Experimental and numerical analysis of the complex permittivity of open-cell ceramic foams

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1   Experimental and numerical analysis of the complex permittivity of open-cell

2   ceramic foams

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15

16   **Abstract**

17   Open-cell ceramic foams are promising materials in the field of microwave heating. They

18   can be manufactured from susceptor materials and can, therefore, be used as selective

19   heating elements. In this study, the complex permittivities of ceramic foam materials,

20   including silicon-infiltrated silicon carbide (SiSiC), pressureless sintered silicon carbide

21   (SSiC), silicate bonded silicon carbide (SBSiC), and cordierite were determined. The

22   dielectric properties of the foams were determined by the cavity perturbation technique

23   using a TE_{104} WR340 waveguide resonator at 2.45 GHz. Samples were preheated in a

24   tubular furnace, enabling temperature-dependent permittivity measurements up to 200 °C.

25   The effective dielectric constant and effective loss factor were found to depend on the

26   porosity and material composition of the foam. The SiSiC material had a higher effective

27   dielectric constant than the SSiC and SBSiC ceramics. The effective dielectric constant of

28   the foams showed a trend of gradual increase with increasing temperature. Some selected

29   dielectric mixing relations were then applied to describe the effective permittivity of the

30   foams and compare them with predictions from finite element simulations performed using

31   the CST Studio Suite. The foam morphologies and simple block inclusions were used in the

32   simulations.

33   **Keywords**: A. microwave processing, C. dielectric properties, D. silicon carbide, D.

34   cordierite.

1 Introduction

Open-cell solid foams are interconnected solid networks (also referred to as skeletons), through which fluids can pass (see Figure 1A). They exhibit characteristics that make them extremely attractive for various thermal applications, such as heat exchangers, porous burners, electric heaters [1], and catalyst supports [2,3].

Figure 1. (A) Image of the SiSiC foam morphology obtained by SEM; micro X-ray tomography (µCT) cross-sectional images of the SiSiC foams (B) 30 ppi and (C) 45 ppi.

Microwave heating has been increasingly applied to many industrial processes and can also be considered as a promising approach for the selective heating of foams. The microwave absorbance and the heating characteristics of a material are commonly described by its complex permittivity, which is constituted of real and imaginary parts ($\varepsilon = \varepsilon' - j\varepsilon''$), i.e., the dielectric constant (ε') and dielectric loss factor (ε''), respectively. The effective permittivity (ε_{eff}) is the dielectric permittivity of a system or object that is made of two or more materials. In the case of open-cell foams, which can be manufactured from any sinterable material (e.g., ceramics), ε_{eff} is a function of the permittivities of the fluid (ε_{air}) and solid ($\varepsilon_{ceramic}$) phases.

Silicon carbide (SiC) is a ceramic base material that is suitable for the manufacturing of open-cell solid foams. Its high-temperature stability, hardness, and corrosion resistance make it an outstanding candidate for many engineering applications. Due to its excellent

3

54 microwave susceptibility (i.e., the degree of absorption of the electromagnetic energy and

55 its conversion to heat), SiC is used to improve the heating efficiency in microwave

56 applications, either by blending low-loss materials with SiC particles or by placing those

57 materials close to or in contact with the bulky susceptor portions [4]. The temperature-

58 dependent complex permittivity of SiC powder (bulk density of 1.89 ± 0.06 g/cm^3) was

59 measured at 2.45 GHz by Fernandez et al. [5], as shown in Table 1. However, the

60 permittivity can be significantly influenced by doping or impurities. For example, boron-

61 doping increases both the dielectric constant and loss factor, from $\varepsilon = 6.70 - j0.02$ to $\varepsilon =$

62 $65.44 - j32.40$ (at 2.45 GHz) [6].

63 **Table 1**. Complex permittivity of SiC powder and cordierite.

64 The porosity (P) of solid foams is defined as the ratio of the volume of voids to the total

65 volume of the foam. As P increases, the effective permittivity decreases. For example, an

66 SiC foam with $P = 83.7 \pm 1.0$ % and a linear pore density of 50 pores per inch (ppi), has an

67 $\varepsilon_{\text{eff}} = 4.7 - j0.33$ at 2.45 GHz (at room temperature of approximately 20 °C) [6], which is

68 lower than the ε_{eff} for P = 41.1 ± 0.18 % (SiC powder) [5], as reported in Table 1.

