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Interfering Doorway States and Giant Resonances I: Resonance Energies, Widths and Dipole Strengths

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Abstract

Using a phenomenological schematic model of multipole giant resonances we consider the effects of overlapping of their doorway components. The concept of the partial widths of a giant resonance becomes ambiguous when the escape widths get comparable with the spacings between the components. In such a case, the partial widths determined in terms of the K- and S-matrices differ from each other. The mixing of the doorway components due to the interaction via the common decay channels influences significantly their multipole strengths, widths and positions in energy.

1 Introduction

In spite of much efforts, the nature of giant resonances (GR) is only partly understood nowadays. Commonly accepted is that they are collective excitations formed by coherent superpositions of many correlated predominantly one particle – one hole configurations with given total quantum numbers. These superpositions are usually found by diagonalizing the residual interaction in the $1p-1h$ configuration space in the framework of the random phase (RPA) or Tamm-Dancoff (TDA) approximations. But their detailed microscopic structure still needs further investigation.

The recent progress of high energy accelerators and high precision detectors gives the possibility to analyse in coincident experiments [1, 2, 3, 4, 5] the direct particle decays of the giant resonance states into specific channels. The decay rates extracted from the data contain the desired microscopic information. They are therefore a very useful tool for a careful study of the collective modes of nuclear motion.

At the GR energies, the value which is typical for the escape widths of the configurations contributing to the collective mode exceeds usually their level spacings. The energy continuum effects play therefore an important role and cannot be treated as a perturbation slightly broadening the levels which, nevertheless, remain isolated. A few methods exist to take into account the energy continuum in a straightforward manner. Some of them [6, 7, 8, 9, 10, 11] extend the RPA by including exactly the single particle continuum. The more general approach [12, 13, 14, 15] based on the explicit separation of the intrinsic and channel subspaces is closely related to Feshbach's idea of doorway states [16] which we exploit in the present paper.

Generally, a giant resonance consists of one or a few doorway states on the background of many complicated fine structure states. Even when within the RPA the corresponding doorway states are well isolated from one another, their overlapping with the background states can give rise to interference effects [17, 18] which cannot be described by a sum of independent Breit-Wigner contributions. However, due to the nearly chaotic structure of the background states, the main effect is [19, 20, 21], after energy averaging, the damping of the doorway states described by the spreading width. In contrast, the interference of the overlapping doorway states with each other may significantly influence the form of the energy spectrum of the decay products of giant resonances as shown in [19, 20].

The interference of doorway components of a GR poses, in particular, the question on the physical meaning of its partial widths [22, 23]. It is impossible to separate the contributions from different non-orthogonal [24] overlapping doorway components into a specific decay channel. The partial widths lose, in such a situation, their standard probabilistic interpretation. More than that, in contrast to the case of isolated resonances, one must distinguish the "partial widths" determined in terms of the hermitian K-matrix from those defined by means of the unitary S-matrix when the resonances overlap.

In this paper, we investigate the interference effects caused by the overlapping of doorway states in the case of giant resonances. We use the extension, proposed in [19], of the simple Brown-Bolsterli [25] schematic model for multipole collective nuclear excitations to open (decaying) systems. Although being qualitative, this phenomenological model still maintains the main features of the real situation. The giant resonances emerge out of the interplay between two different kinds of collective behaviour: the synchronized collective intrinsic motion and the cooperative particle emission.

In sect. 2 the formalism we use is briefly described which is based on the general theory of resonance scattering. The difference between K (KPW) and T matrix (TPW) partial

widths is stressed. Further, the integral sum rules for the cross sections of the decay into specific channels are given. Our schematic model along with its formal solution are presented in sect. 3. The two kinds of collectivity appearing in the K- and S-matrix calculations are analysed analytically in sects. 4 and 5. Also the connection between the resonance spectrum and the multipole strengths of the doorway states is established. The transition strengths are essentially redistributed between the overlapping doorway states when the interaction via the energy continuum is strong. In Sect. 6, we show numerical results obtained in the same model with and without the restrictions introduced into the analytical study. The results confirm the main features of the interference picture. At strong external coupling, they can be understood in the two-level approximation. In the last section, some conclusions are drawn from the results obtained. The influence onto the cross section pattern will be studied in a forthcoming paper.

We use the matrix shorthands throughout the paper. The capital letters are used for matrices in the Hilbert space of the internal motion; matrices in the space of the scattering channels are marked by the hat symbol. The *column* vectors in the internal space as well as the *row* vectors in the channel one are represented by bold letters.

2 General Formalism

In the vicinity of each doorway component (dw) of a GR the transition matrix $\hat{T}(E)$ is usually parametrized in the framework of the random phase approximation with the standard single-resonance Breit-Wigner formula

$$\hat{T}_{dw}(E) = \frac{\hat{K}(E)}{1 + \frac{i}{2}\hat{K}(E)} = \frac{\hat{\mathbf{A}}_{dw}^T \hat{\mathbf{A}}_{dw}}{E - E_{dw} + \frac{i}{2}\Gamma_{dw}} \,. \tag{2.1}$$

Here, the row vector $\hat{A}_{dw}$ is composed of the k real decay amplitudes A_{dw}^c of the doorway state into the individual channels $c = 1, 2, ..., k$. The superscript T means transposition. Contrary to the matrix $\hat{T}(E)$ which possesses the pole $E = \mathcal{E}_{dw} = E_{dw} - \frac{i}{2}\Gamma_{dw}$ in the complex energy plane, the pole of the hermitian matrix

$$\hat{K}(E) = \frac{\hat{\mathbf{A}}_{dw}^T \hat{\mathbf{A}}_{dw}}{E - E_{dw}} \tag{2.2}$$

lies on the real energy axis at the energy E_{dw} of the resonance state. The corresponding residues are however the same for both matrices. The residues $\Gamma_{dw}^c = (A_{dw}^c)^2$ of the diagonal elements of these matrices are the partial escape widths of the state dw relative to the channels c. The hermiticity of the K-matrix automatically provides the unitarity of the scattering matrix $\hat{S}(E) = I - i\hat{T}(E)$ implying the well known connection

$$\Gamma_{dw} = \hat{\mathbf{A}}_{dw}^2 = \sum_c \Gamma_{dw}^c \tag{2.3}$$

between the total, Γ_{dw}, and the partial widths of the resonance dw. In what follows we omit all nonresonant effects. They can, if necessary, be easily taken into account by standard methods.

Using the parametrization (2.1), the partial widths of the resonance state can be extracted from the experimental data. Averaging the cross section of the reaction $c' \to c$ over

all initial channels c', one obtains, with the help of the unitarity condition, the strength

$$\sigma^c(E) = -\frac{\sigma_0}{\pi}\,\mathrm{Im}\,T^{cc}_{dw}(E) = \sigma_0\,\frac{1}{2\pi}\,\frac{\Gamma_{dw}}{(E-E_{dw})^2+\frac{1}{4}\Gamma^2_{dw}}\,\Gamma^c_{dw} \tag{2.4}$$

of the transition into the channel c. The first two factors describe the cross section of the doorway state excitation. Below we set the factor σ_0 to unity measuring all cross sections in units of this quantity. The maximal value

$$\sigma^c(E_{dw}) = \frac{2}{\pi}\,\frac{\Gamma^c_{dw}}{\Gamma_{dw}} \equiv \frac{2}{\pi}\,\mathcal{B}^c_{dw} \tag{2.5}$$

of the transition strength (2.4) is proportional to the branching ratio corresponding to the decay in the channel c. The integration over the whole resonance region gives the partial width itself,

$$\int_{-\infty}^{\infty} dE\;\sigma^c(E) = \Gamma^c_{dw}\;. \tag{2.6}$$

The above discussion implies a good separation of the different resonance states dw so that any interference between them can be neglected. A more careful analysis is however needed when the widths of the relevant doorway states become comparable with their spacings. In this case one has to use the formulae of the general theory of resonance reactions [26, 27, 28, 29]. Here, the transition matrix

$$\hat{T}(E) = A^T\,\frac{1}{E-\mathcal{H}}\,A \tag{2.7}$$

is composed of the three matrix factors which describe the formation of the intermediate unstable system, its propogation and subsequent desintegration. If there are N_{dw} doorway resonance states near the excitation energy E coupled to k decay channels, the matrix A consists of k N_{dw}-dimensional column vectors $\mathbf{A}^c$ connecting all internal states with each channel c. These vectors are real because of time-reversal invariance. In the following we neglect a possible smooth energy dependence of the components A^c_n over the whole energy domain considered. The validity of such an assumption is not always obvious and deserves a special consideration. It may lead to further complications.

The evolution of the intermediate open system is described by the Green's matrix

$$\mathcal{G}(E) = \frac{1}{E-\mathcal{H}} \tag{2.8}$$

corresponding to the non-hermitian effective Hamiltonian

$$\mathcal{H} = H - \frac{i}{2}AA^T\,. \tag{2.9}$$

Its antihermitian part

$$W = AA^T \qquad \Longrightarrow \qquad W_{mn} = \sum_{c(open)} A^c_m A^c_n \tag{2.10}$$

originates from the on-shell self-energy contributions of the decays. The factorized form (2.10) of the interaction via the continuum ensures the unitarity of the scattering matrix for arbitrarily overlapping resonances [28, 29]. However, the simple Breit-Wigner parametrization (2.1) loses its validity in general.

