-plane, respectively. x_A, y_A and the constant Z_A denote the coordinates of the fixed point of the rotations of cylinder A. In fig.2 $Z_A = Z_B = L/2$. The degrees of freedom $x_B, y_B, \psi_{xB}, \psi_{yB}$ of cylinder B have to be understood analogously. The sign of the angles follows the usual mathematical convention.

The following properties of the system are presumed:

- P1 The fluid is incompressible.
- P2 The width S of the annular gap is small compared with any of the two radii R_A, R_B of the cylinders A, B. $S := R_A - R_B$. $S \ll R_B$.
- P3 The displacements of the mechanical structure are small in comparison with the gap width S.
- P4 The products $q_i \dot{q}_j$ and $q_i \ddot{q}_j$ are small enough to be neglected.
- P5 At the outlet of the annular gap the fluid escapes into a large volume.
- P6 The mean radius $R := (R_A + R_B)/2$ is small in comparison with the length L of the cylinders. $R < L$.

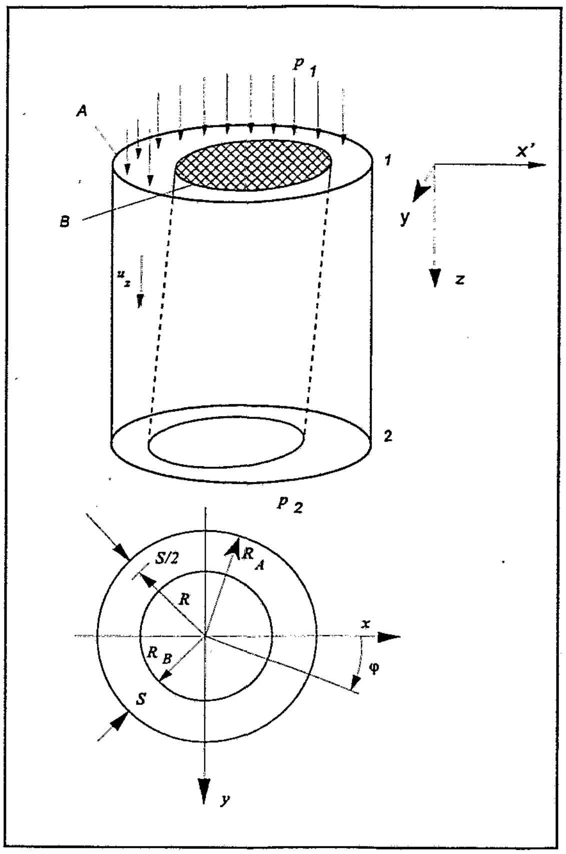

Fig. 1: Geometry of the system

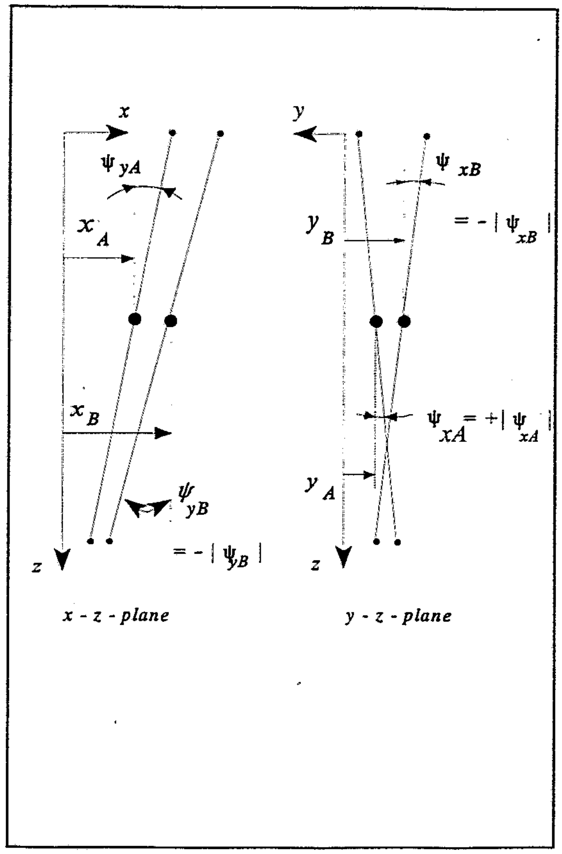

Fig. 2: Structural degrees of freedom

In order to investigate the added stiffness effect caused by the basic stream, both cylinders are fixed at a certain position. At least one of the cylinders is fixed out of rest position. Stationary flow is considered. This treatment of the problem will be sufficient if the products $q_i \dot{q}_j$ and $q_i \ddot{q}_j$ are small (P4).

The governing Navier-Stokes equations and the conservation of mass equation are written in cylindrical coordinates (r, φ, z) with $\vec{u}$ as the velocity field of the fluid and p as the static pressure.

$$p_{,r} = -\rho\left[u_r u_{r,r} + \frac{1}{r} u_\varphi u_{r,\varphi} + u_z u_{r,z} - \frac{1}{r} u_\varphi^{2}\right] + \mu\left[u_{r,rr} + \frac{1}{r^2} u_{r,\varphi\varphi} + u_{r,zz} + \frac{1}{r} u_{r,r} - \frac{1}{r^2} u_r - \frac{2}{r^2} u_{\varphi,\varphi}\right] . \quad \textbf{(2)}$$

$$\frac{1}{r} p,_\varphi = -\rho [u_r u_\varphi,_r + \frac{1}{r} u_\varphi u_\varphi,_\varphi + u_z u_\varphi,_z + \frac{1}{r} u_r u_\varphi]$$
$$+ \mu [u_\varphi,_{rr} + \frac{1}{r^2} u_\varphi,_{\varphi\varphi} + u_\varphi,_{zz} + \frac{1}{r} u_\varphi,_r - \frac{1}{r^2} u_\varphi + \frac{2}{r^2} u_r,_\varphi] . \quad (3)$$

$$p,_z = -\rho [u_r u_z,_r + \frac{1}{r} u_\varphi u_z,_\varphi + u_z u_z,_z]$$
$$+ \mu [u_z,_{rr} + \frac{1}{r^2} u_z,_{\varphi\varphi} + u_z,_{zz} + \frac{1}{r} u_z,_r] . \quad (4)$$

$$u_r,_r + \frac{1}{r} u_\varphi,_\varphi + u_z,_z + \frac{1}{r} u_r = 0 . \quad (5)$$

On the cylinder jackets J_A and J_B the fluid velocity must be zero.

$$[\vec{u}]_{J_{A/B}} = \vec{0} . \quad (6)$$

At the inlet I_1 with $z = 0$ and the outlet O_2 with $z = L$ the pressure values averaged over the circumference are constant.

$$\frac{1}{2\pi} \left[\int_0^{2\pi} [p(r,\varphi,z)]_{z=0} \, d\varphi \right] = p_1 . \quad (7)$$

$$\frac{1}{2\pi} \left[\int_0^{2\pi} [p(r,\varphi,z)]_{z=L} \, d\varphi \right] = p_2 . \quad (8)$$

At first the investigation is restricted to the case $p_1 \geq p_2$. In this case the condition

$$[p]_{z=L} \equiv p_2 \quad (9)$$

must be fulfilled in agreement with experimental results [9]. In Appendix C the case $p_1 < p_2$, this is upward flow direction, is considered.