69 The objective of this study was the analysis of the relationship between the morphology and

70 effective permittivities (at 2.45 GHz) of SiC-based and cordierite-based open-cell solid

71 foams. A low-loss magnesium-aluminum-cyclosilicate ($Mg_2Al_3(Si_5AlO_{18})$) ceramic

72 material known as cordierite was compared with the SiC-based foams; the permittivity of

73 cordierite bulk as indicated in the literature is provided in Table 1. Thus, a systematic study

74 of the dielectric parameters was performed for ceramic-based solid foams with different

75   porosities, pore densities, and compositions. Furthermore, the effective permittivities that

76   were predicted by some mixing relations were compared against those derived from

77   performed electromagnetic simulations.

78   **2   Experimental methods and materials**

79   **2.1   Experimental dielectric measurements**

80   Different techniques can be used to measure the dielectric properties of solid materials [9].

81   Among them, the cavity perturbation technique compares the resonance frequency (f_1) and

82   quality factor (Q_1) of a cavity without perturbation against the resonance frequency (f_2) and

83   quality factor (Q_2) of the same cavity after perturbation by a small material sample. The

84   complex permittivity of the perturbation sample is then calculated according to

$$\varepsilon' = \frac{V_c}{V_s}\frac{(f_1-f_2)}{A*f_2} + 1, \text{ and} \tag{1}$$

$$\varepsilon'' = \frac{V_c}{B*V_s}\left(\frac{1}{Q_1} - \frac{1}{Q_2}\right), \tag{2}$$

85   where V_c is the cavity volume and V_s is the sample volume [10]. The coefficients A and B

86   can be determined by calibration, either using samples of an identical size and well-known

87   permittivities that are similar to the material of interest or by employing a numerical

88   calibration procedure. The latter is based on full-wave 3D electromagnetic simulations of

89   the forward problem. By variation of the sample permittivity within the anticipated interval,

90 these simulations allow the correlation of the calculated frequency shift $(f_1 - f_2)$ and the

91 changes in the quality factor $\left(1/Q_1 - 1/Q_2 \right)$ with a given permittivity. The complex

92 permittivity is then calculated from the measured scattering parameters (*S*-parameters)

93 using this correlation [10]. The transmission factor S_{21}, which is the ratio of the transmitted

94 power (at output port 2) to the incident power (at input port 1) characterizes the resonance

95 behavior.

96 **Figure 2.** Schematic of the dielectric measurement system based on the cavity perturbation

97 technique. $S_{21}(f)$ is the frequency-dependent *S*-parameter response at port 2 to the signal at

98 port 1.

99 A schematic of the experimental system used to acquire the complex permittivity of the

100 samples is shown in Figure 2. A more detailed description of the dielectric measurement

101 system developed by Soldatov et al. can be found in [11]. The resonator was based on a

102 standard rectangular WR-340 waveguide cavity, and the length of the cavity was chosen to

103 be resonant at a frequency of 2.45 GHz in the TE_{104} mode. Thus, the electric field standing

104 wave pattern had four maxima along the longest side of the cavity. The microwave signals

105 were coupled using inductive coupling ports with a 10 mm width and 43.18 mm height on

106 both ends. The cavity was connected to a vector network analyzer (VNA Agilent E5071C)

107 with coaxial-to-waveguide adaptors and coaxial cables to measure the S_{21} parameter. Two

108 small opposing holes were made in the walls of the cavity at the position of one of the four

109 electric field maxima to enable the positioning of the sample inside the cavity (within a

110 quartz tube sample holder). A pyrometer (CellaTemp-PA) at the opposite end measured the

111 sample surface temperature inside the resonator. A resistive tubular furnace (Carbolite

112 MFT) was placed in front of the waveguide, allowing the sample to be heated to 1200 °C

113 before entering the cavity. In this study, the temperature range for the characterization was

114 30 °C–200 °C. The parameters S_{21} and the temperature were recorded by a computer, which

115 provided automatic control of the entire data acquisition process as well as predefined

116 heating scenarios. The acquired data (i.e., resonance frequency shifts and Q-factors) were

117 then post-processed to obtain the real and imaginary parts of the effective material

118 permittivity (at different temperatures), via Equations (1) and (2).

119 **2.2   Ceramic foams**

120 Very high temperatures or pressures (or both) are required to densify pure SiC powder by

121 sintering due to its covalent bond characteristics [12]. Therefore, the use of additives to

122 make the sintering process easier is preferred. In this study, the following types of SiC

123 ceramics were investigated [13]:

124 ▪ Silica-bonded silicon carbide (SBSiC) is an oxide-bonded SiC ceramic manufactured

125 from coarse and medium-grained SiC powders by sintering at standard pressure, during

126 which amorphous silica or cristobalite (or both) bond the SiC particles.