The propagator $\mathcal{G}(E)$ of the unstable system satisfies the Dyson equation

$$\mathcal{G}(E) = G(E) - \frac{i}{2}\, G(E)\, W\, \mathcal{G}(E) \tag{2.11}$$

where

$$G(E) = \frac{1}{E - H} \tag{2.12}$$

is the resolvent of the hermitian part H of the effective Hamiltonian (2.9). Subsequent iterations in the antihermitian part of the effective Hamiltonian lead to

$$\mathcal{G}(E) = G(E) - \frac{i}{2}\, G(E)\, A\, \frac{1}{1 + \frac{i}{2}\hat{K}(E)}\, A^T\, G(E) \tag{2.13}$$

by using the factorized form (2.10) [30]. Now

$$\hat{K}(E) = A^T\, \frac{1}{E - H}\, A = A^T\, G(E)\, A\,. \tag{2.14}$$

The relation (2.13) casts the transition matrix (2.7) into the explicitly unitary form

$$\hat{T}(E) = \frac{\hat{K}(E)}{1 + \frac{i}{2}\hat{K}(E)}\,. \tag{2.15}$$

The elements of both the K- and T- channel space matrices are presented by

$$K^{cc'}(E) = \mathrm{Tr}\, \left(G(E)\, \mathbf{A}^c\, (\mathbf{A}^{c'})^T \right)\,, \tag{2.16}$$

$$T^{cc'}(E) = \mathrm{Tr}\, \left(\mathcal{G}(E)\, \mathbf{A}^c\, (\mathbf{A}^{c'})^T \right) \tag{2.17}$$

as the matrix traces in the Hilbert space of the internal motion. Therefore, they are invariant with respect to any equivalency transformation of this space. These transformations connect different parametrizations of the reaction amplitudes. The concrete choice of the parametrization is dictated by physical reasons as well as by convenience. For instance, one can use the eigenbasis of the hermitian part H of the effective Hamiltonian (2.9) to represent the K-matrix as the sum

$$\hat{K}(E) = \sum_r \frac{\hat{\mathbf{A}}_r^T \hat{\mathbf{A}}_r}{E - \varepsilon_r} \tag{2.18}$$

over the internal states r corresponding to all N overlapping resonances. Each term of this sum is directly analogous to the single-resonance expression (2.2). The row vectors $\hat{\mathbf{A}}_r$ consist of the real components

$$A_r^c = \mathbf{\Phi}^{(r)} \cdot \mathbf{A}^c \tag{2.19}$$

where the eigenvector $\mathbf{\Phi}^{(r)}$ of the hermitian matrix H belongs to the eigenenergy ε_r. The positive residues

$$\Gamma_r^c = (A_r^c)^2 \tag{2.20}$$

at the poles of the diagonal elements of the matrix (2.18) may be called the K-matrix partial widths (KPW). They characterize the coupling of the internal states $\mathbf{\Phi}^{(r)}$ to the continuum. It must be stressed however that, contrary to the case of isolated resonances, they differ from the residues at the poles of the matrix $\hat{T}(E)$ when the resonances overlap.

The pole (resonance) parametrization of the transition matrix (2.7),

$$\hat{T}(E) = \sum_{dw} \frac{\hat{\mathbf{A}}^T_{dw} \hat{\mathbf{A}}_{dw}}{E - \mathcal{E}_{dw}} \tag{2.21}$$

is achieved by diagonalizing the total effective Hamiltonian (2.9) with the help of a transformation Ψ which is complex since the Hamiltonian $\mathcal{H}$ is not hermitian. Its complex eigenvalues

$$\mathcal{E}_{dw} = E_{dw} - \frac{i}{2}\Gamma_{dw} \tag{2.22}$$

determine the energies and total widths of the overlapping resonance states. The decay amplitudes of these states are (compare with (2.19))

$$A^c_{dw} = \mathbf{\Psi}^{(dw)} \cdot \mathbf{A}^c \tag{2.23}$$

with $\Psi^{(dw)}$ being the eigenvectors of the effective Hamiltonian $\mathcal{H}$. Together with these eigenvectors, the residues at the resonance poles are also complex. Therefore, the resonances are mixed with nonzero relative phases. In particular, the residues are equal to

$$(A^c_{dw})^2 = |A^c_{dw}|^2 \exp(2i\phi^c_{dw}) \tag{2.24}$$

in the elastic scattering amplitudes. Here, the resonance mixing phases ϕ^c_{dw} are introduced. The quantities

$$\Gamma^c_{dw} = |A^c_{dw}|^2 \tag{2.25}$$

are usually interpreted as the partial widths of the resonance states dw. In analogy with (2.20) we will refer to them as the T-matrix partial widths (TPW).

The transformation matrix Ψ satisfies the matrix equation

$$\mathcal{H}\Psi = \Psi\mathcal{E} \tag{2.26}$$

where $\mathcal{E}$ is the diagonal matrix of resonance energies $\mathcal{E}_{dw}$. This transformation is complex orthogonal [30],

$$\Psi^T\Psi = \Psi\Psi^T = 1\,. \tag{2.27}$$

However, for the hermitian matrix

$$U = \Psi^\dagger\Psi \tag{2.28}$$

the inequality $U \neq I$ holds so that the overlapping resonance states are not orthogonal (for illustration see [31]). The matrix U appears in the well-known Bell-Steinberger relation [24] (see also a compact matrix version of this relation in [30])

$$\hat{\mathbf{A}}^*_{dw} \cdot \hat{\mathbf{A}}_{dw'} = i\, U_{dw\,dw'} \left(\mathcal{E}_{dw'} - \mathcal{E}^*_{dw}\right)\,. \tag{2.29}$$

Its diagonal part gives the relation

$$\Gamma_{dw} = \frac{1}{U_{dw}} |\hat{\mathbf{A}}_{dw}|^2 = \frac{1}{U_{dw}} \sum_c |A^c_{dw}|^2 \tag{2.30}$$

between the total widths and TPW (2.25). Here

$$U_{dw} = 1 + 2\sum_n \left(\mathrm{Im}\Psi^{(dw)}_n\right)^2 > 1 \tag{2.31}$$

is the corresponding diagonal matrix element of the matrix U.

Because of eqs. (2.30) and (2.31), the inequality condition

$$\Gamma_{dw} < \sum_c \Gamma^c_{dw} \tag{2.32}$$

holds in contrast to the equality (2.3) characteristic for an isolated resonance.

As it follows from eq. (2.30), the TPW can be renormalized as

$$\tilde{\Gamma}^c_{dw} = \frac{1}{U_{dw}} \Gamma^c_{dw} \,, \tag{2.33}$$

[32, 31] leading to the equality

$$\Gamma_{dw} = \sum_c \tilde{\Gamma}^c_{dw} \tag{2.34}$$

also for overlapping resonances. It should be emphasized however that neither the Γ^c_{dw} nor the renormalized quantities $\tilde{\Gamma}^c_{dw}$ coincide with the KPW Γ^c_τ from eq. (2.20) in the case of overlapping resonances. The only relation between them,

$$(\mathbf{A}^c)^2 = \sum_\tau \Gamma^c_\tau = \sum_{dw} \Gamma^c_{dw} \exp(2i\phi^c_{dw}) \leq \sum_{dw} \Gamma^c_{dw} \,, \tag{2.35}$$

follows from the completeness of the sets of the corresponding eigenvectors. Similarly, the energies ε_τ differ from the energies E_{dw} of the resonance eigenstates. In the second equality (2.35) additional phase factors appear in the sum over the resonance states. The imaginary part of this sum vanishes since the contributions of different resonances perfectly compensate one another.

The condition (2.35) results in the integral sum rule

$$\int_{-\infty}^{\infty} dE\, \sigma^c(E) = -\frac{1}{\pi} \int_{-\infty}^{\infty} dE\, \mathrm{Im}\, T^{cc}(E) = (\mathbf{A}^c)^2 = \sum_\tau \Gamma^c_\tau \tag{2.36}$$

instead of eq. (2.6) for an isolated resonance. The integration is extended here over the whole energy region, occupied by the overlapping resonance states. It leads to the sum of the KPW Γ^c_τ, (2.20), rather than to the sum of the TPW Γ^c_{dw}, (2.25). Therefore, one cannot learn much on the latter or even on their sum $\sum_{dw} \Gamma^c_{dw}$ from the integral (2.36) despite of the expectation sometimes being expressed in the scientific literature. Still less information can be drawn from the maxima of the total cross section since their heights and positions are connected with the widths and energies of the overlapping resonances in a very complicated way.

A useful generalization of the sum rule (2.36) reads

$$-\frac{1}{\pi} \int_{-\infty}^{\infty} dE\, \mathrm{Im}\, T^{cc'}(E) = \mathbf{A}^c \cdot \mathbf{A}^{c'} = X^{cc'} \,. \tag{2.37}$$

In this relation the $k \times k$ matrix [33, 30]

$$\hat{X} = A^T A \tag{2.38}$$

of the scalar products of the real amplitude vectors $\mathbf{A}^c$ appears. We will use this matrix in the following sections.

3 The Model

As usual (see for example [27]), we suggest that a hierarchy of complexities of the internal states of the system under consideration exists. The first class contains the simplest states presumably of $1p-1h$ nature which are directly connected to the continuum by appreciable transition matrix elements A_n^c. The states of the other classes of complexity have no direct connection to the continuum, at least to the same set of channel states as the first class. They can decay into these channels only through the states of the first class if they are connected to them by some residual internal interaction.

In this paper, we restrict ourselves mainly to the consideration of the dynamics in the subspace of the states of the class 1. In our schematic model, the hermitian part

$$H = H_0 + \mathbf{D}\mathbf{D}^T \tag{3.1}$$

of the effective Hamiltonian (2.9) is chosen to be equal to that in the pioneering work by Brown and Bolsterli [25]. It consists of the unperturbed part H_0 containing N discrete intrinsic levels $e_n (n = 1, 2, ..., N)$, and of the factorized residual internal interaction of, let us say, dipole-dipole type. Adding to H the external interaction (2.10) via common decay channels, we get the effective Hamiltonian

$$\mathcal{H} = H_0 + \mathcal{H}^{(int)} = H_0 + \mathbf{D}\mathbf{D}^T - \frac{i}{2} A A^T \equiv H - \frac{i}{2} W \equiv \mathcal{H}_0 + \mathbf{D}\mathbf{D}^T \tag{3.2}$$

which is the many-channel version of the effective Hamiltonian considered in ref. [19]. Having omitted the coupling to the sea of more complicated states, we neglect in particular the damping and spreading widths of the collective exitations formed by superpositions of the states of the class 1. They can be taken into account in a next step by using statistical methods (see subsection 6.3).