The properties P1-P6 fall into two groups: Properties P1-4 enable the approximate analytical solution described in this report. Properties P5-6 are connected with the boundary conditions at the inlet and the outlet of the annular gap. P5 sets the boundary condition (9). Boundary condition (7) does not require a certain pressure field close to the inlet. As will be seen later, property P6 ensures that the effect of this uncertainty is small.

3 Conclusions from the Geometry of the System

3.1 Approximations of the Governing Equations

Laminar flow regime is presumed. In the turbulent flow regime the argumentation is valid for mean velocity components averaged in time.

In the calculations of this report all terms being quadratic in the displacements q_i are omitted because of property P3, which states that $(q_i/S) \ll 1$ in the case that i refers to a degree of freedom with parallel translation and $(q_i L/S) \ll 1$ in the case that i refers to a rotational degree of freedom.

If both cylinders are fixed at rest position, the velocity components u_r and u_φ are vanishing. Therfore in a power series expansion of these velocity components with respect to the displacements, the lowest non-vanishing order can be the linear one. Hence in eqs. (2)-(4) all products $u_r u_{r},_r$ or $u_\varphi u_{r},_\varphi$ or u_φ^2 or $u_r u_{\varphi},_r$ or $u_\varphi u_{\varphi},_\varphi$ or $u_\varphi u_r$ are at least quadratic in the displacements.

In eq.(2) the order of magnitude of the term $u_{r},_{rr}$ is (u_r/S^2), the order of magnitude of the term $(1/r)u_{r},_r$ is $(u_r/(RS))$ and $(1/r^2)u_r \approx (u_r/R^2)$. Therefore $(1/r)u_{r},_r$ and $(1/r^2)u_r$ can be neglected against $u_{r},_{rr}$ because of P2, which states that $S \ll R$. By analogy with this argumentation it is concluded that in the viscosity terms of eqs. (3)-(4) every term with a factor $1/r$ is negligible. In the continuity equation (5) the term u_r/r is negligible in comparison with $u_{r},_r$.

In summary eqs. (10)-(13) are treated instead of eqs. (2)-(5).

$$p,_r = -\rho[u_z u_{r},_z] + \mu[u_{r},_{rr} + u_{r},_{zz} - \frac{2}{r^2} u_{\varphi},_\varphi] . \qquad (10)$$

$$\frac{1}{r} p,_\varphi = -\rho[u_z u_{\varphi},_z] + \mu[u_{\varphi},_{rr} + u_{\varphi},_{zz}] . \qquad (11)$$

$$p,_z = -\rho[u_r u_{z},_r + \frac{1}{r} u_\varphi u_{z},_\varphi + u_z u_{z},_z] + \mu[u_{z},_{rr} + u_{z},_{zz}] . \qquad (12)$$

$$u_{r,r} + \frac{1}{r} u_{\varphi,\varphi} + u_{z,z} = 0 \ . \tag{13}$$

3.2 Separation of the Pressure Field and the Velocity Field into Fourier Components

The dependence of the pressure field and the fluid velocity field on the circumferential angle φ is expanded in a Fourier series. The uniqueness of these fields requires that the Fouier series only contains terms with a spatial frequency $(k/2\pi)$ with $k \in \{0,1,2,...\}$. Among these terms those with a frequency larger than $(1/2\pi)$ are omitted, because in linear approximation with respect to the displacements q_i the motion of any cylinder only produces disturbances of period 2π. This is a pure geometrical fact which is expressed in the φ-dependence of the shape functions g_i listed in appendix A.

The component u_z averaged over the circumference is denoted by $\bar{u}_z$. The corresponding components $\bar{u}_r$ and $\bar{u}_\varphi$ are set zero, because there is no external pressure gradient in radial or circumferential direction. On this condition it is concluded from the continuity equation (13)

$$\bar{u}_{z,z} = 0 \ . \tag{14}$$

In order to describe the boundaries of the deformed gap with constant radial coordinates, the following coordinate transformation is introduced:

$$\begin{aligned} \xi &:= r - \underline{q}^T \underline{N}(r,\varphi,z) \ , \\ \phi &:= \varphi \ , \\ \zeta &:= z \ , \end{aligned} \tag{15}$$

with $\underline{q}$ as in eq.(1) and real functions N_i with

$$\begin{aligned} &N_i = h_i(r)\, g_i(\varphi,z) \ , \\ &h_i(r{=}R_A) = 1, \quad h_i(r{=}R_B) = 0 \qquad \text{for} \quad q_i \in \underline{q}_A \ , \quad i = 1..4 \ , \\ &h_i(r{=}R_A) = 0, \quad h_i(r{=}R_B) = 1 \qquad \text{for} \quad q_i \in \underline{q}_B \ , \quad i = 5..8 \ , \\ &g_i(\varphi,z) = (a_{i0} + a_{i1} z) \cos\varphi + (b_{i0} + b_{i1} z) \sin\varphi \ . \end{aligned} \tag{16}$$

The functions g_i are the shape functions relating the radial positions of points on the cylinder jackets with the displacements q_i in linear approximation. A list of the functions g_i is given in appendix A. Up to now it is not necessary to determine the smooth functions h_i in the interior of the annular gap.

Any point on the jacket of the inner cylinder has the coordinate $\xi = R_B$ and on the other hand any point on the jacket of the outer cylinder has the coordinate $\xi = R_A$ for every displacement $\underline{q}$.

The velocity components are expressed by

$$\begin{aligned} u_r(\xi,\phi,\zeta) &= \underline{q}^T \underline{c}_r(\xi,\zeta)\cos(\phi) + \underline{q}^T \underline{s}_r(\xi,\zeta)\sin(\phi) , \\ u_\varphi(\xi,\phi\zeta) &= \underline{q}^T \underline{c}_\varphi(\xi,\zeta)\cos(\phi) + \underline{q}^T \underline{s}_\varphi(\xi,\zeta)\sin(\phi) , \\ u_z(\xi,\phi,\zeta) &= \overline{u}_z(\xi) \\ &\quad + \underline{q}^T \underline{c}_z(\xi,\zeta)\cos(\phi) + \underline{q}^T \underline{s}_z(\xi,\zeta)\sin(\phi) , \end{aligned} \tag{17}$$

and the Fourier expansion of the pressure reads

$$\begin{aligned} p(\xi,\phi,\zeta) &= p_1 - \frac{(p_1 - p_2)}{L}\zeta \\ &\quad + \underline{q}^T \underline{c}_p(\xi,\zeta)\cos(\phi) + \underline{q}^T \underline{s}_p(\xi,\zeta)\sin(\phi) . \end{aligned} \tag{18}$$