127 ▪ Pressureless sintered silicon carbide (SSiC) is produced using very fine SiC powder that

128 contains sintering additives.

129 ▪ Silicon-infiltrated silicon carbide (SiSiC) is produced by the reaction of molten silicon

130 with compact SiC powder, containing a high amount of free carbon. Initially, the carbon

131 reduces the native silica layer on the SiC particles. The infiltrating melted silicon then

132 reacts with the remaining free carbon to produce secondary SiC, leading to SiC bonding

133 between the SiC grains. The remaining melted silicon fills the inter-particle voids

134 creating a somewhat dense microstructure.

135 The compositions and structures of SBSiC, SSiC, and SiSiC are shown in Table 2 and Figure

136 3, respectively.

137 **Figure 3.** Schematic structure of (A) SBSiC, (B) SSiC, and (C) SiSiC.

138 **Table 2.** Material compositions.

139 Solid foams have a macro-porous open-cell structure composed of stiff vertices

140 interconnected with struts, which form a continuous grid. Images example of an SiSiC solid

141 foam and its pore structure, obtained by scattering electron microscopy (SEM) and X-ray

142 micro-computed tomography (µCT), is presented in Figure 1. The struts have small cavities

143 inside them, which are isolated from the exterior. As the pore density increases, the pore

144 size decreases. The foam structure results from the replication technique, wherein the

145 polymeric template defines the cell size and shape. The open-cell polymer foams were

146 impregnated with a ceramic slurry. The excess slurry was then removed by squeezing and

147 centrifugation, resulting in the uniform coating of the polymer foam struts. During the

148   subsequent heat treatment, the polymeric template was decomposed (by pyrolysis), and the

149   ceramic material became sintered. The same SiC powders were used to manufacture the

150   SSiC and SiSiC test samples, which were used to identify the influences of the various

151   compositional materials and binding types (see Table 2). These powders were fine and

152   reactive, with a median grain diameter $d_{50} \approx 2$ µm. The SBSiC also included a second,

153   coarser SiC powder ($d_{50} \approx 20$ µm) to achieve a fine to coarse ratio of 2:3.

154   Cylindrical samples of the bulk material and the open-cell foams having nominal pore

155   densities of 30 ppi, 45 ppi, and 60 ppi and porosities in the range of 86.9 % to 96.5 % were

156   produced (IKTS Fraunhofer, Germany) for dielectric characterization based on the cavity

157   perturbation technique. The average cell diameters, densities, porosities, and dimensions of

158   bulk and open-cell ceramic foam samples are summarized in Table 3. The average cell

159   diameters of the foams were determined by optical 2D-image analysis of light micrographs

160   using a Pore!Scan-System from Goldlücke GmbH [40]. The density of the foam strut

161   material was lower than the maximum feasible density because no subsequent compaction,

162   excepting the eventual sintering shrinkage, can be applied to open-cell ceramic foams. The

163   same suspension was used for both sample types to identify the structure-related differences

164   between the foam and bulk samples.

165   **Table 3.** Bulk and foam sample properties.

| 166 | **3** | **Calculations** |

| 167 | **3.1** | **Mesh construction for the numerical simulations** |

168 Two morphology classes, i.e., foams and blocks, were analyzed to compare their influences

169 on the effective complex permittivity. The foam morphology chosen for the simulation was

170 obtained from numerous 2D digital cross-sectional images of a rectangular 45 ppi SiSiC

171 foam sample (size: 40 mm × 40 mm × 25 mm, porosity: 85 %) recorded by µCT. The back-

172 projection algorithm [14] was used for the 3D sample reconstruction. The spatial resolution

173 of the reconstructed tomographs was 56 µm per voxel. The foam stack was first cut to a

174 smaller representative cubic volume with a 15 mm edge length to reduce the computational

175 burden of the simulations. A representative cube is a rectangular cuboidal representation

176 that utilizes a particular volume of the foam that has the same porosity as the sample.