The resonance spectrum of the model is given by the secular equation in the complex energy plane

$$\mathrm{Det}\,(\mathcal{E} - \mathcal{H}) = \mathrm{Det}\,(\mathcal{E} - \mathcal{H}_0) \left[1 - \mathbf{D}^T \frac{1}{\mathcal{E} - \mathcal{H}_0} \mathbf{D}\right] = 0 \tag{3.3}$$

or

$$1 - \mathbf{D}^T \frac{1}{\mathcal{E} - \mathcal{H}_0} \mathbf{D} =$$

$$1 - \mathbf{D}^T G_0(\mathcal{E})\, \mathbf{D} + \frac{i}{2} \mathbf{D}^T G_0(\mathcal{E})\, A \frac{1}{1 + \frac{i}{2} \hat{K}_0(\mathcal{E})} A^T G_0(\mathcal{E})\, \mathbf{D} = 0 \tag{3.4}$$

where $G_0(E) = (E - H_0)^{-1}$ is the Green's matrix describing the unperturbed intrinsic motion and $\hat{K}_0(E) = A^T G_0(E) A$ is the K-matrix without internal residual interaction. We have used in eq. (3.3) the factorized structure $\mathbf{D}\mathbf{D}^T$ of this interaction and in eq. (3.4) the relation (2.13). The last term on the l.h.s. of the second line of eq. (3.4) describes the influence of the external interaction via the continuum onto the energy spectrum of the internal motion. This equation can easily be reduced to an algebraic equation of the order N producing the N complex energies of the resonance states.

In a similar manner we obtain

$$\hat{K}(E) = A^T \frac{1}{E - H} A = \hat{K}_0(E) + A^T G_0(E)\, \mathbf{D} \frac{1}{1 - \mathbf{D}^T G_0(E)\, \mathbf{D}} \mathbf{D}^T G_0(E)\, A\,. \tag{3.5}$$

Together with the expression (2.15) for the reaction matrix $\hat{T}(E)$, the eqs. (3.4) and (3.5) present the explicit solution of our model. In the following we investigate this solution analytically as well as numerically.

Before finishing this section, let us consider the limit of a very strong internal interaction (see below for the explicit condition). It is well known that such an interaction leads to the creation of a collective vibration mode which is shifted in energy from the location of the original unperturbed (parental) levels by a distance of the order of magnitude of $Tr(\mathbf{D}\mathbf{D}^T) = \mathbf{D}^2$. Characterizing this location by some average position $\varepsilon_0 = \langle e \rangle$, we find the complex energy

$$\mathcal{E}_{gr} = E_{gr} - \frac{i}{2}\Gamma_{gr} = \varepsilon_0 + \mathbf{D}^2 - \frac{i}{2}\hat{\mathbf{A}}_d^2 \tag{3.6}$$

of the isolated giant resonance state from the secular equation (3.4). In this limit, the K-matrix (3.5) simplifies to the expression (2.2) and the transition matrix (2.15) acquires the standard Breit-Wigner form (2.1). The partial widths

$$\Gamma_{gr}^c = (\mathbf{d} \cdot \mathbf{A}^c)^2 \tag{3.7}$$

of the giant resonance (3.6) are determined by the projections

$$A_d^c \equiv (\mathbf{d} \cdot \mathbf{A}^c) \tag{3.8}$$

of the decay amplitudes $\mathbf{A}^c$ onto the unit dipole vector $\mathbf{d} = \mathbf{D}/\sqrt{\mathbf{D}^2}$. These projections are the components of the row vector $\hat{\mathbf{A}}_d$.

Generally, eqs. (3.5, 2.15) lead however to a more complicated energy dependence of the cross section as that described by (2.1) because of the interference of the resonance states. In the following we investigate this behaviour in detail.

4 Collective Phenomena

It is convenient to treat the factorized internal residual interaction $\mathbf{D}\mathbf{D}^T$ formally in the same manner as the external one introducing an additional imaginary "channel" with the "decay amplitudes" $\mathbf{A}^0 \equiv \sqrt{2i}\mathbf{D}$. Defining the new matrix

$$\mathcal{A} = \left(\mathbf{A}^0 \;\; \mathbf{A}^1 \;\; \ldots \;\; \mathbf{A}^k\right) , \tag{4.1}$$

we will consider the following two matrices in the enlarged channel space:

$$\hat{\mathcal{K}}(E) = \mathcal{A}^T G(E) \mathcal{A} \tag{4.2}$$

and

$$\hat{\mathcal{T}}(E) = \mathcal{A}^T \mathcal{G}(E) \mathcal{A} . \tag{4.3}$$

Besides the $k \times k$ blocks $\hat{K}(E)$ and $\hat{T}(E)$ along the main diagonals, these matrices contain additionally the following functions:

$$\mathcal{K}^{00}(E) \equiv 2i\, P(E) = 2i\, \mathbf{D}^T G(E)\, \mathbf{D} \tag{4.4}$$

and

$$\mathcal{T}^{00}(E) \equiv 2i\, \mathcal{P}(E) = 2i\, \mathbf{D}^T \mathcal{G}(E)\, \mathbf{D} \tag{4.5}$$

respectively in their upper left corners. The function $P(E)$ carries information on the degree of collectivity of the internal motion whereas $\mathcal{P}(E)$ describes the interplay of both kinds of the collective behaviour. These functions are closely connected to the photoemission. The exact solution (2.15, 3.5) can now be compressed in the very compact expression

$$\hat{T}(E) = \frac{\mathcal{A}^T G_0(E)\,\mathcal{A}}{1 + \frac{i}{2}\,\mathcal{A}^T G_0(E)\,\mathcal{A}} \tag{4.6}$$

in terms of the amplitudes (4.1).

4.1 Internal Collectivity

The degree of collectivity of an internal state described by the eigenvector $\mathbf{\Phi}^{(\tau)}$ of the hermitian part H of the effective Hamiltonian (3.2) is characterized by the residue $\left(\mathbf{\Phi}^{(\tau)} \cdot \mathbf{D}\right)^2$ of the function $P(E)$ eq. (4.4) at its pole at the energy $E = \varepsilon_\tau$. To estimate this degree, let us note first that the collectivity can become appreciable only if the internal interaction is sufficiently strong and dominates the internal dynamics. Under such a condition it is natural to start with the diagonalization of the interaction matrix $\mathbf{D}\mathbf{D}^T$. Because of its factorized structure, this matrix possesses the only nonzero eigenvalue $\mathbf{D}^2$ belonging to the eigenvector

$$\mathbf{\Upsilon}^{(1)} = \mathbf{d} = \mathbf{D}/\sqrt{\mathbf{D}^2}\,. \tag{4.7}$$

The rest of the eigenstates $\mathbf{\Upsilon}^{(\mu)}$, $\mu = 2, 3, ..., N$ remains arbitrary because of the degeneracy of the zero eigenvalue. These states span a basis in the $(N-1)$-dimensional subspace orthogonal to the vector $\mathbf{d}$. We will fix this basis a little bit later.

In the new basis the function $P(E)$ reduces to

$$P(E) = \mathrm{Tr}\left(\frac{1}{E - H}\,\mathbf{D}\mathbf{D}^T\right) = \mathbf{D}^2\, G_{coll}(E) \tag{4.8}$$

where $G_{coll}(E)$ is the upper diagonal matrix element of the internal Green's matrix. The Hamiltonian matrix H looks as follows:

$$H = \begin{pmatrix} \varepsilon_0 + \mathbf{D}^2 & \mathbf{h}^T \\ \mathbf{h} & \tilde{H} \end{pmatrix}. \tag{4.9}$$

Here the energy

$$\varepsilon_0 \equiv \sum_n e_n\, d_n^2 = \langle e \rangle \tag{4.10}$$

is the weighted mean position of the parental levels e_n. The $(N-1)$-dimensional vector $\mathbf{h}$ has the components

$$h_\mu = \sum_n e_n\, d_n\, \Upsilon_n^{(\mu)} \tag{4.11}$$

while the matrix elements of the $(N-1) \times (N-1)$ submatrix $\tilde{H}$ are equal to

$$\tilde{H}_{\mu\nu} = \sum_n e_n\, \Upsilon_n^{(\mu)}\, \Upsilon_n^{(\nu)}\,. \tag{4.12}$$

Representing similar to (4.9) the resolvent $G(E)$ as

$$G(E) = \begin{pmatrix} G_{coll}(E) & \mathbf{F}^T(E) \\ \mathbf{F}(E) & \tilde{G}(E) \end{pmatrix}, \tag{4.13}$$

one finds

$$G_{coll}(E) = \frac{1}{E - \varepsilon_0 - \mathbf{D}^2 - \mathbf{h}^T \frac{1}{E-\tilde{H}} \mathbf{h}} \tag{4.14}$$

when

$$\mathbf{F}(E) = \frac{1}{E - \tilde{H}} \mathbf{h} \; G_{coll}(E) \tag{4.15}$$

and

$$\tilde{G}(E) = \frac{1}{E - \tilde{H}} + \frac{1}{E - \tilde{H}} \mathbf{h}\mathbf{h}^T \frac{1}{E - \tilde{H}} \; G_{coll}(E) \, . \tag{4.16}$$

Each eigenvalue ε_r of the Hamiltonian H satisfies the equation

$$\lambda(E) \equiv E - \varepsilon_0 - \mathbf{D}^2 - \mathbf{h}^T \frac{1}{E - \tilde{H}} \mathbf{h} = 0 \tag{4.17}$$

and the value

$$f^r = \left(\mathbf{d} \cdot \mathbf{\Phi}^{(r)}\right)^2 = ResP(\varepsilon_r)/\mathbf{D}^2 = \left(\frac{d\lambda(E)}{dE}\right)^{-1}_{E=\varepsilon_r} = \left[1 + \mathbf{h}^T \left(\frac{1}{\varepsilon_r - \tilde{H}}\right)^2 \mathbf{h}\right]^{-1} , \tag{4.18}$$

subject to the condition

$$\sum_r f^r = 1 \, , \tag{4.19}$$

describes the part of the dipole strength carried by the eigenstate $\mathbf{\Phi}^{(r)}$.