The first component of the column matrix $\underline{c}_p$ is denoted by c_{p1}, as an example. Throughout this report the following definitions are valid:

$$\partial_\xi := \left(\frac{\partial}{\partial \xi}\right)_{\phi,\zeta\, =\, const.} , \tag{19}$$

$$\partial_\phi := \left(\frac{\partial}{\partial \phi}\right)_{\xi,\zeta\, =\, const.} , \tag{20}$$

$$\partial_\zeta := \left(\frac{\partial}{\partial \zeta}\right)_{\xi,\phi\, =\, const.} . \tag{21}$$

4 Added Stiffness Related to the Pendular Motion of the Inner Cylinder

In this section the outer cylinder is fixed at rest, the inner cylinder is fixed at a position where only the pendular displacement ψ_{yB} is different from zero. This situation corresponds to

$$\underline{q}^T = [0,0,0,0,0,0,0,\psi_{yB}] , \qquad \psi_{yB} \neq 0 , \tag{22}$$

and

$$\begin{aligned} \xi &= r - \psi_{yB}\, h_8 \left(z - Z_B\right) \cos\varphi , \\ \phi &= \varphi , \\ \zeta &= z . \end{aligned} \tag{23}$$

Eq. (23) means with respect to partial derivatives

$$\begin{aligned} ,_r &= \partial_\xi - \psi_{yB}\, (\zeta - Z_B)\, \frac{dh_8}{d\xi} \cos(\phi)\partial_\xi , \\ ,_\varphi &= \partial_\phi + \psi_{yB}\, (\zeta - Z_B)\, h_8 \sin(\phi)\partial_\xi , \\ ,_z &= \partial_\zeta - \psi_{yB}\, h_8 \cos(\phi)\partial_\xi . \end{aligned} \tag{24}$$

Inserting the ansatz (17) and (18) into the momentum equations (10) - (12) and into the continuity equation (13), one obtains the following separate systems of basic equations.

Equation of the basic stream

Momentum equation in axial direction:

Average component:

$$\frac{d^2 \overline{u}_z}{d\xi^2} = - \frac{(p_1 - p_2)}{\mu \ L} . \tag{25}$$

System I (Containing the unknown functions c_{p8} , c_{r8} , $s_{\varphi 8}$ and c_{z8} .)

Momentum equation in axial direction

$\cos\phi$ - component:

$$\partial_\zeta c_{p8} = -\rho\left[\bar{u}_z\,\partial_\zeta\,c_{z8} - \bar{u}_z\,h_8\,\frac{d\bar{u}_z}{d\xi} + c_{r8}\,\frac{d\bar{u}_z}{d\xi}\right] + \mu\left[-(\zeta - Z_B)\,\frac{d^2h_8}{d\xi^2}\,\frac{d\bar{u}_z}{d\xi} - 2(\zeta - Z_B)\,\frac{dh_8}{d\xi}\,\frac{d^2\bar{u}_z}{d\xi^2} + \partial_\xi^2\,c_{z8} + \partial_\zeta^2\,c_{z8}\right] . \tag{26}$$

Momentum equation in radial direction:

$\cos\phi$ - component:

$$\partial_\xi\,c_{p8} = -\rho\,\bar{u}_z\,\partial_\zeta\,c_{r8} + \mu\left[\partial_\xi^2\,c_{r8} + \partial_\zeta^2\,c_{r8} - \frac{2}{\xi^2}\,s_{\varphi 8}\right] . \tag{27}$$

Momentum equation in circumferential direction:

$\sin\phi$ - component:

$$c_{p8} = \rho\,\xi\,\bar{u}_z\,\partial_\zeta\,s_{\varphi 8} - \mu\,\xi\left[\partial_\xi^2\,s_{\varphi 8} + \partial_\zeta^2\,s_{\varphi 8}\right] . \tag{28}$$

Continuity equation

$\cos\phi$ - component:

$$s_{\varphi 8} = \xi\,(h_8\,\frac{d\bar{u}_z}{d\xi} - \partial_\zeta\,c_{z8} - \partial_\xi\,c_{r8}) . \tag{29}$$

System II (Containing the unknown functions s_{p8} , s_{r8} , $c_{\varphi 8}$ and s_{z8} .)
Momentum equation in axial direction:

$\sin\phi$ - component:

$$\partial_\zeta \, s_{p8} = - \rho \left[\bar{u}_z \, \partial_\zeta \, s_{z8} + s_{r8} \, \frac{d\bar{u}_z}{d\xi} \right] + \mu \left[\partial_\xi^2 \, s_{z8} + \partial_\zeta^2 \, s_{z8} \right] \; . \tag{30}$$

Momentum equation in radial direction:

$\sin\phi$ - component:

$$\partial_\xi \, s_{p8} = - \rho \, \bar{u}_z \, \partial_\zeta \, s_{r8} + \mu \left[\partial_\xi^2 \, s_{r8} + \partial_\zeta^2 \, s_{r8} + \frac{2}{\xi^2} \, c_{\varphi 8} \right] \; . \tag{31}$$

Momentum equation in circumferential direction:

$\cos\phi$ - component:

$$s_{p8} = - \rho \, \xi \, \bar{u}_z \, \partial_\zeta \, c_{\varphi 8} + \mu \, \xi \left[\partial_\xi^2 \, c_{\varphi 8} + \partial_\zeta^2 \, c_{\varphi 8} \right] \; . \tag{32}$$

Continuity equation:

$\sin\phi$ - component:

$$\partial_\xi \, s_{r8} + \partial_\zeta \, s_{z8} - \frac{1}{\xi} \, c_{\varphi 8} = 0 \; . \tag{33}$$

Only eq.(25) is of order zero in ψ_{yB}. The other equations are of order one in ψ_{yB}. In any of the eqs.(26)-(33) it holds $\xi = r$ and $\partial_\xi = {}_{,r}$, because deviations of order two in the displacements are neglected throughout this investigation.

Sytem II is a homogeneous system in the unknown functions s_{p8}, s_{r8}, $c_{\varphi 8}$ and s_{z8} . With the boundary conditions (6), (7) and (9) system II is solved by

$$s_{p8} = s_{r8} = s_{z8} = c_{\varphi 8} = 0 \; . \tag{34}$$

In order to gain an approximate solution of (26) - (29), the following calculation steps will be done.

(i) The basic flow $\bar{u}_z$ will be calculated from eq. (25).

(ii) The basic flow $\bar{u}_z$ will be inserted into the continuity equation (29). Additional boundary conditions of the velocity component u_r will be gained. It will be concluded from the boundary conditions, that u_r can be approximated as a function of ξ and ϕ.