177 Additional meshes with lower porosities were produced using a 3D dilatation filter to study

178 the porosity over a wide range (see Section 4.3) [15]. This filter dilates the cross-sectional

179 structure of the foam using a spherical kernel diameter of 3 voxels, which increases the

180 skeleton volume, and, consequently, decreases the porosity. The mesh reconstruction (see

181 Figure 4) was then achieved by applying the marching cubes algorithm [16], following

182 which the small non-connected structures were eliminated. Further, mesh simplification

183 (i.e., the transformation of a given polynomial mesh into another with fewer faces, edges,

184 and vertices), mesh smoothing (i.e., the removal of noise from the mesh surface), and

185   uniform mesh resampling (to optimize the point distribution) were performed [39]. A

186   detailed description of the mesh construction can be found in [17].

187   **Figure 4.** Foam morphology reconstruction and meshing.

188   Finally, the meshes were imported into the CST Microwave Studio Suite and incorporated

189   as inclusions in a box. The outer box represented the air and the inclusions for the ceramic

190   foams. The resulting boxes with the inclusions had porosities of 11 %, 30 %, 50 %, 70 %,

191   and 85 %, based on the ceramic and air domains.

192   For the block morphology inclusions, a series of blocks were used instead. The blocks were

193   generated in the directions of the box corners, resulting in an arrangement of eight blocks.

194   The block size was selected to obtain porosities ranging from 0 % to 100 % at 10 %

195   increments. Additionally, the porosities were adjusted to facilitate direct comparison with

196   the porosities of the foam morphologies mentioned previously. Two approaches were used

197   for the block model generation, wherein the properties of the blocks and matrix were

198   interchanged. The first approach involved block-shaped ceramic inclusions in an air matrix,

199   while the second used block-shaped air inclusions in a ceramic matrix. These arrangements

200   facilitated the comparison of the effects of the non-connected and connected ceramic

201   structures to those of real foam structures. Examples of the foam and block inclusions are

202   shown in Figure 5.

203   **Figure 5.** Examples of the foam morphology inclusions (upper row), ceramic block

204   inclusions (middle row), and air block inclusions (lower row), within the boxes. Ceramic

205   and air materials are color-coded in red and blue, respectively.

206   **3.2   Numerical modeling and complex permittivity calculation**

207   Electromagnetic wave propagation was simulated at 2.45 GHz using the CST Microwave

208   Studio Suite and the model shown in Figure 6 to estimate the effective permittivities of the

209   different foams. Simulations of the hexagonal meshes representing the foam morphologies

210   (minimum 1 000 000 cells) and block morphologies (minimum 300 000 cells) were

211   performed using a transient domain solver. The boundary conditions were defined as

212   follows: the pairs of normal faces in the x, y, and z directions were taken as the electric

213   walls, magnetic walls, and free space boundaries with no reflections (i.e., open walls),

214   respectively. The input and output microwave ports were defined at reference planes shifted

215   to the normal faces in the z-direction of the models. The bulk permittivity from the

216   effective-medium approximation of the SiSiC was used for the ceramic inclusions (foams

217   and blocks). Air properties were set for the waveguide domain. Finally, the scattering

218   parameters were acquired by numerical simulation. The simulation setup and sequential

219   procedure are visually summarized in Figure 6.

220   **Figure 6.** Sequential procedure for the foam morphology (upper row) and block

221   morphology (lower row) inclusions used for computing the scattering parameters using

222   numerical simulations.

223   In the next step, the effective permittivities were determined using the S-parameters [18].

224   The S-parameter retrieval method [19,20] was used to calculate the permittivity from the

225   reflected (S_{11}) and transmitted scattering signals (S_{21}). Initially, the impedance (z) was

226   obtained as

$$z = \pm \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}}. \tag{3}$$

227   The refractive index (n) was then calculated by applying the obtained impedance and

228   Equation (4):

$$n = \frac{1}{k_0 L}\left[Im\left\{ln\left(\frac{S_{21}}{1 - \frac{(z-1)S_{11}}{(z+1)}}\right)\right\} + 2m\pi - i \cdot Re\left\{ln\left(\frac{S_{21}}{1 - \frac{(z-1)S_{11}}{(z+1)}}\right)\right\}\right], \tag{4}$$

229   where k_0 is the wavenumber, L is the sample length in the wave propagation direction

230   between ports 1 and 2, m is the fundamental branch of the sinusoidal function periodicity

231   (in this work, $m = 0$ because $L < \lambda_{foam}/4$), and λ_{foam} is the wavelength in the foam at