Further, we diagonalize the submatrix (4.12),

$$\sum_n e_n \, \Upsilon_n^{(\mu)} \, \Upsilon_n^{(\nu)} = \tilde{\varepsilon}_\mu \, \delta_{\mu\nu} \tag{4.20}$$

by choosing the till now unspecified basic vectors $\mathbf{\Upsilon}^{(\mu)}$ to coincide with its eigenvectors. Using the completeness condition

$$\sum_\nu \Upsilon_m^{(\nu)} \Upsilon_n^{(\nu)} = \delta_{mn} - d_m \, d_n \tag{4.21}$$

one easily obtains

$$\mathbf{\Upsilon}^{(\mu)} = -h_\mu \frac{1}{\tilde{\varepsilon}_\mu - H_0} \mathbf{d} \, . \tag{4.22}$$

The matrix elements h_μ play the role of the normalization coefficients of the eigenvectors and are equal to

$$h_\mu = \left[\mathbf{d}^T \left(\frac{1}{\tilde{\varepsilon}_\mu - H_0}\right)^2 \mathbf{d}\right]^{-\frac{1}{2}} \, . \tag{4.23}$$

The orthogonality condition $\mathbf{d} \cdot \mathbf{\Upsilon}^{(\mu)} = 0$ immediately leads to the equation

$$\mathbf{d}^T \frac{1}{\tilde{\varepsilon} - H_0} \mathbf{d} = \sum_n \frac{d_n^2}{\tilde{\varepsilon} - e_n} = 0 \tag{4.24}$$

for the eigenvalue spectrum of the submatrix $\tilde{H}$. Obviously, each eigenvalue $\tilde{\varepsilon}_\mu$ lies between two neighboring parental levels e. Therefore, the levels $\tilde{\varepsilon}$ are shifted, with respect to the original ones, by distances of the order of magnitude of the unperturbed mean level spacing.

This is much smaller than the energy shift $\sim \mathbf{D}^2$ of the collective level in the upper corner of the Hamiltonian matrix (4.9).

The collective level $\varepsilon_{coll} = \varepsilon_0 + \mathbf{D}^2$ is mixed with the $N-1$ levels $\tilde{\varepsilon}_\mu$ via the matrix elements h_μ. The square length of the vector $\mathbf{h}$ turns out to be equal to the variance

$$\mathbf{h}^2 = \sum_n e_n^2\, d_n^2 - \left(\sum_n e_n d_n^2\right)^2 = \sum_n (e_n - \langle e\rangle)^2\, d_n^2 = \langle (e - \langle e\rangle)^2\rangle = \Delta_e^2 \qquad (4.25)$$

of the distribution of the parental levels. This leads to the estimation $|h_\mu| \sim \Delta_e/\sqrt{N-1}$ of the individual matrix elements. Therefore, the mixing is governed by the parameter $\kappa = \Delta_e/\mathbf{D}^2$. Suggesting that this parameter is small, $\kappa \ll 1$, one can use the standard perturbation expansion which gives

$$\varepsilon_1 = \varepsilon_0 + \left(1+\kappa^2\right)\mathbf{D}^2\,, \qquad |\varepsilon_r - \tilde{\varepsilon}_r| \sim \frac{\kappa^2}{N-1}\,\mathbf{D}^2 \quad (r \neq 1) \qquad (4.26)$$

for the levels and

$$f^1 = 1 - \kappa^2\,, \qquad f^r \sim \frac{\kappa^2}{N-1} \quad (r \neq 1) \qquad (4.27)$$

for the dipole strengths. The first level accumulates the lion's share of both the total dipole strength and the energy displacement. In the limit $\kappa \to 0$ the collectivity of the first level becomes perfect while the rest of the levels carries no collectivity at all.

4.2 External Collectivity

Let us now turn to the properties of the K-matrix eq. (3.5) in our model. In the absence of the internal interaction the KPW would be equal to the squares $(A_n^c)^2$ of the original amplitudes. However, the strong internal interaction causes a remarkable redistribution of these widths. In this case, the Υ-basis (4.7, 4.22) constructed above becomes a preferential one. Taking into account eqs. (4.15, 4.16) one gets in this basis

$$\hat{K}(E) = \left[\hat{\mathbf{A}}_d + \mathbf{h}^T \frac{1}{E-\tilde{H}}\, A_\perp\right]^T \left[\hat{\mathbf{A}}_d + \mathbf{h}^T \frac{1}{E-\tilde{H}}\, A_\perp\right] G_{coll}(E) + A_\perp^T \frac{1}{E-\tilde{H}}\, A_\perp\,. \quad (4.28)$$

The rectangular submatrix $A_\perp$ is composed of $(N-1)$-dimensional column vectors $\mathbf{A}_\perp^c$ orthogonal to the dipole vector,

$$(\mathbf{d}\cdot\mathbf{A}_\perp^c) = 0\,, \qquad A_\mu^c = \left(\Upsilon^{(\mu)}\cdot\mathbf{A}^c\right), \qquad (4.29)$$

whereas the row vector $\hat{\mathbf{A}}_d$ of the longitudinal components A_d^c is defined in (3.8). It can easily be checked that the contributions of the poles at the energies $E = \tilde{\varepsilon}_\mu$ in the two terms on the r.h.s. perfectly cancel each other. The actual poles of the K-matrix are given by the roots of the equation (4.17).

It immediately follows from (4.28) that the KPW are equal to

$$\Gamma_r^c = f^r \left[A_d^c + \mathbf{h}^T \frac{1}{\varepsilon_r - \tilde{H}}\, \mathbf{A}_\perp^c\right]^2 \qquad (4.30)$$

and depend on the relative strength κ of the residual mixing. In particular, by using condition (4.21) one finds

$$\left|\mathbf{h}^T \frac{1}{\varepsilon_1 - \tilde{H}} \mathbf{A}_\perp^c\right| \approx \frac{1}{\mathbf{D}^2} |(\mathbf{h} \cdot \mathbf{A}_\perp^c)| \approx \frac{1}{\mathbf{D}^2} \left|\sum_n (e_n - \varepsilon_0) d_n \left(A_\perp^c\right)_n\right| \leq \kappa \sqrt{(\mathbf{A}_\perp{}^c)^2} \qquad (4.31)$$

for the collective level if $\kappa \ll 1$.

In the square bracket of eq. (4.30) the first term dominates for the collective level $r = 1$ as long as $|A_d^c|/|A_\perp^c| \gg \kappa$. Therefore, $\Gamma_1^c \approx (A_d^c)^2$ under such a condition. The remaining part $(\mathbf{A}_\perp^c)^2$ is distributed over the $N-1$ levels lying in the energy interval $\sim \Delta_e$ around the point ε_0. In this region the pattern turns out to depend crucially on the ratio $(\mathbf{A}_\perp^c)^2/\Delta_e^2$. Each state acquires the partial width $\sim (\mathbf{A}_\perp^c)^2/(N-1)$ if this ratio is small while a strong redistribution of the widths occurs in the opposite case $(\mathbf{A}_\perp^c)^2/\Delta_e^2 \gg 1$. It is called "width collectivization" [34, 30, 35] or "trapping effect" [36, 37, 38, 39, 40, 41]: k eigenstates $\Phi^{(r)}$ get large components along the vector $\mathbf{A}_\perp^c$ and accumulate almost the total value $(\mathbf{A}_\perp^c)^2$ (see also [42, 43, 44]). This phenomenon was first observed in realistic numerical simulations of nuclear reactions in [45, 46, 47]. In the limit $\kappa = 0$, when the internal collectivity is maximal, the expression (4.28) reduces to

$$\hat{K}(E) = \frac{\hat{\mathbf{A}}_d^T \hat{\mathbf{A}}_d}{E - \varepsilon_{coll}} + \frac{\hat{X}_\perp}{E - \varepsilon_0}\,. \qquad (4.32)$$

Here the matrix

$$\hat{X}_\perp = \hat{X} - \hat{\mathbf{A}}_d^T \hat{\mathbf{A}}_d = A_\perp^T A_\perp \qquad (4.33)$$

is composed of the scalar products $(\mathbf{A}_\perp^c \cdot \mathbf{A}_\perp^{c'})$ in the orthogonal subspace (compare with eq. (2.38)).

One can immediately see from eq. (4.32) that the KPW of the only collective state with the energy $\varepsilon_{coll} = \varepsilon_0 + \mathbf{D}^2$ is equal to:

$$\Gamma_{coll}^c = (A_d^c)^2 \qquad (4.34)$$

in agreement with eq. (3.7). However, as it follows from the general consideration given in section 2 (see also below), the TPW does not coincide with the KPW if the collective state overlaps the states jointly presented in the given limit by the single pole at the energy $E = \varepsilon_0$.

The residues at the pole ε_0 of the matrix (4.32) do not factorize contrary to that at the collective pole. This means that different linear superpositions of the original states are excited via different channels at the same energy ε_0. To find their KPW, one has first to diagonalize the matrix $\hat{X}_\perp$. The corresponding partial widths are then expressed as

$$\Gamma_r^c = \gamma_\perp^r \left(\xi_{\perp c}^{(r)}\right)^2, \qquad (r = 1, 2, ..., k) \qquad (4.35)$$

in terms of the eigenvalues $\gamma_\perp^r$ and the left eigenvectors $\xi_\perp^{(r)}$ of the matrix $\hat{X}_\perp$. One sees that in the considered limit of very strong internal collectivity only k superpositions out of the $N-1$ ones with the energy ε_0 possess nonzero KPW. They absorb the part

$$\sum_{r=1}^{r=k} \Gamma_r^c = (\mathbf{A}_\perp^c)^2 = (\mathbf{A}^c)^2 - \Gamma_{coll}^c \qquad (4.36)$$

of the total original value $(\mathbf{A}^c)^2$. We conclude that the KPW of the states presented in the K-matrix (4.32) are formed by coherent contributions of all parental states.