(iii) The ζ - derivative of eq. (28) will be compared with eq. (26). The ζ-derivative of the right hand side of the continuity equation (29) will be substituted for the ζ-derivative of the Fourier coefficient $s_{\varphi 8}$. The resulting differential equation will be solved with respect to c_{z8}.

(IV) The ξ - derivative of eq. (28) will be compared with eq. (27). The ξ-derivative of the right hand side of the continuity equation (29) will be substituted for the ξ-derivative of the Fourier coefficient $s_{\varphi 8}$. The coefficient c_{r8} and in turn the coefficients $s_{\varphi 8}$ and c_{p8} will be calculated.

4.1 Calculation of the Basic Flow

Eq. (25) is solved by

$$\bar{u}_z(\xi) = \left(1 - \frac{4}{S^2}(\xi - R)^2\right) \frac{(p_1 - p_2)\, S^2}{8\, \mu\, L} \quad . \tag{35}$$

The basic flow $\bar{u}_z$ is similar to plane Poiseuille flow. The annular gap can be locally approximated by a plane gap because of $S \ll R$ (P2). As an abbreviation it is defined

$$U_o = \frac{(p_1 - p_2) S^2}{8 \mu\, L} \quad . \tag{36}$$

4.2 Continuity of Mass on the Cylinder Jackets

Boundary conditions (6) require

$$\left[\partial_\phi\, u_\varphi\right]_{\xi = R_B} = \left[\partial_\zeta\, u_z\right]_{\xi = R_B} = 0 \quad . \tag{37}$$

$$\left[\partial_\phi\, u_\varphi\right]_{\xi=R_A} = \left[\partial_\zeta\, u_z\right]_{\xi=R_A} = 0 \ . \tag{38}$$

On the jacket of the inner cylinder the continuity equation (13) is reduced to

$$\left[\partial_\xi\, u_r\right]_{\xi=R_B} - \psi_{yB}\,(\zeta - Z_B)\,\frac{dh_8}{d\xi}\cos(\phi)\left[\partial_\xi\, u_r\right]_{\xi=R_B} = \psi_{yB}\cos\phi\left[\partial_\xi\, u_z\right]_{\xi=R_B} \tag{39}$$

by inserting eqs. (24) and (37). On the left hand side of the equation above the second term of the sum is of order two in ψ_{yB} because u_r does not contain any term of order zero in ψ_{yB}. Hence this term of the sum is omitted. Taking into account eq.(35) and the abbreviation (36) one obtains

$$\left[\partial_\xi\, c_{r8}\right]_{\xi=R_B} = \left[\partial_\xi\, \bar{u}_z\right]_{\xi=R_B} = 4\,\frac{U_o}{S} \ . \tag{40}$$

On the outer cylinder the corresponding equation is

$$\left[\partial_\xi\, c_{r8}\right]_{\xi=R_A} = 0 \ . \tag{41}$$

The boundary conditions (6), (40) and (41) of u_r do not depend on ζ. It is concluded that the dependence of u_r on ζ is weak and can be neglected.

$$\partial_\zeta\, u_r \equiv 0 \ . \tag{42}$$

The above equation can be used to eliminate Fourier coefficients of u_r by applying the operator ∂_ζ.

4.3 Calculation of the Disturbance of the Axial Fluid Velocity

Applying the operator ∂_ζ to eq. (28), one obtains an expression for $\partial_\zeta\, c_{p8}$. This expression is set equal to that from eq. (26). The coefficient $s_{\varphi 8}$ is replaced by the right hand side of eq. (29). The resulting differential equation is

$$\Big\{ \mu\, \xi^2 \left[\partial_\zeta^4 + \partial_\zeta^2\, \partial_\xi^2 \right] + \rho\, \bar{u}_z\, \xi^2 \partial_\zeta^3 - 2\mu\,\xi\,\partial_\zeta^2 \partial_\xi$$
$$+ \mu \left[\partial_\zeta^2 + \partial_\xi^2 \right] - \varrho\, \bar{u}_z \partial_\zeta \Big\}\, c_{z8}$$
$$= \mu \left[2 \frac{dh_8}{d\xi}\, \frac{d^2 \bar{u}_z}{d\xi^2} + \frac{d^2 h_8}{d\xi^2}\, \frac{d\bar{u}_z}{d\xi} \right] (\zeta - Z_B)$$
$$+ \varrho\, (c_{r8} - \bar{u}_z h_8)\, \frac{d\bar{u}_z}{d\xi} \;. \tag{43}$$

A particular solution of eq. (43) is

$$c_{z8}^{part} = c_{z8,0}\,(\xi) + c_{z8,1}\,(\xi, \zeta) \tag{44}$$

with

$$c_{z8,1} = (\zeta - Z_B) \left[h_8(\xi) \frac{d\bar{u}_z}{d\xi} + 2 \frac{U_0}{S} \left(\frac{\xi - R}{S/2} \right) - 2 \frac{U_0}{S} \right] , \tag{45}$$

$$\partial_\zeta\, c_{z8,0} \equiv 0 \;, \tag{46}$$

$$\partial_\xi^2\, c_{z8,0} = \frac{d^2 c_{z8,0}}{d\xi^2}$$
$$= \frac{\varrho}{\mu}\, c_{r8} \frac{d\bar{u}_z}{d\xi} + \frac{\varrho}{\mu}\, \bar{u}_z\, U_0 \left[\frac{2}{S} \left(\frac{\xi - R}{S/2} \right) - \frac{2}{S} \right] . \tag{47}$$

In eq.(43) the term

$$-\mu\, \xi^2\, \partial_\zeta^2\, \partial_\xi^2\, c_{z8} = -\mu\, \xi^2 \partial_\xi\, (\partial_\zeta^2\, \partial_\xi\, c_{z8}) \tag{48}$$

has the order of magnitude $|\mu\,(R^2/S)\;\partial_\zeta^2\,\partial_\xi\, c_{z8}|$, which is larger than

$$|- 2\mu\,\xi\,\partial_\zeta^2 \partial_\xi\, c_{z8}| \;\approx\; 2\mu R\,|\partial_\zeta^2 \partial_\xi\, c_{z8}| \tag{49}$$

because of $S \ll R$ (P2). Therefore the term $-2\mu\,\xi\,\partial_\zeta^2 \partial_\xi\, c_{z8}$ can be neglected and

$$c_{z8}^{hom} = \gamma_1(\xi)\,\exp\left(\frac{\zeta}{\xi}\right)$$

$$+ \gamma_2(\xi)\,\exp\left(-\frac{\zeta}{\xi}\right)$$

$$= \Gamma_1(\xi)\,\exp\left(\frac{\zeta - L}{\xi}\right) + \Gamma_2(\xi)\,\exp\left(-\frac{\zeta}{\xi}\right) \qquad \textbf{(50)}$$

is an approximate homogeneous solution of eq. (43). Γ_1 and Γ_2 are undetermined functions of ξ. At first the further investigation is restricted to the particular solution, $c_{z8} = c_{z8}^{part}$. The homogeneous solution (50) will play an important role only near the fringes $\zeta \equiv 0$, $\zeta \equiv L$.