232   2.45 GHz. The effective permittivity was then calculated as

$$\varepsilon_{eff} = \frac{n}{z}. \tag{5}$$

233

4   Results and discussion

4.1   Determination of the complex permittivity

Figure 7 provides the effective dielectric constants ($\varepsilon'_{\text{eff}}$) and the effective loss factors ($\varepsilon''_{\text{eff}}$) of the cordierite and SiC-based samples obtained from the temperature dependence measurements using the cavity perturbation method. Notably, the bulk did not necessarily refer to samples having a 100 % theoretical density (as listed in Table 3) because of the previously described preparation technique. In contrast to the SiSiC, the other bulk materials had significant residual porosities (SBSiC: 29.1 %, SSiC: 8.7 %, cordierite: 29.3 %). The permittivity of the bulk SiSiC sample is not shown because it was detected uncertainty regarding its composition and porosity. Further, the high losses of the bulk SiSiC, SSiC, and SBSiC samples resulting from excessive perturbation caused the solution of the inverse problem associated with the cavity perturbation technique to provide unreliable results. Thus, the results for these bulk materials are also not shown. For all materials, the samples had a range of complex permittivity values (represented by the error bars in Figure 7), which was primarily caused by the spread in the sample geometry (see Table 3).

No clear trend could be observed for the relationship of the permittivity to the skeleton structure or pore size (in terms of the pore density). Notably, the pore density was provided as the mean ppi-value and is a measure of the interface density [21], which corresponded to the inverse of the mean chord length. The unit of ppi is not recommended for use in

254   scientific papers because the inch is not a unit recognized in the international system of

255   units. However, it is used in various industrial standards for foam production [22] (as was

256   the case in this study). Further, it can be observed that, in most of the samples, the

257   permittivity decreased with increasing porosity because the contribution of the air to the

258   effective permittivity increased ($\varepsilon_{air} < \varepsilon_{ceramic}$).

259   **Figure 7.** Effective dielectric constants (left) and effective loss factors (right) of cordierite

260   (first row), SiSiC (second row), SBSiC (third row), and SSiC (fourth row) at a frequency of

261   2.45 GHz.

262   As seen in Figure 7, the permittivity of the SiSiC differed significantly from that reported

263   for the SSiC and SBSiC, highlighting the impact of the free silicon infiltration. The

264   dielectric properties of SiC ceramic foams depend strongly on the characteristics of their

265   doping, additives, or impurities.

266   From Figure 7, it is evident that the SBSiC and SSiC had similar effective dielectric

267   constants. In comparison, the SiSiC exhibited a significantly higher effective dielectric

268   constant, presumably related to the free silicon that infiltrated the SiSiC foam structure

269   (Figure 3). Given that silicon is a semiconductor with a dielectric constant of approximately

270   11.9 [23] at 2.45 GHz, the resulting effective dielectric constant was higher as compared to

271   the SiC foams that did not contain free silicon.

272 The effect of the temperature on the loss factor was considerable for the SSiC but not the

273 other foams, presumably related to the comparably higher fraction of silicon carbide and

274 the excess carbon that was not completely consumed by the native silica layer of the SiC

275 particles during the sintering process (see Table 2). When adding carbon to SiC (C-SiC)

276 [24], its conductivity changes from n-type to p-type, so a greater amount of carbon would

277 increase the conductivity and, therefore, its loss factor. Furthermore, increasing

278 temperatures result in increased carbon-based conductivity and permittivity [25]. Moreover,

279 after the manufacturing process is complete, some additive materials used in the

280 manufacturing process may remain as reacted materials or binders. In this study, boron was

281 employed as the sintering aid. Boron is a trivalent element that creates p-type electron

282 vacancies and, thus, further increases the electrical conductivity in the SSiC.

283 Highly porous materials always contain some water in their pores due to the ambient

284 atmosphere. Being exposed to elevated temperature causes water to leave the material, thus

285 leading to a decrease in its effective permittivity. This effect, however, coincides with the

286 change of permittivity in the skeleton material of the foam. The effect of water depletion on

287 the effective permittivity competes in foams made from typical dielectrics like oxides,

288 nitrides, or carbides, which have increasing trends in permittivity versus temperature.

289 Semiconducting additives, such as silicon and carbon (in this case conducting), in SiSiC

290 and SSiC foams, respectively, could cause increased effective loss factors, which correlates

291 with the SiC skeleton temperature behavior and inversely with water depletion. The

292 interplay of these three factors that define the resulting temperature behavior of the

293 permittivity of the foams in this study is clearly different between SiSiC and SSiC.