5 Interplay of Two Kinds of Collectivity. Interference of Doorway Resonances

5.1 The Doorway Basis

Now we turn to the matrix $\hat{T}(E)$ containing both kinds of collectivity on equal footing. Since the interaction plays the dominant role in the dynamics studied, we start with the consideration of the interaction matrix

$$\mathcal{H}^{(int)} = \mathbf{D}\mathbf{D}^T - \frac{i}{2} A A^T \,. \tag{5.1}$$

The manifold of the $k+1$ linearly independent vectors $\mathbf{D}$ and $\mathbf{A}^c$ forms a $(k+1)$-dimensional subspace in the internal Hilbert space the total dimension of which is N. It is convenient to choose the first $k+1$ basis vectors of the total Hilbert space in such a manner that they belong entirely to this subspace. Then only the upper left $(k+1) \times (k+1)$ block of the interaction matrix will contain non-zero matrix elements. We proceed in the following three steps:

(i) Let us first orthogonalize the set of the k vectors $\mathbf{A}^c$. For this purpose, we diagonalize the matrix $\hat{X}$, eq. (2.38), of the scalar products of these vectors. Let $\hat{\xi}$ be the matrix of the (left) eigenvectors,

$$\hat{\xi}\,\hat{X} = \hat{\gamma}\,\hat{\xi} \tag{5.2}$$

where

$$\hat{\gamma} = diag\,(\gamma^1\ \gamma^2 \ldots \gamma^k) \tag{5.3}$$

is the diagonal matrix of the eigenvalues. It is then obvious that the rectangular matrix

$$a = A\hat{\xi}^T \gamma^{-\frac{1}{2}} \tag{5.4}$$

consists of k mutually orthogonal unit vectors

$$\mathbf{a}^c = \frac{1}{\sqrt{\gamma^c}} \sum_{c'} \xi^{(c)}_{c'}\, \mathbf{A}^{c'} \,. \tag{5.5}$$

Adding to this set an extra unit vector $\mathbf{a}^0$ which is orthogonal to all of them, one obtains a new basis in the non-trivial part of the total Hilbert space. One can easily see that the vectors $\mathbf{a}^0$, $\mathbf{a}^c$ are just the eigenvectors of the antihermitian part $W = A\,A^T$. Therefore, this matrix becomes diagonal,

$$W = diag\,(0\ \ \hat{\gamma}) \,. \tag{5.6}$$

Its nonzero eigenvalues γ^c coincide with those of the matrix $\hat{X}$ [34, 30]. For the present, we drop the matrix blocks and the vector components which belong to the complementary $(N-(k+1))$-dimensional subspace and consist of zero elements.

In the chosen basis the unit dipole vector $\mathbf{d}$ has the components

$$d_0 = (\mathbf{a}^0 \cdot \mathbf{d}) = \sin\Theta\,, \qquad d_c = (\mathbf{a}^c \cdot \mathbf{d}) = \cos\Theta\cos\varphi_c\,;$$

$$\sum_c \cos^2\varphi_c = 1 \,. \tag{5.7}$$

Here we have introduced the angle $\Theta, (0 \le \Theta \le \pi/2)$, between the dipole vector $\mathbf{D}$ and the k-dimensional subspace *spanned* by the decay vectors $\mathbf{A}^c$. This angle is an important parameter which governs the interference effects under consideration.

The matrix of the internal interaction reads

$$\mathbf{D}^2 \begin{pmatrix} \sin^2\Theta & \sin\Theta\cos\Theta\, \mathbf{l}^T \\ \sin\Theta\cos\Theta\, \mathbf{l} & \cos^2\Theta\, \mathbf{l}\mathbf{l}^T \end{pmatrix} \tag{5.8}$$

where $\mathbf{l}$ stands for the unit vector with the components $l_c = \cos\varphi_c$.

(ii) Next we diagonalize the $k \times k$ submatrix $\mathbf{l}\mathbf{l}^T$ with the help of a k-dimensional orthogonal matrix $\hat{\eta}$ the first column of which coincides with the vector $\mathbf{l}$. This transformation resembles that described in subsection 4.1.

Now only two nonzero components of the unit dipole vector $\mathbf{d}$ are left which are equal to

$$d_0 = \sin\Theta\,, \qquad d_1 = \cos\Theta\,, \tag{5.9}$$

and only the 2×2 upper block of the internal interaction matrix (5.8) remains non-trivial. The vectors $\mathbf{A}^c$ are transformed into

$$A_0^c = 0\,, \qquad A_1^c = \sum_{c'} \sqrt{\gamma^{c'}} \cos\varphi_{c'}\, \xi_c^{(c')}\,, \qquad A_\alpha^c = \sum_{c'} \sqrt{\gamma^{c'}}\, \eta_{c'}^{(\alpha)}\, \xi_c^{(c')}\,, \tag{5.10}$$

so that

$$A_d^c = \cos\Theta\, A_1^c = \cos\Theta \sum_{c'} \sqrt{\gamma^{c'}} \cos\varphi_{c'}\, \xi_c^{(c')} \tag{5.11}$$

when the lower diagonal submatrix $\hat{\gamma}$ in eq. (5.6) is replaced by

$$\hat{\gamma} \to \begin{pmatrix} \langle\gamma\rangle & \mathbf{w}^T \\ \mathbf{w} & \tilde{W} \end{pmatrix}. \tag{5.12}$$

Here

$$\langle\gamma\rangle = \sum_c (A_1^c)^2 = \sum_c \gamma^c \cos^2\varphi_c\,, \qquad w^{(\alpha)} = \sum_c \gamma^c \cos\varphi_c\, \eta_c^{(\alpha)}\,, \quad (\alpha = 2, 3, ..., k) \tag{5.13}$$

and

$$W_{\alpha\alpha'} = \sum_c \gamma^c\, \eta_c^{(\alpha)}\, \eta_c^{(\alpha')}\,. \tag{5.14}$$

(iii). Returning now into the total N-dimensional Hilbert space, the two consecutive transformations just described are part of the global transformation produced by the orthogonal matrix

$$\Omega = \left(\Omega^{(0)} = \mathbf{a}^{(0)} \quad \Omega^{(1)} = \sum_c \cos\varphi_c\, \mathbf{a}^{(c)} \quad \Omega^{(2\le\alpha\le k)} = \sum_c \eta_c^{(\alpha)}\, \mathbf{a}^{(c)} \quad \Omega^{(k+1\le s\le N-1)} \right). \tag{5.15}$$

In (5.15), the two groups of vectors, the $N-(k+1)$ vectors $\Omega^{(k+1\le s\le N-1)}$ in the full space and the $k-1$ ones $\eta^{(2\le s=\alpha\le k)}$ in the $k+1$ - dimensional subspace, can still be chosen arbitrarily. We will fix them later.

In the Ω-basis, the diagonal matrix elements of the unperturbed Hamiltonian H_0 are given by the weighted mean positions

$$\tilde{e}_s = \sum_n e_n \left(\Omega_n^{(s)}\right)^2\,, \qquad (s = 0, 1, ..., N-1) \tag{5.16}$$

when the off-diagonal elements

$$V_{ss'} = \sum_n e_n \, \Omega_n^{(s)} \, \Omega_n^{(s')} \,, \qquad (s \neq s') \tag{5.17}$$

obey the general sum rules

$$\sum_{s' \neq s} V_{ss'}^2 = \sum_{s'} (e_{s'} - \tilde{e}_s)^2 \, \left(\Omega_n^{(s)}\right)^2 \sim \Delta_e^2 \tag{5.18}$$

(compare with eq. (4.25)). Since we do not expect any special relation between the original basis and the doorway one, all off-diagonal matrix elements are suggested to be of the same order of magnitude. This leads to the estimation

$$\left| V_{s \neq s'} \right| \sim \Delta_e / \sqrt{N-1} \tag{5.19}$$

similar to that found in subsection 4.1.

We now use the last $N-(k+1)$ vectors $\mathbf{\Omega}^{(s)}$, $(s \equiv tr = k+1, k+2, ..., N-1)$ in (5.15) in order to diagonalize the lower block of the unperturbed Hamiltonian [30, 35],

$$\sum_n e_n \, \Omega_n^{(tr)} \, \Omega_n^{(tr')} = \tilde{\varepsilon}_{tr} \, \delta_{tr \, tr'} \,. \tag{5.20}$$

The $N_{tr} = N - k - 1$ eigenstates with the energies $\tilde{\varepsilon}_{tr}$ (which lie within the original energy region Δ_e [35]) are "trapped" [47, 34, 42], i.e. they do not have a direct access to the continuum. These states can decay only via the first $N_{dw} = k + 1$ doorway states due to the hermitian residual interaction

$$V_{dw \; tr} = \sum_n e_n \, \Omega_n^{(dw)} \, \Omega_n^{(tr)} \tag{5.21}$$

which appears from the initial unperturbed hamiltonian being transformed into the doorway basis.

A typical value for the widths of the doorway states is $\langle \gamma \rangle \approx \frac{1}{k} Tr \, W \sim \langle (\mathbf{A}^c)^2 \rangle$. Only one of them can, under certain conditions, become almost stable (see subsection 5.2) but then it is displaced by a distance $\sim \mathbf{D}^2$. Therefore, the admixture of all the trapped states to the N_{dw} doorway states is small as one of the ratios

$$\kappa = \frac{\Delta_e}{\mathbf{D}^2} \qquad \kappa' = \frac{\Delta_e}{\langle \gamma \rangle} \,. \tag{5.22}$$

According to the estimation (5.19) the trapped states acquire the widths $\sim \frac{N_{dw}}{N_{tr}} \kappa'^2 \langle \gamma \rangle$. The energy shifts of the trapped states are of the same order of magnitude. These states are responsible therefore for the fine structure effects (with the characteristic energy scale $\Delta_e/(N_{tr})$) in the energy domain of the parental levels. They become irrelevant when $(\kappa \, , \kappa') \to 0$.