In order to proceed with the investigation, it is not necessary to calculate the function $c_{z8,0}$. Insertion of eqs. (44) and (45) into eq. (29) results in

$$s_{\varphi 8}^{part} = -\xi \left[\frac{dc_{r8}}{d\xi} + 2\frac{U_0}{S}\left(\frac{\xi - R}{S/2}\right) - 2\frac{U_0}{S}\right] \qquad \textbf{(51)}$$

and insertion of the above eq.(51) into eq. (28) provides

$$c_{p8}^{part} = \mu\,\xi^2 \frac{d^3 c_{r8}}{d\xi^3} + 2\mu\,\xi \frac{d^2 c_{r8}}{d\xi^2} \quad . \qquad \textbf{(52)}$$

4.4 Calculation of the Radial Fluid Velocity

Applying the operator ∂_ξ to eq. (28), one obtains an expression for $\partial_\xi c_{p8}$. This expression is set equal to that from eq. (27). The coefficient $s_{\varphi 8}$ is replaced by $s_{\varphi 8,p}$ as written in eq. (51). The resulting differential equation is

$$\left\{\frac{d^4}{d\xi^4} + \frac{4}{\xi}\frac{d^3}{d\xi^3} + \frac{1}{\xi^2}\frac{d^2}{d\xi^2} - \frac{2}{\xi^3}\frac{d}{d\xi}\right\} c_{r8}$$

$$= -8\frac{U_0}{S^2}\frac{1}{\xi^2} + 4\frac{U_0}{S}\frac{1}{\xi^3} + 4\frac{U_0}{S}\frac{1}{\xi^3}\left(\frac{\xi - R}{S/2}\right) \quad . \qquad \textbf{(53)}$$

Eq.(53) has to be solved taking into account the boundary conditions (6), (40) and (41). Approximate solutions were gained by inserting a polynomial $c_{r8} = \alpha_0 + \alpha_1 \eta + \alpha_2 \eta^2 + ...$ with

$$\eta := \left(\frac{\xi - R}{S/2} \right) \quad . \tag{54}$$

and unknown coefficents $\alpha_0, \alpha_1, \alpha_2, ...$ into eqs. (6),(40), (41), and (53) and solving the resulting linear equation system with respect to the coefficients $\alpha_0, \alpha_1, ...$.

Here approximate solutions are presented for two ratios S/R .

Case of $\frac{S}{R} = 0.01$:

$$\begin{aligned} c_{r8} = U_0 \cdot \{ & 0.497499 - 0.499988\ \eta \\ & - 0.494988\ \eta^2 + 0.499976\ \eta^3 \\ & - 0.002501\ \eta^4 + 0.000012\ \eta^5 \} \ . \end{aligned} \tag{55}$$

Case of $\frac{S}{R} = 0.1$:

$$\begin{aligned} c_{r8} = U_0 \cdot \{ & 0.474899 - 0.498806\ \eta \\ & - 0.449853\ \eta^2 + 0.497615\ \eta^3 \\ & - 0.024990\ \eta^4 + 0.001189\ \eta^5 \\ & - 0.000055\ \eta^6 + 3 \cdot 10^{-6} \eta^7 \} \ . \end{aligned} \tag{56}$$

The above solutions of eq. (53) can be inserted into eq. (52) and the coefficient c_{p8}^{part} is obtained. Only the first term of the sum on the right hand side of eq. (52) is taken into account. The second term of the sum is one order of magnitude smaller because of $S \ll R$. For any ratio $S/R \leq 0.15$ the result is

$$c_{p8}^{part} = 3\ \frac{R^2}{S\ L}\ (p_1 - p_2) \quad . \tag{57}$$

More exactly one calculates

$$\left[c_{p8}^{part}\right]_{\xi=R_B} = (3 + \delta)\,\frac{R^2}{S\,L}\,(p_1 - p_2)\,, \qquad \text{with} \quad 0 < \delta < 0.5 \tag{58}$$

and increasing δ with increasing ratio S/R . But in view of the approximations made, the term δ is not significant. In formula (57) small radial pressure gradients are omitted.

4.5 Fitting the Pressure to the Boundary Conditions at the Outlet

Now the homogenous solution (50) of differential equation (43) is added to the particular solution (44).

$$\begin{aligned} c_{z8} &= c_{z8}^{part} + c_{z8}^{hom} \\ &= c_{z8}^{part} + \Gamma_1(\xi)\,\exp\left(\frac{\zeta - L}{\xi}\right) + \Gamma_2(\xi)\,\exp\left(\frac{-\zeta}{\xi}\right). \end{aligned} \tag{59}$$

Then a homogeneous part of $s_{\varphi 8}$ has to balance eq. (29).

$$s_{\varphi 8} = s_{\varphi 8}^{part} - \Gamma_1 \exp\left(\frac{\zeta - L}{\xi}\right) + \Gamma_2 \exp\left(\frac{-\zeta}{\xi}\right). \tag{60}$$

Eq. (60) ist inserted into the momentum equation (28).

$$\begin{aligned} c_{p8} &= c_{p8}^{part} + c_{p8}^{hom} \\ &= c_{p8}^{part} + \varrho\bar{u}_z\left(-\Gamma_1\exp\left(\frac{\zeta-L}{\xi}\right) - \Gamma_2\exp\left(\frac{-\zeta}{\xi}\right)\right) \\ &+ \mu\left\{\xi\Gamma_1'' - 2\Gamma_1'\frac{\zeta-L}{\xi} + \Gamma_1\left(\frac{1}{\xi} + 2\frac{\zeta-L}{\xi^2} + \frac{(\zeta-L)^2}{\xi^3}\right)\right\}\exp\left(\frac{\zeta-L}{\xi}\right) \\ &+ \mu\left\{-\xi\Gamma_2'' - 2\Gamma_2'\frac{\zeta}{\xi} + \Gamma_2\left(2\frac{\zeta}{\xi^2} - \frac{\zeta^2}{\xi^3} - \frac{1}{\xi}\right)\right\}\exp\left(\frac{-\zeta}{\xi}\right). \end{aligned} \tag{61}$$

Γ_1', Γ_2' stand for $\dfrac{d\Gamma_1}{d\xi}, \dfrac{d\Gamma_2}{d\xi}$, respectively.

Everywhere in the annular gap it is demanded that

$$\left[\partial_\xi \, c_{p8}^{hom}\right]_{(R_B \le \xi \le R_A, 0 \le \zeta \le L)} \approx 0 \,. \tag{62}$$

Then the added homogeneous part c_{p8}^{hom} does not violate the differential equation (53) with the approximate solutions (55) and (56).