294   Compared to SSiC, SBSiC has no carbon or boron source added, and includes alumina

295   (Al_2O_3) and silica (SiO_2). Fused amorphous silica (SiO_2) has a dielectric constant of

296   approximately $\varepsilon'_{SiO_2} \sim 3.8$ throughout the frequency range of 1×10^{-5} to 40 GHz [26], and

297   is known for its very low loss factor of $\varepsilon''_{fused\ silica} < 1 \times 10^{-3}$ [27]. Thus, the competition

298   of only two substances, i.e., SiC and water, could be assumed. Interestingly, the water

299   depletion resulted in a declining trend of the loss factor with temperature, but an increasing

300   trend with the dielectric constant. This is likely a consequence of the relatively higher

301   contribution of the water loss factor than its dielectric constant.

302   The notable increase of the cordierite permittivity for P = 29 % is a result of its domination

303   of the skeleton material composition. In contrast, at P ~ 85 %, the water depletion almost

304   fully compensated for the trend of increasing permittivity. As seen in Figure 7, only the loss

305   factor of SSiC increased significantly with temperature. Thus, the microwave absorbance of

306   SiSiC and cordierite foams could be considered to be constant within the studied

307   temperature range.

308   The assessment of whether the foams behaved as homogeneous effective media is also

309   important. Foam can be considered to behave as an effective medium when the wavelength

310   of the incident electromagnetic radiation λ is much larger than the mean cell size.

311   Therefore, the criterion used to determine whether the system behaves as an effective

312   medium is based on its inclusion size parameter, which is calculated as $x = 2\pi r/\lambda$, where r

313   represents the inclusion or particle size. Mishchenko et al. proposed a threshold value of

314    $x < 0.15$ [37].

315   The permittivity of a medium affects the wavelength according to $\lambda = \lambda_0 / \sqrt{\varepsilon_{eff}}$, where λ_0

316   is the wavelength in a vacuum. In this study, the cells are considered as spheres, and their

317   diameters ranged between ~ 1.46 mm (30 ppi) and < 1 mm (60 ppi). As shown in Table 4,

318   inclusion size parameters $x < 0.15$ were obtained for all foam types; i.e., they can be

319   treated as effective homogeneous media.

320   **Table 4.** Maximum inclusion size parameter (x) of foams; 30 ppi foams were used for the

321   calculations because they have the largest cell diameter.

322   **4.2   Permittivity mixing relations**

323   As in the effective-medium approximations and empirical mixing relations (see Table 5),

324   two-phase mixing relations were used to describe the effective permittivity of the ceramic

325   foams in terms of their porosities and bulk permittivities. The bulk permittivities of the

326   SiSiC, SBSiC, and SSiC were not determined and, accordingly, were considered as

327   parameters in the mixing relations that were fitted to the experimental data using iterative

328   least-squares estimation.

329   When the porosity reaches 100 %, the effective permittivity should approach the

330   permittivity of the material occupying the pores, in this case, air; $\varepsilon'_{\text{eff}}(P = 1) = \varepsilon'_{\text{air}} \approx$

331   1.00, which corresponds to $\varepsilon'_{\text{air}}$ at 25 °C and is almost constant within the studied

332   temperature range. In contrast, if the porosity is 0 %, $\varepsilon'_{\text{eff}}(P = 0) = \varepsilon'_{\text{cer}}$ represents the

333   dielectric constant of the ceramic material of the skeleton or the fully dense ceramic

334   samples.

335   **Table 5.** Mixing relations for the description of effective permittivities. ([1] The exponential

336   relation proposed by Pabst and Gregorová [29] is not identical to the simple exponential

337   relation promoted by Rice [36]).

338   **4.2.1   Effective-medium approximations**

339   The most popular effective-medium approximations [28–30,35] were used for simultaneous

340   data fitting: the Maxwell-Garnett approximation (Maxwell type relation), Bruggeman-

341   Landauer approximation (self-consistent relation), Differential Effective Medium (DEM,

342   differential relation), the exponential relation (from Rice [35]), and the Pabst-Gregorová

343   exponential relation [29] (see Table 5), as shown in Figure 8. The dimensional parameter

344   (d) for the spheres was set as $d = 3$. The adjustable parameter (B) of the Pabst-Gregorová

345   exponential relation was set to $B = 2$ for the spherical pores [29], and B in the exponential

346   relation (from Rice [35]) was estimated from experimental data fitting.

347   **Figure 8**. Comparison of the measured and predicted effective dielectric constants using

348   various effective medium approximation relation for SiSiC (upper-row, left), SBSiC

349   (upper-row, right), SSiC (lower-row, left) and cordierite (lower-row, right) at 2.45 GHz and

350   100 °C. The SSiC permittivities from power extrapolated to 100 °C powder* [5] and

351   foam** [6] were used for comparison.