The doorway $N_{dw} \times N_{dw}$ part of the effective Hamiltonian

$$\mathcal{H}^{(dw)} = \begin{pmatrix} \mathcal{H}^{(coll)} & \chi^T \\ \chi & \tilde{\mathcal{H}} \end{pmatrix} \tag{5.23}$$

includes two different blocks along the main diagonal. Only the upper 2×2 block

$$\mathcal{H}^{(coll)} = \begin{pmatrix} \tilde{e}_0 + \sin^2\Theta \, \mathbf{D}^2 & V_{01} + \sin\Theta\cos\Theta \, \mathbf{D}^2 \\ V_{10} + \sin\Theta\cos\Theta \, \mathbf{D}^2 & \tilde{e}_1 + \cos^2\Theta \, \mathbf{D}^2 \end{pmatrix} - \frac{i}{2}\langle\gamma\rangle \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{5.24}$$

contains, along with the collectivity via the continuum, the collective effects induced by the internal residual interaction. Due to the mixing described by the off-diagonal matrix elements, the widths, energy shifts and dipole strengths of the two eigenstates of this block are generally comparable to each other provided that the angle Θ differs from 0 and $\pi/2$ so that $\sin^2\Theta \sim \cos^2\Theta$.

The $(k-1)\times(k-1)$ block

$$\tilde{\mathcal{H}}_{\alpha\,\alpha'} = \sum_{c,c'} \eta_c^{(\alpha)} \left[\sum_n e_n\, a_n^{(c)}\, a_n^{(c')} - \frac{i}{2}\,\gamma^c\,\delta_{c\,c'} \right] \eta_{c'}^{(\alpha')} \tag{5.25}$$

(see eq. (5.14) for the antihermitian part) describes the states without both dipole strengths (according to (5.9)) and collective energy shifts. They are, generally, strongly mixed with one another. The set of vectors $\eta^{(\alpha)}$ can be used to diagonalize either the hermitian or the antihermitian part of $\tilde{\mathcal{H}}$ depending on which of them dominates the dynamics inside this block. The remaining part may then be treated as a weak perturbation. When however both, the hermitian and the antihermitian parts, are of equal importance the full Hamiltonian $\tilde{\mathcal{H}}$ must be diagonalized.

The two doorway blocks just described are coupled by the complex interaction

$$\chi = \left(\mathbf{v}^{(0)}\ \mathbf{v}^{(1)}\right) - \frac{i}{2}\,(\mathbf{0}\ \mathbf{w})\ ; \tag{5.26}$$

$$v_\alpha^{(0)} \equiv V_{0\alpha}\ , \qquad v_\alpha^{(1)} \equiv V_{1\alpha}\ , \qquad w^{(\alpha)} = \sum_c \gamma^c \cos\varphi_c\, \eta_c^{(\alpha)}\ . \tag{5.27}$$

The influence of the hermitian part is weak as κ^2, κ'^2 again. The strength of the antihermitian coupling can be estimated by using the identity

$$\mathbf{w}^2 \equiv \sum_\alpha \left(w^{(\alpha)}\right)^2 = \sum_c (\gamma^c)^2 \cos^2\varphi_c - \left(\sum_c \gamma^c \cos^2\varphi_c\right)^2 = \Delta_\gamma^2 \tag{5.28}$$

which is the counterpart of the eq. (4.25).

5.2 Resonance Spectrum and Dipole Strengths of Doorway States

The interaction W of the resonance states via the continuum causes a strong redistribution of the dipole strength when the doorway states overlap. One may try, similar to eq. (4.18), to use the residues $\left(\mathbf{d}\cdot\Psi^{(s)}\right)^2$ of the function $\mathcal{P}(E)$, eq. (4.5), as a measure of the dipole strengths of the individual resonance states. These residues are however complex. Therefore, although they satisfy themselves the condition (4.19), their moduli can be arbitrarily large. It is natural and appropriate to extend the definition of the dipole strength f^r of the internal state $\Phi^{(r)}$ to that of the unstable state $\Psi^{(s)}$ in the following manner (compare with (2.30))

$$f^s = \frac{1}{U_s}\left|\mathbf{d}\cdot\Psi^{(s)}\right|^2 \tag{5.29}$$

where $U_s = \left(\Psi^{(s)^*}\cdot\Psi^{(s)}\right)$ are the diagonal matrix elements of the Bell-Steinberger non-orthogonality matrix (2.28). These quantities are directly linked with the resonance spectrum. Multiplying equation (2.26) by the matrix $\Psi^\dagger$ from the left side and adding the

hermitian conjugate of the relation thus received, one obtains

$$f^s = \frac{1}{\mathbf{D}^2}\left(E_s - \sum_n e_n \frac{|\Psi_n^{(s)}|^2}{U_s}\right) \tag{5.30}$$

where E_s is the resonance energy.

In the doorway basis introduced in subsection 5.1, the upper $N_{dw} \times N_{dw}$ block of the effective Hamiltonian is totally decoupled from the lower block of the trapped states if one neglects the small matrix elements (5.21) the contributions of which are of the order of magnitude κ^2 or κ'^2. Omitting them, one neglects the fine structure variations of the transition amplitudes in the energy domain of the parental levels as mentioned above (sec. 5.1). In such an approximation, the trapped states remain stable and are entirely excluded from all further calculations. Then relation (5.30) becomes especially simple

$$f^{dw} = \frac{1}{\mathbf{D}^2}\left(E_{dw} - \varepsilon_0\right) . \tag{5.31}$$

Taking into account eq. (5.23), the secular equation (3.3) can now be reduced to

$$\mathrm{Det}\left(\mathcal{E} - \mathcal{H}^{(coll)} - \mathcal{Q}(\mathcal{E})\right) = 0 . \tag{5.32}$$

The second order self-energy matrix

$$\mathcal{Q}(\mathcal{E}) = \chi^T \frac{1}{\mathcal{E} - \tilde{\mathcal{H}}} \chi \tag{5.33}$$

describes the virtual transitions between the two types of doorway states. Its explicit form depends on the interference regime inside the second group. We further assume that the antihermitian part dominates in the Hamiltonian submatrix (5.25). Therefore we diagonalize first this part by demanding the vectors $\eta^{(\alpha)}$ to satisfy the conditions

$$W_{\alpha\alpha'} = \sum_c \gamma^c \eta_c^{(\alpha)} \eta_c^{(\alpha')} = \tilde{\gamma}^\alpha \delta_{\alpha\alpha'} . \tag{5.34}$$

The opposite case with dominating hermitian part can be treated in an analogous manner.

Going further along the same line as in subsection 4.1, one finds for the eigenvectors

$$\eta_c^{(\alpha)} = w^{(\alpha)} \frac{\cos\varphi_c}{\gamma^c - \tilde{\gamma}^\alpha} \tag{5.35}$$

with the normalization condition

$$w^{(\alpha)} = \left[\sum_c \frac{\cos^2\varphi_c}{(\gamma^c - \tilde{\gamma}^\alpha)^2}\right]^{-\frac{1}{2}} . \tag{5.36}$$

The corresponding eigenvalues $\tilde{\gamma}^\alpha$ are the roots of the equation

$$\sum_c \frac{\cos^2\varphi_c}{\gamma^c - \tilde{\gamma}} = 0 . \tag{5.37}$$

Each of the $k-1$ eigenvalues $\tilde{\gamma}^\alpha$ lies between two neighbouring values γ^c. The last three equations should be compared with eqs. (4.22) - (4.24).

Considering the hermitian part of the hamiltonian (5.25) to be a weak perturbation, one obtains

$$\tilde{\mathcal{H}}_{\alpha\alpha'} \approx \left(\tilde{e}_\alpha - \frac{i}{2}\tilde{\gamma}^\alpha\right)\delta_{\alpha\alpha'} = \tilde{\mathcal{E}}_\alpha\,\delta_{\alpha\alpha'} \tag{5.38}$$

in first approximation. The corrections are proportional to the ratio $\Delta_e^2/\Delta_\gamma^2$ of the variances of the unperturbed levels e_n and of the collective widths γ^c. The approximation is justified when this ratio is small.

Under the last condition, one can also neglect the hermitian part of the coupling matrix χ, eq. (5.26). The only nonzero matrix element in the right lower corner of the self-energy matrix $\mathcal{Q}$ reads then

$$\mathcal{Q}_{11}(E) = -\frac{1}{4}\sum_\alpha \frac{w^{(\alpha)^2}}{E - \tilde{\mathcal{E}}_\alpha} \equiv -\frac{1}{4}\,q(E) \tag{5.39}$$

and the secular equation (5.32) reduces to

$$\Lambda(\mathcal{E}) \equiv (\mathcal{E} - \varepsilon_0)\,(\mathcal{E} - \varepsilon_{coll}) + \frac{i}{2}\omega(\mathcal{E})\,(\mathcal{E} - \varepsilon_0 - \sin^2\Theta\,\mathbf{D}^2) = 0 \tag{5.40}$$

where the notation

$$\omega(\mathcal{E}) = \langle\gamma\rangle - \frac{i}{2}\,q(\mathcal{E}) \tag{5.41}$$

has been introduced. Here we neglected the matrix elements $V_{01} = V_{10}$ and set $\tilde{e}_0 = \tilde{e}_1 = \varepsilon_0$. The corresponding corrections are again proportional to κ^2, κ'^2. The equation (5.40) is equivalent to an algebraic equation of $(k+1)$th order. It determines the complex energies of the $k+1$ doorway resonances.