Eq. (61) is considered in the region near $\zeta = L$. Terms with the factor $\exp(-\zeta/\xi)$ or with factor $(\zeta - L)$ are neglected here. From eq. (61) with boundary condition (4) it is concluded that

$$- \varrho \bar{u}_z \Gamma_1 + \mu \left(\xi \, \Gamma_1'' + \frac{1}{\xi} \, \Gamma_1 \right) = - c_{p8}^{part} \,. \tag{63}$$

The function Γ_2 is set to zero, because at the inlet an exact boundary condition is not imposed.

$$\Gamma_2 \equiv 0 \,. \tag{64}$$

Combined with eqs. (63) and (64), eq. (61) reads

$$\begin{aligned} c_{p8} = {} & c_{p8}^{part} \left(1 - \exp\left(\frac{\zeta - L}{\xi} \right) \right) \\ & + \mu \left[- 2 \, \frac{\zeta - L}{\xi} \, \Gamma_1' + \left(2 \frac{\zeta - L}{\xi^2} + \frac{(\zeta - L)^2}{\xi^3} \right) \Gamma_1 \right] \exp\left(\frac{\zeta - L}{\xi} \right) \,. \end{aligned} \tag{65}$$

The second term of the sum on the right hand side of eq. (65) is omitted, because either the exponential factor or the factors linear or quadratic in $(\zeta - L)$ are small. With the same argument c_{p8}^{hom} fulfills the condition (62). The relation $\xi \approx R$ is valid in the whole annular gap, hence the result is

$$c_{p8} = 3 \, \frac{R^2}{S \;\; L} \, (p_1 - p_2) \left(1 - \exp\left(\frac{\zeta - L}{R} \right) \right) \,. \tag{66}$$

4.6 Calculation of the Added Stiffness Acting on the Pendulum

The result (66) and $\underline{q} = [0,0,0,0,0,0,0,\psi_{yB}]$ are inserted into the eq.(18).

$$p\ (\varphi, z; \psi_{yB})$$

$$= p_1 - \frac{(p_1 - p_2)}{L} z + \psi_{yB}\, 3\, \frac{R^2}{S \cdot L} (p_1 - p_2) \left(1 - \exp\left(\frac{z-L}{R}\right)\right) \cos(\varphi)$$

$$:= p_{disp}\ . \tag{67}$$

In order to include the added mass and the added damping effects, the terms which are linear in $(d\psi_{yB}/dt)$ or linear in $(d^2\psi_{yB}/dt^2)$ in eq. (4.22) in [2] must be added to p_{disp}. The pressure field $p(z,\varphi,t)$ from eq. (4.22) in [2] is denoted by p_t here. An approximation of the total transient pressure field is

$$p\ (\varphi, z; (d^2\psi_{yB}/dt^2), (d\psi_{yB}/dt), \psi_{yB})$$

$$= p_1 - \left(\frac{p_1 - p_2}{L}\right) z + (d^2\psi_{yB}/dt^2) \frac{\partial p_t}{\partial (d^2\psi_{yB}/dt^2)}$$

$$+ (d\psi_{yB}/dt) \frac{\partial p_t}{\partial (d\psi_{yB}/dt)} + \psi_{yB} \frac{\partial p_{disp}}{\partial \psi_{yB}}\ . \tag{68}$$

The equation of motion of the pendulum is

$$\Theta \frac{d^2\psi_{yB}}{dt^2} + \delta_H \frac{d\psi_{yB}}{dt} + (m_B^* g\, l_S)\, \psi_{yB}$$

$$+ R_B \left(\int_0^L dz \int_0^{2\pi} d\varphi\ p(\varphi, z; (d^2\psi_{yB}/dt^2), (d\psi_{yB}/dt), \psi_{yB})\ z \cos\varphi \right) = 0\ . \tag{69}$$

The notation in eq. (69) is

Θ Moment of inertia of the pendulum

$m_B^* g$ weight of the pendulum reduced by the buoyancy.

l_s distance between the center of gravity of cylinder B and the hinge, $l_S := L/2 - Z_B$ with $Z_B \leq L/2$.

δ_H damping of the pendulum caused by the hinge.

Inserting the pressure field (68) into eq. (69) one obtains

$$(\Theta + \Theta_F) \frac{d^2 \psi_{yB}}{dt^2} + (\delta_H + \delta_R + \delta_L) \frac{d \psi_{yB}}{dt} + (m_B^* g l_S + C_{F8}) \psi_{yB} = 0 . \qquad \textbf{(70)}$$

Θ_F and δ_R, δ_L are the added inertia and damping coefficients caused by $\partial p_t / \partial (d^2 \psi_{yB} / dt^2)$ and $\partial p_t / \partial (d \psi_{yB} / dt)$, respectively. $\Theta_F > 0$, $\delta_L > 0$ and $\delta_R > 0$ are given by eqs. (4.32) - (4.34) in [2]. C_{F8} is the added stiffness caused by p_{disp} in eq. (67). It holds

$$C_{F8} = - \frac{\partial F_8}{\partial \psi_{yB}} , \qquad \textbf{(71)}$$

with

$$F_8 = - R_B \left[\int_0^L dz \int_0^{2\pi} d\varphi \; p_{disp} \; g_8(\varphi, z) \right] . \qquad \textbf{(72)}$$

Due to $R_B = R - S/2 \approx R$ because of $S \ll R$ (P2) the integral (72) provides

$$F_8 = - \psi_{yB} \frac{R^3}{S} (p_1 - p_2) \left\{ \frac{L}{2} - R + \frac{R^2}{L} \left(1 - \exp(-\frac{L}{R}) \right) \right\} \quad \text{for } Z_B = 0 . \qquad \textbf{(73)}$$

In the case of a fluid with a density as small as the density of air, Θ_F and δ_L can be omitted in eqn. (70). Further the mass m_B of the pendulum is approximately equal to m_B^*. Then, presumed the damping ist not too large, the eigenfrequency f of the pendulum is

$$f = \frac{1}{2\pi} \sqrt{\frac{m_B g l_s + C_{F8}}{\Theta}} . \qquad \textbf{(74)}$$

5 Investigation of the Stiffness Effect Related to the Parallel Displacement of the Inner Cylinder

Here it is assumed that the outer cylinder is fixed at rest, the inner cylinder is fixed at a position where only the parallel displacement x_B is different from zero, i.e.

$$\underset{\sim}{q}^T = [0,0,0,0,x_B,0,0,0], \qquad x_B \neq 0 \ . \tag{75}$$

Again the ansatz (17) - (18) is inserted into the momentum equations (10) - (12) and into the continuity equation (13). Eq. (25) and the formula of the basic stream, eq. (35), are not influenced by the displacements $\underset{\sim}{q}$. Therefore eqs. (25) and (35) are valid throughout this report. The equations in the unknown functions c_{r5}, $s_{\varphi 5}$, c_{z5}, c_{p5}, c_{r5}, $s_{\varphi 5}$, c_{z5}, and c_{p5} are listed in Appendix B and can be solved by analogy with section 4. The result is

$$s_{p5} = s_{r5} = c_{\varphi 5} = s_{z5} \equiv 0 \ , \tag{76}$$

$$c_{z5} = \left[h_5\ (\xi)\ \frac{d\bar{u}_z}{d\xi} + 2\left(\frac{\xi - R}{S/2}\right) U_0 - 2\,U_0 \right] \ , \tag{77}$$

$$c_{r5} = s_{\varphi 5} = 0 \ , \tag{78}$$

$$c_{p5} = 0 \ . \tag{79}$$

Therefore,

$$F_5 = -R_B\, x_B \left[\int_0^L dz \int_0^{2\pi} d\varphi\ c_{p5}\ g_5(\varphi,z) \right] = 0 \ , \tag{80}$$

and axial displacements do not cause an added stiffness describable in the frame of this approximation.