352   The estimated effective dielectric constants of the fully dense materials were: $\varepsilon'_{SiSiC} =$

353   95.05 ± 0.31, $\varepsilon'_{SBSiC} = 19.34 \pm 0.10$, $\varepsilon'_{SSiC} = 20.26 \pm 0.40$, and $\varepsilon'_{cord} = 5.35 \pm 0.21$.

354   Among the compared relations, the best agreement was obtained using the Maxwell-

355   Garnett relation for the SiC-based foams. The Maxwell-Garnett approach is a non-

356   symmetric relation wherein the sphere inclusions are well-separated from each other.

357   Because $\varepsilon'_{air} < \varepsilon'_{cer}$, the Maxwell-Garnett approach corresponds to the Hashin-Shtrikman

358   upper bound, as shown in Figure 8.

359   Additionally, there was some agreement between the extrapolated permittivities (at 100 °C)

360   of the SiC powder [5] and foam [6] with boron impurities and the Maxwell-Garnett

361   approach relation for the SSiC, despite these materials having a slightly different

362   composition. Further, the dielectric constant of fully dense SSiC ($\varepsilon'_{SSiC} = 20.26 \pm 0.40$)

363   agrees with the extrapolated value of $\varepsilon'_{SSiC} = 20.3 \pm 0.3$ (at 100 °C) from a fully dense tile

364   sample [38] of SiC with boron impurities.

365   **4.2.2   Fixed-parameter weighted mean relations**

366   Fixed-parameter weighted mean relations [28], such as those based on the arithmetic,

367   harmonic, and geometric means (see Table 5) were also used for simultaneous data fitting.

368   The Hashin-Shtrikman (HS) bounds were used along the fixed-parameter weighted mean

369   relations because the foams were considered to be anisotropic. The results are shown in

370   Figure 9.

371   **Figure 9**. Comparison of the measured and predicted effective dielectric constants using

372   the weighted mean relations with Hashin-Shtrikman bounds for SiSiC (upper-row, left),

373   SBSiC (upper-row, right), SSiC (lower-row, left), and cordierite (lower-row, right) at 2.45

374   GHz and 100 °C.

375   The estimated effective dielectric constants of the fully dense materials for the fixed-

376   parameter weighted mean relations were: $\varepsilon'_{\text{SiSiC}} = 91.07 \pm 0.31$, $\varepsilon'_{\text{SBSiC}} = 19.73 \pm 0.10$,

377     $\varepsilon'_{\text{SSiC}} = 20.63 \pm 0.40$, and $\varepsilon'_{\text{cord}} = 5.69 \pm 0.21$, which agreed exceptionally well with the

378   values estimated by the effective-medium approximations.

379   For the SiC-based foams, the weighted mean relations were practically reduced to the fitted

380   upper bound because the data were primarily concentrated at the right side of the graph

381   ($P > 0.8$). Thus, the lines for the upper Wiener or HS bounds ($\Psi_{\text{arithm}} \approx \Psi_{\text{harm}} \approx$

382     $\Psi_{\text{geom}} \approx 0$) nearly overlapped. Conversely, the measured data for the cordierite was

383   between the HS bounds. All weighted mean relations agreed well with the measured data.

384   The weight parameters estimated for cordierite using the HS bounds were

385     $\Psi_{\text{geom/arithm/harm}} \approx 0.25$ and $\varepsilon'_{\text{cord}} = 5.69$.

386   ### 4.2.3   General sigmoidal mean relations

387   The N-parameter (N) in the general sigmoidal mean relation [28] denotes the type of

388   volume-weighting (which is subharmonic for $N < -1$, harmonic for $N = -1$, geometric

389   for $N = 0$, arithmetic for $N = 1$, and superarithmetic for $N > 1$). Figure 10 shows the

390   comparison of the measured and predicted effective dielectric constants for different N-

391   parameters; one parameter was fitted to the experimental data with the condition of $N \neq 0$,

392   and the other was fitted by setting $N = 0$.

393   **Figure 10**. Comparison of the measured and predicted effective dielectric constants using

394   the general sigmoidal mean relations for different N-parameters and bounds for SiSiC

395   (upper-row, left), SBSiC (upper-row, right), SSiC (lower-row, left), and cordierite (lower-

396   row, right) at 2.45 GHz and 100 °C.