5.3 The Two-Level Approximation

Let us temporarily omit also the second term $q(E)$ in eq. (5.41). Then the secular equation (5.40) reduces to the same quadratic one which appears in the single-channel problem investigated in [19]. In this approximation, one is left with two interfering collective levels only. The latter problem can be easily solved exactly. (See for example [32] and [48] where different aspects of the problem are treated. It has much in common with the physics of the text-book systems of the neutral kaons [28, 49], the ρ and ω mesons [50] or the 2^+ doublet in ^8Be [29, 51].) Using the notation

$$z = \frac{\mathcal{E} - \varepsilon_0}{\mathbf{D}^2}\,, \tag{5.42}$$

one obtains explicitly

$$z_0 = \frac{1}{2}(1 - |x|) - \frac{i}{2}\,\lambda\,\frac{1}{2}(1 - |y|)\,, \qquad z_1 = \frac{1}{2}(1 + |x|) - \frac{i}{2}\,\lambda\,\frac{1}{2}(1 + |y|) \tag{5.43}$$

where

$$|x| = \frac{1}{\sqrt{2}}\left[\sqrt{\left(1 - \frac{1}{4}\lambda^2\right)^2 + \lambda^2\cos^2 2\Theta} + \left(1 - \frac{1}{4}\lambda^2\right)\right]^{\frac{1}{2}} \leq 1 \tag{5.44}$$

and

$$|y| = \frac{1}{\sqrt{2}}\left[\sqrt{\left(1 - \frac{4}{\lambda^2}\right)^2 + 4\,\frac{4}{\lambda^2}\cos^2 2\Theta} + \left(1 - \frac{4}{\lambda^2}\right)\right]^{\frac{1}{2}} \leq 1\,. \tag{5.45}$$

Apart from the angle Θ, the interference of the collective states depends on the ratio

$$\lambda \equiv \frac{\langle\gamma\rangle}{\mathbf{D}^2} \tag{5.46}$$

of the strengths of the external and internal interactions. The solution (5.43) is valid when $0 < \Theta < \pi/4$; for $\pi/4 < \Theta < \pi/2$ the imaginary parts of the two roots are to be replaced by each other. To be definite, we consider the first possibility below.

The quantity $|x|$ measures the energy distance between the two resonances,

$$E_1 - E_0 = |x|\,\mathbf{D}^2\,, \tag{5.47}$$

whereas $|y|$ measures the difference of their total widths,

$$\left|\Gamma_1 - \Gamma_0\right| = |y|\,\langle\gamma\rangle\,. \tag{5.48}$$

According to (5.31, 5.44), one further has

$$f^{0,1} = \frac{1}{2}(1 \mp |x|) \tag{5.49}$$

in the same approximation. The latter expression shows that the closer the resonances are to each other the more similar are their dipole strengths.

The situation is especially simple for the angle $\Theta = \pi/4$. In this case

$$|x| = \begin{cases} \sqrt{1-\lambda^2/4}, & \lambda < 2 \\ 0\,, & \lambda > 2 \end{cases}\;; \qquad |y| = \begin{cases} 0\,, & \lambda < 2 \\ \sqrt{1-4/\lambda^2} & \lambda > 2 \end{cases}\;. \tag{5.50}$$

In the limit $\lambda \ll 2$, the two collective levels $dw = 0$ and $dw = 1$ are separated by a large distance $\sim \mathbf{D}^2$ but have the same widths. The level $dw = 1$ carries the whole dipole strength. With growing λ the levels are getting closer and finally merge when λ reaches the value 2. For $\lambda > 2$, the dipole strengths as well as the energies of both resonances remain equal to each other while their widths differ more and more with increasing λ.

The transition at the point $\lambda = 2$ gets smoother for other values of the angle Θ but still exists as long as Θ is not too close to 0 or $\pi/2$. If the internal interaction prevails and $\lambda \ll 2$, both collective levels have comparable widths,

$$\mathcal{E}_0 = \varepsilon_0 - \frac{i}{2}\sin^2\Theta\,\langle\gamma\rangle\,, \qquad \mathcal{E}_1 \equiv \mathcal{E}_{gr} = \varepsilon_0 + \mathbf{D}^2 - \frac{i}{2}\cos^2\Theta\,\langle\gamma\rangle\,, \tag{5.51}$$

while only the second one is displaced by the distance $\mathbf{D}^2$ and carries the whole dipole strength. According to eqs. (5.9, 5.10), the total width of this level is equal to

$$\Gamma_1 = \cos^2\Theta\,\langle\gamma\rangle = \hat{\mathbf{A}}_d^2 = \Gamma_{gr} \tag{5.52}$$

in full agreement with eq. (3.6). In the opposite case of the dominating external coupling, $\lambda \gg 2$, the energy displacement $\mathbf{D}^2$ is shared by the two collective resonances,

$$\mathcal{E}_0 = \varepsilon_0 + \sin^2\Theta\,\mathbf{D}^2 - \frac{i}{2}\langle\gamma\rangle\,\frac{1}{\lambda^2}\sin^2 2\Theta\,, \qquad \mathcal{E}_1 = \varepsilon_0 + \cos^2\Theta\,\mathbf{D}^2 - \frac{i}{2}\langle\gamma\rangle\left(1 - \frac{1}{\lambda^2}\sin^2 2\Theta\right)\,. \tag{5.53}$$

(Here we omitted the small corrections $\sim \lambda^{-2}$ to the positions of the resonances). The corresponding dipole strengths are, in this case, equal to

$$f^0 \approx \sin^2\Theta\,, \qquad f^1 \approx \cos^2\Theta\,. \tag{5.54}$$

The nucleon width of the level $dw = 0$ decreases with growing λ. Finally this resonance practically disappeares from the decay spectrum in the particle channels. In agreement with eq. (5.54) it gets however a nonvanishing radiation width and contributes in the photoemission process (see subsection 6.2).

Let us now consider the role of the other doorway resonances $\tilde{\mathcal{E}}_\alpha \approx \varepsilon_0 - \frac{i}{2}\langle\gamma\rangle$, eq. (5.38). Substituting the complex energies $\mathcal{E}_{0,1}$ found above into the sum (5.39), one sees that the denominators of the terms of this sum contain, as a rule, one of the large quantities $\mathbf{D}^2$ or $\langle\gamma\rangle$. According to eq. (5.28), such terms are of the order of magnitude τ^2/k or τ'^2/k where the parameters

$$\tau = \frac{\Delta_\gamma}{\mathbf{D}^2}\,, \qquad \tau' = \frac{\Delta_\gamma}{\langle\gamma\rangle} \tag{5.55}$$

may be expected to be reasonably small. Therefore, these doorway states acquire a relatively small dipole strength. The interference of the first two collective states remains most important and the above two-resonance approximation gives a description which is at least qualitatively satisfactory. When, however, some of the levels $\tilde{\mathcal{E}}_\alpha$ fall by chance anomolously close to one of the former two, these doorway states take part in the interference and the picture becomes more complicated.

6 Numerical Results and Discussion

To simplify the analytical study, we restricted ourselves in the foregoing sections to the case of a very strong interaction $\mathcal{H}^{(int)}$ (5.1) so that the energy range Δ_e of the unperturbed levels e_n could be neglected. In such an approximation, the $N_{tr} = N - k - 1$ trapped states are almost fully decoupled from the continuum. Only the $N_{dw} = k + 1$ collective doorway states remain relevant in studying the cross section pattern. In this case, the interference picture is determined essentially by the ratio $\lambda = \langle\gamma\rangle/\mathbf{D}^2$ of the strengths of external and internal interactions and by the angle Θ between the dipole vector $\mathbf{D}$ and the k-dimensional subspace of decay vectors $\mathbf{A}^c$.

Assuming further that all the vectors $\mathbf{A}^c$ are pairwise orthogonal and have the same lengths, the matrix $\hat{X}$ of the scalar products becomes proportional to unity and the problem is reduced to the case with only two decaying states which are mixed and share the total dipole strength. The picture arising from the interference of two resonance states is governed by the effect of avoided resonance crossing. According to eq. (5.31), the dipole strengths of the two resonances behave very much like their positions in energy when considered as functions of λ. While the sums of the two resonance energies and of the two strengths remain constant, the corresponding differences decrease as functions of increasing λ up to certain minimum values which depend on the angle Θ only. The widths of the two states increase first with increasing λ but bifurcate for large λ.

This behaviour is illustrated in Fig.1. The angle Θ is chosen to give $\cos^2\Theta \approx 0.65$ and ε_0 is set to zero. The energies of the two collective resonances (measured in units of the total energy displacement $\mathbf{D}^2$) and their dipole strengths plotted in dependence on λ in Fig. 1(a) coincide perfectly. Fig.1(b) displays the behaviour of the widths of the two resonance states versus their energies and/or dipole strengths when λ changes in the interval $0 \div 5$.

In the following, we check the relevance of the analytical results obtained by performing numerical calculations under less restrictive assumptions. We have chosen $N = 10$ levels e_n distributed more or less homogeneously and coupled to $k = 3$ open particle decay channels. The extension of the parental spectrum of the N discrete levels e_n is from -0.2 to 0.2 in relative units of the total energy displacement $\mathbf{D}^2$. This implies that $\kappa \equiv \Delta_e/\mathbf{D}^2 \approx 0.4$. As in Fig. 1, we set $\Theta \approx 36.3°$ but the lengths of the vectors $\mathbf{A}^c$ differ from one another within 10%. The angles $\theta_{cc'}$ between the pairs $\mathbf{A}^c$ and $\mathbf{A}^{c'}$ are confined to $0.17 \leq |\cos\theta_{cc'}| \leq 0.31$.

In Fig.2, the energies and dipole strengths of all 10 resonances are plotted as a function of λ while the changes of the widths with λ are shown in the representation of the Γ_s versus the positions E_s and dipole strengths f^s, respectively. For small λ, there is only one displaced state the dipole strength of which is very close to unity. With λ increasing, first $N_{dw} = 4$ doorway states appear three of which are formed according to the three open decay channels from the group of $N - 1$ states lying around $E = 0$, while the fourth state with large dipole strength lies at the energy $E/\mathbf{D}^2 \approx 1$. These four states almost exhaust the whole sum of widths $Tr\,W$ and the total dipole strength while the internal as well as external collectivity of the $N_{tr} = 6$ trapped states remain small. Fig. 2(b) shows that the total dipole strength is distributed mainly over two states: the original dipole state and one out of the group around $E = 0$. The width of the latter state is smaller than those of the other three broad states. Finally, it will be trapped at very large λ (Fig. 2(c)). This behaviour of the two states is qualitatively quite similar to that in the two-level approximation (compare Fig. 1). However, an appreciable part of the dipole strength is moved to the other low-lying doorway components which will not be trapped.