6 Results

Every structural degree of freedom q_i can be treated like ψ_{yB} in section 4. The results obtained for different isolated degrees of freedom can be superimposed. This can be done because - at first - nonlinear interactions of the displacements q_i are neglected and - secondly - all calculations are linear in the q_i.

The superposition results in the pressure field

$$p\,(\varphi,z;\underline{q}) \;=\; \left(p_1 - \frac{(p_1-p_2)}{L}\,z\right) \;+\; \underline{q}^T\,\underline{c}_p\,\cos\varphi \;+\; \underline{q}^T\,\underline{s}_p\,\sin\varphi$$

with

$$\underline{c}_p^{\,T} \;=\; f(z)\;[0,0,0,-1,0,0,0,1]\quad ,$$

$$\underline{s}_p^{\,T} \;=\; f(z)\;[0,0,1,0,0,0,-1,0]\quad ,$$

$$f(z) \;=\; 3\,\frac{R^2}{S\;L}\,(p_1-p_2)\left(1-\exp\left(\frac{z-L}{R}\right)\right)\;. \qquad \textbf{(81)}$$

The force or the torque F_i acting on the i-th degree of freedom q_i is obtained from eq.(81) by

$$F_i \;=\; R_A\left[\int_0^L dz \int_0^{2\pi} d\varphi\;\; p(\varphi,z;\underline{q})\;\; g_i(\varphi,z)\right] \qquad \text{for } i=1,\ldots,4\quad ,$$

$$F_i \;=\; -R_B\left[\int_0^L dz \int_0^{2\pi} d\varphi\;\; p(\varphi,z;\underline{q})\;\; g_i(\varphi,z)\right] \qquad \text{for } i=5,\ldots,8\quad . \qquad \textbf{(82)}$$

7 Comparison with Experimental Data

Grunwald et. al. [9] measured eigenfrequencies of a rigid cylinder performing pendular motions inside a cylindrical duct, see fig. 3. The pendular motion has one degree of freedom completely corresponding to the problem in section 4. In the duct air flows from the top to the bottom. At the bottom of the annular gap near $z = L$ the air streamed into the atmosphere.

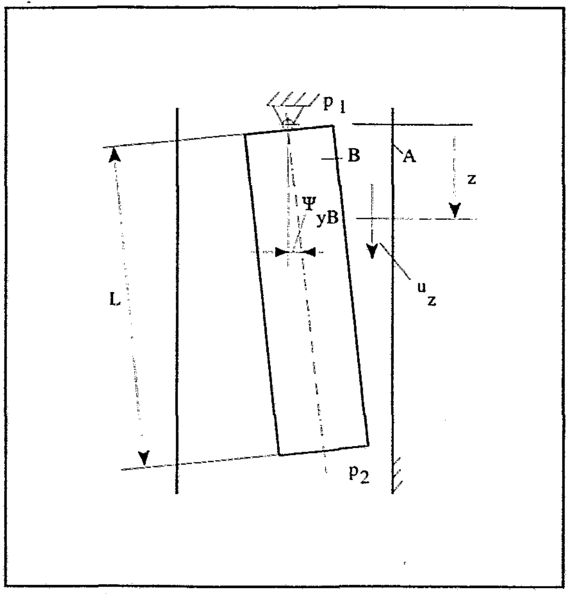

Fig. 3: Scheme of the test system.

These experiments were characterized by the following main parameters: Moment of inertia of the pendulum with respect to the hinge $\Theta = 3.7 \cdot 10^{-3}\, kg\, m^2$; mass of the pendulum $m_B = 0.44 kg$; mean radius of the two cylinders $R = 2.6 \cdot 10^{-2} m$; gap width $S = 2.5 \cdot 10^{-3} m$; length of the pendulum $L = 0.16 m$; distance between the hinge and the center of gravity of the pendulum $l_S \approx L/2$; Reynolds number range $0 \le Re \le 9000$. Here the Reynolds number Re is defined as

$$Re = \frac{2\, S\, \rho\, U}{\mu} \tag{83}$$

with

$$U := \frac{1}{2 \pi S} \int_0^{2\pi} \left(\int_{R_B}^{R_A} u_z\, d\xi \right) d\varphi \quad . \tag{84}$$

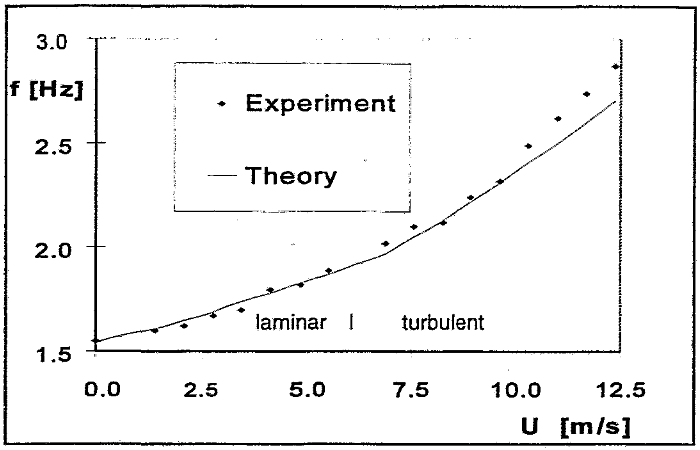

Fig. 4: Eigenfrequencies of a pendulum.

Fig. 4 depicts the dependence of the pendulums eigenfrequency on the mean velocity U. Experimental data from reference [9] are compared with theoritical eigenfrequencies calculated according to eqs.(74) and (73).

In the laminar flow regime the predicted eigenfrequencies of the pendulum agree very well with the experimental data. This statement is still valid for the turbulent flow regime with Reynolds numbers not too large. For large Reynolds numbers the added stiffness effect is underestimated. The underestimate is due to the fact that laminar theory is applied. However in ref. [9] the experimental data can even be estimated by using eq. (81) in the case of turbulent flow regime.

8 The Influence of the Boundary Conditions at the Inlet and the Outlet

The investigation [9] shows that the boundary condition (9) is well posed in the physical sense provided that at the end of the annular gap the fluid can escape into a large volume. On the other hand reference [9] describes pressure measurements which give hints that at the inlet of the annular gap a boundary condition $p = const.$ is not adequate to physical reality. If a boundary condition more restrictive than eq. (7) was demanded, the pressure field (81) could be strongly disturbed from $z=0$ downward to a depth with the magnitude of the mean radius R. According to the approximate homogenous solution (61) this disturbance would vanish in a distance of some multiples of R from the points with $z=0$.

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Appendix A: List of the Shape Functions g_i

List of the functions g_i (φ,z) in eq. (16):

$$g_1 = g_5 = \cos\varphi \quad ,$$
$$g_2 = g_6 = \sin\varphi \quad ,$$
$$g_3 = -\left(z - Z_A\right)\sin\varphi \quad ,$$
$$g_4 = \left(z - Z_A\right)\cos\varphi \quad ,$$
$$g_7 = -\left(z - Z_B\right)\sin\varphi \quad ,$$
$$g_8 = \left(z - Z_B\right)\cos\varphi \quad . \qquad \textbf{(85)}$$

Z_A and Z_B are the z - coordinates of the fixed points of the rotations ψ_{xA} , ψ_{yA} and ψ_{xB}, ψ_{yB} respectively. In fig.2 the case $Z_A = Z_B = L/2$ is depicted. In reference [9] the experiments were performed with fixed cylinder A and $Z_B \approx 0$.

Appendix B: List of Equations Related to the Parallel Displacement of the Inner Cylinder in x-Direction

Here the equations are listed which belong to the problem in section 5.

System III (Containing the unknown functions c_{r5} , $s_{\varphi 5}$, c_{z5} and c_{p5})
Momentum equation in axial direction:

cosϕ - component:

$$\partial_\zeta\, c_{p5} = -\,\rho \left[\bar{u}_z \partial_\zeta c_{z5} + c_{r5} \frac{d\bar{u}_z}{d\xi} \right]$$

$$+\,\mu \left[-\,\frac{d^2 h_5}{d\xi^2}\,\frac{d\bar{u}_z}{d\xi} - 2\,\frac{dh_5}{d\xi}\,\frac{d^2\bar{u}_z}{d\xi^2} + \partial_\xi^2 c_{z5} + \partial_\zeta^2 c_{z5} \right] . \qquad \textbf{(86)}$$

Momentum equation in radial direction:

cosϕ - component:

$$\partial_\xi\, c_{p5} = -\,\rho \bar{u}_z\, \partial_\zeta\, c_{r5} + \mu \left[\partial_\xi^2\, c_{r5} + \partial_\zeta^2\, c_{r5} - \frac{2}{\xi^2}\, s_{\varphi 5} \right] . \qquad \textbf{(87)}$$

Momentum equation in circumferential direction:

sinϕ - component:

$$c_{p5} = \rho\, \xi\, \bar{u}_z\, \partial_\zeta\, s_{\varphi 5} - \mu\, \xi \left[\partial_\xi^2\, s_{\varphi 5} + \partial_\zeta^2\, s_{\varphi 5} \right] . \qquad \textbf{(88)}$$

Continuity equation:

cosϕ - component:

$$s_{\varphi 5} = -\,\xi\, (\partial_\zeta\, c_{z5} + \partial_\xi\, c_{r5}) . \qquad \textbf{(89)}$$

System IV (Containing the unknown functions c_{r5}, $s_{\varphi 5}$, c_{z5} and c_{p5}.)

Momentum equation in axial direction:

sinϕ - component:

$$\partial_\zeta\, s_{p5} = -\,\rho \left[\bar{u}_z\, \partial_\zeta\, s_{z5} + s_{r5}\, \frac{d\bar{u}_z}{d\xi} \right] + \mu \left[\partial_\xi^2\, s_{z5} + \partial_\zeta^2\, s_{z5} \right] . \qquad \textbf{(90)}$$

Momentum equation in radial direction

sinϕ - component:

$$\partial_\xi\, s_{p5} = \rho\, \bar{u}_z\, \partial_\zeta\, s_{r5} + \mu \left[\partial_\xi^2\, s_{r5} + \partial_\zeta^2\, s_{r5} + \frac{2}{\xi^2}\, c_{\varphi 5} \right] . \qquad \textbf{(91)}$$

Momentum equation in circumferential direction:

cosϕ - component:

$$s_{p5} = -\rho\,\xi\,\bar{u}_z\,\partial_\zeta\,c_{\varphi 5} + \mu\,\xi\left[\partial_\xi^2\,c_{\varphi 5} + \partial_\zeta^2\,c_{\varphi 5}\right]. \tag{92}$$

Continuity equation:

sinϕ - component:

$$\partial_\xi\,s_{r5} + \partial_\zeta\,s_{z5} - \frac{1}{\xi}\,c_{\varphi 5} = 0\,. \tag{93}$$

Appendix C: Negative Added Stiffness Caused by a Basic Stream Flowing in Upward Direction

A situation is considered similar to that in section 4,

$$q^T = [0,0,0,0,0,0,0,\psi_{yB}]; \qquad \psi_{yB} \neq 0\,, \tag{94}$$

but

$$p_1 < p_2 \tag{95}$$

is assumed. Boundary condition (4) is omitted and

$$[p]_{z=0} \equiv p_1 \tag{96}$$

is required. The problem can be treated in the same manner as it is done in section 4 up to eq. (61). In this state of the investigation the functions $\Gamma_1(\xi)$, $\Gamma_2(\xi)$ have to be chosen such that the boundary conditions (7), (8) and (96) are fulfilled. This can be done by complete analogy with subsection 4.5. However, in the case considered here, the function Γ_1 is identically set to zero and Γ_2 is used to adapt the pressure field to the boundary condition (96). The result is

$$p(\varphi,z;\,\psi_{yB}) = p_1 + \frac{(p_2-p_1)}{L}\,z - \psi_{yB}\,3\,\frac{R^2}{S\;L}\,(p_2-p_1)\left(1-\exp\left(-\frac{z}{R}\right)\right). \tag{97}$$

In the more general case with displacements in eight degrees of freedom the result is

$$p(\varphi,z;\underset{\sim}{q}) = (p_1 + \frac{p_2 - p_1}{L} z) + \underset{\sim}{q}^T \underline{c}_p \cos\varphi + \underset{\sim}{q}^T \underline{s}_p \sin\varphi$$

with

$$\underline{c}_p^{\,T} = f_2(z)\ [0,0,0,-1,0,0,0,1]\ , \qquad (98)$$

$$\underline{s}_p^{\,T} = f_2(z)\ [0,0,1,0,0,0,-1,0]\ ,$$

$$f_2(z) = -3\ \frac{R^2}{S\ L}\ (p_2 - p_1) \left(1 - \exp\left(\frac{-z}{R} \right) \right) \quad .$$