Thus, the numerical results confirm the interplay of two kinds of collectivity in the nuclear motion. The coherent internal dipole-dipole residual interaction together with the external interaction via k common decay channels creates a concentration of the dipole strength and full escape width $Tr\,W$ on $k + 1$ collective doorway states out of the $N > k$ resonance states.

Two very different energy scales are formed due to the internal dipole-dipole interaction: In the limit of zero coupling to the continuum all levels with the exception of the collective one are confined to the energy interval Δ_e while the latter is displaced far away by the distance $\mathbf{D}^2 \gg \Delta_e$. With increasing external interaction, the width collectivization takes place if TrW exceeds the interval Δ_e. This happens when λ is still small. As a result, $k+1$ states get escape widths being comparable to one another while $N - k - 1$ states become trapped. The $k+1$ states absorbing the total width TrW are the collective doorway states.

When TrW approaches the value $\mathbf{D}^2$, with further increasing λ a second stage begins: the widths are redistributed once more and the width of one of the doorway states starts to decrease and becomes finally trapped in the limit of very large λ. This redistribution of the widths is accompanied by a redistribution of the dipole strength and an energy shift of mainly two doorway states: the state with large internal collectivity and that which becomes trapped finally.

7 Summary

Summarizing, we state the following. On the basis of a phenomenological schematic model we have investigated the overlapping of the different doorway components of a giant multipole resonance. The internal damping of the collective motion due to the coupling to complicated compound states has been omitted on this stage. The mixing phases are large

when both the internal and external interaction are of comparable strength. The interplay and competition of the two kinds of collective behaviour induced by the internal and external coupling, respectively, give rise to nontrivial interferences between the doorway resonances.

The concept of the partial widths of a giant resonance becomes ambiguous because of the interferences: the partial widths determined in terms of the hermitian K-matrix and of the unitary S-matrix must be distinguished. The first ones appear in the integral sum rule (2.36) for the decay strengths into specific channels while the second ones are connected with the decay into the individual channels in a quite nontrivial manner (section 2). Therefore, the parameters of the K- rather than S-matrix are generally extracted when the escape widths of a GR are measured.

In a forthcoming paper, we study the influence of the interferences discussed in this paper onto the cross section in order to allow an at least qualitative comparison with experimental data.

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Figure Captions

Fig.1
The λ-dependence of resonance energies and dipole strengths (a) and the logarithm of the widths versus energies or strengths (b) with λ varying from 0 to 5 in the two-level approximation.

Fig.2
The λ-dependence of resonance energies (a) and dipole strengths (b). The logarithm of the widths versus energies (c) and dipole strengths (d) with λ varying from 0 to 5 in steps of 0.02. (Inset of (d) shows the magnified region of small f^s in the double log-scale). For parameters see text.

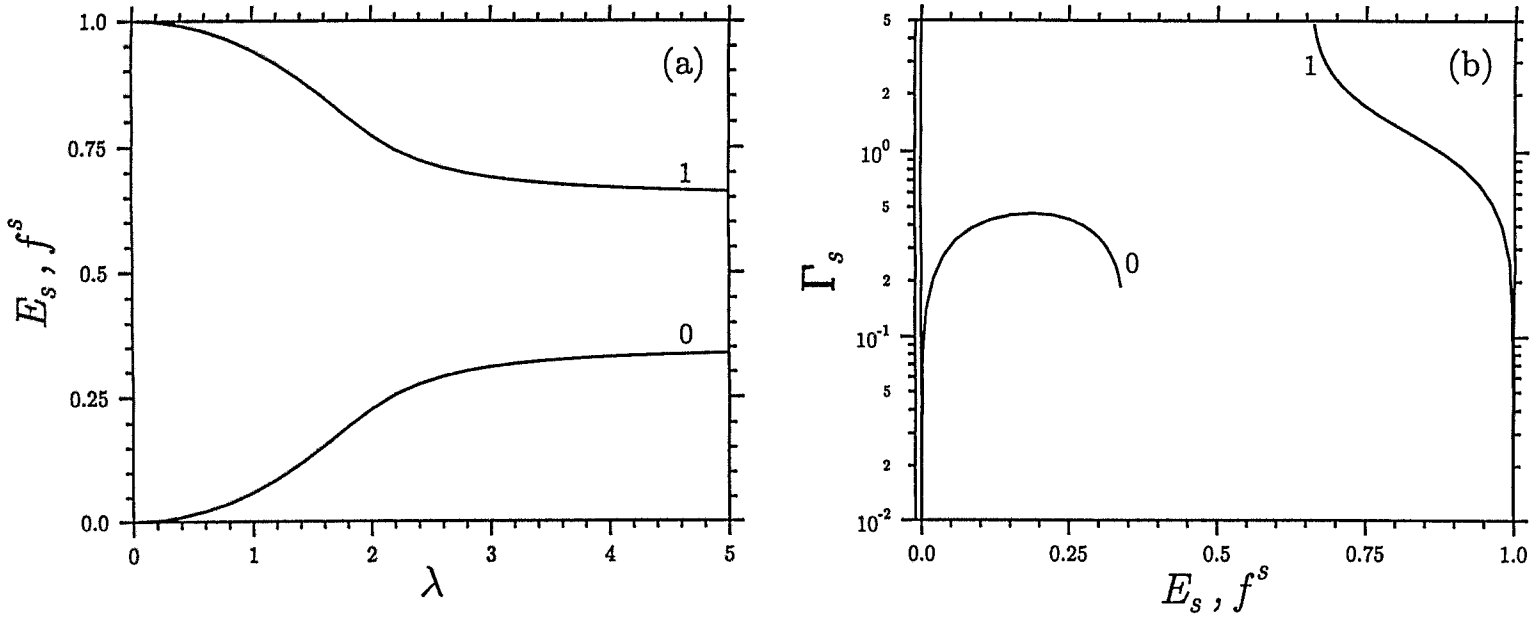

Fig. 1

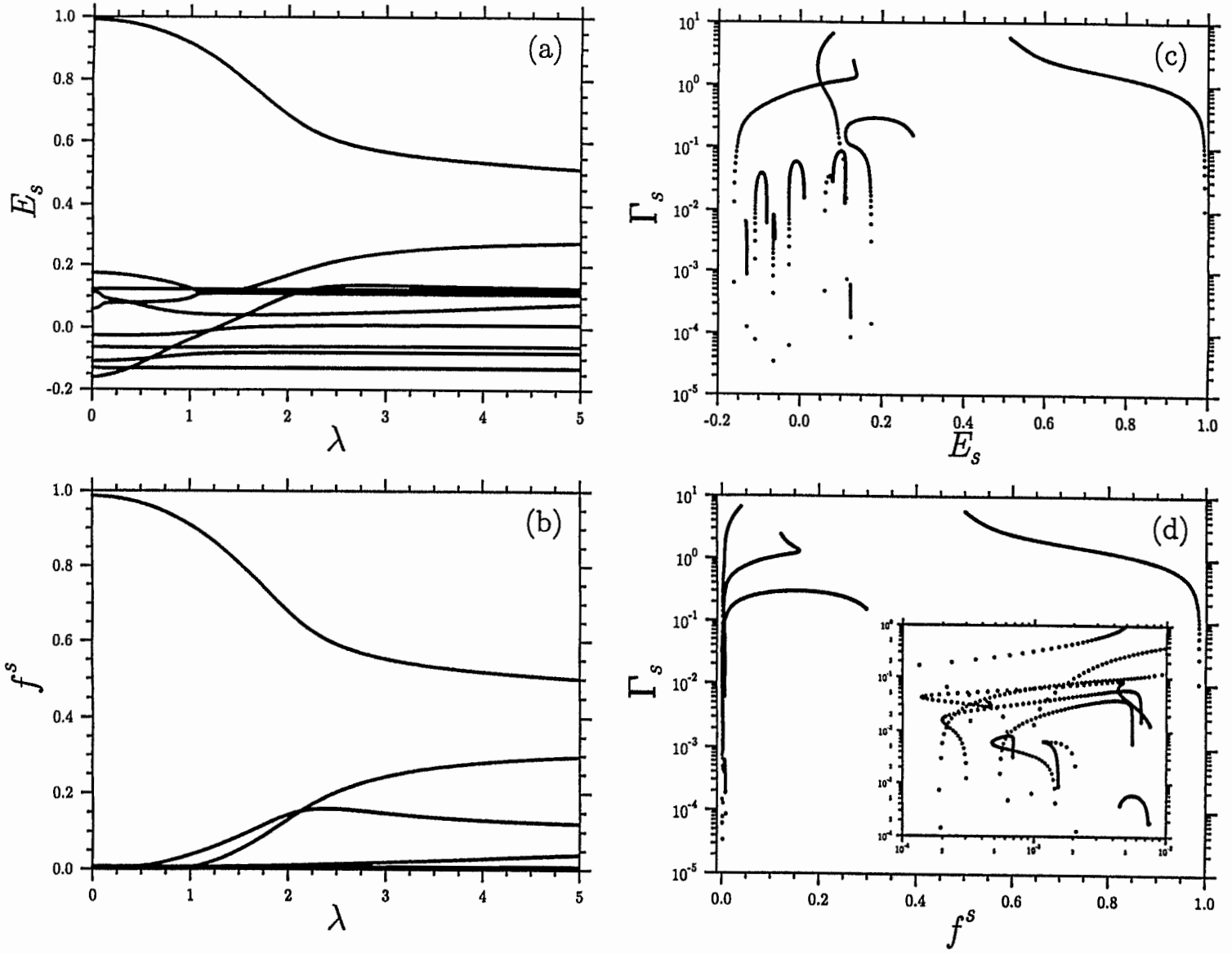

Fig. 2