397   The estimated effective dielectric constants of the fully dense materials for the general

398   sigmoidal mean relation were essentially equal to those obtained from the weighted mean

399   relation, which is probably related to both relations using the HS bounds. Good agreement

400   was obtained for the non-zero N-parameter ($N \neq 0$) for all materials $N \geq 60.83$, which

401   corresponds with the volume-weighted superarithmetic mean relations coinciding with the

402   Hashin-Shtrikman upper bound.

403   **4.2.4   General power mean relations**

404   Similar to the previous relation, the n-parameter in the general power mean relation [28]

405   denotes the type of volume-weighting, which is harmonic for $n = -1$, geometric for $n =$

406   0, and arithmetic for $n = 1$. The comparison of the predicted and measured effective

407   dielectric constants is shown in Figure 11.

408 **Figure 11**. Comparison of predicted and measured effective dielectric constants using the

409 general power mean relations (solid line: $n = 0$, dashed line: $n \neq 0$); and bounds for the

410 SiSiC (upper-row, left), SBSiC (upper-row, right), SSiC (lower-row, left), and cordierite

411 (lower-row, right) at 2.45 GHz and 100 °C.

412 Sigmoidal relations did not well represent the effective dielectric constant of the SiC-based

413 materials because the data were mainly located at the right side of the graphs, which caused

414 unrealistically high values of the fully dense effective dielectric constant ($\varepsilon'_{SiSiC} \gg 1000$)

415 and the n-parameters to be almost zero. In contrast, $n = 0.327$ was determined for the

416 cordierite to ensure satisfactory agreement between the experimental and predicted results.

417 This value was close to the n-parameter value (n = 1/3) predicted by Landau-Lifshitz and

418 Looyenga [33–34] for the formula $(\varepsilon'_{eff})^{1/3} = (1 - P)(\varepsilon'_{cer})^{1/3} + P(\varepsilon'_{air})^{1/3}$. Interestingly,

419 this formula was developed for the finely dispersed mixing of isotropic particles, which

420 would seem a reasonable assumption for cordierite foams as well. The dielectric constant

421 estimated for the fully dense cordierite was $\varepsilon'_{cord} = 6.038$.

422 **4.3   Complex permittivity calculated by the numerical simulations**

423 The effective permittivities were also calculated from the scattering parameters obtained

424 via electromagnetic simulations for the SiSiC foam inclusions (porosities of 11 %, 30 %, 50

425 %, 70 %, and 85 %) and their corresponding block morphologies at 2.45 GHz. The

426 effective permittivities of the fully dense SiSiC that were obtained from the effective

427 medium approximations were used. Predictions from the simulations in comparison with

428   the effective medium approximation relations are shown in Figure 12. The simulated

429   effective dielectric constants of the foam morphologies demonstrated a tendency similar to

430   the Bruggeman-Landauer relation. Further, the effective dielectric constants of the block

431   morphologies with ceramic and air inclusions behaved similarly to the DEM-complement

432   relation [35] and Maxwell-Garnett relations, respectively; the DEM-complement relation

433   refers to the mixing wherein the phases and their volume fractions are interchanged, as is

434   the case for ceramic inclusions: $\frac{\varepsilon'_{\text{eff}} - \varepsilon'_{\text{cer}}}{\varepsilon'_{\text{air}} - \varepsilon'_{\text{cer}}} \left(\frac{\varepsilon'_{\text{air}}}{\varepsilon'_{\text{eff}}} \right)^{1/d} = P.$

435   **Figure 12.** Comparison of the effective dielectric constants for the SiSiC open-cell foams

436   and blocks at 2.45 GHz and 100 °C obtained via simulations with the experimental data and

437   effective medium approximation relations.

438   The Bruggeman-Landauer relation handles each phase symmetrically, making it invariant

439   to the simultaneous interchanges of the phase permittivities and porosities. The host

440   medium or inclusions are not required to be defined as ceramic or air for the entire porosity

441   range. Therefore, phase-inversion symmetry of the material is allowed, as is expected from

442   the foam morphology models. However, the relation predicts a percolation threshold at $P =$

443    $2/3$ (because air is an insulator), which results in an inflection at this point. This does not

444   hold completely in the case of the simulated foam with $P \geq 0.70$ (see Table 9), indicating

445   that the struts of the ceramic material were still connected at even high porosity values. The

446   permittivity of the simulated foam further explains the apparent agreement in the trends

447   with $P = 0.50$, which matches the bi-continuous [30] segment $(1/3 < P < 2/3)$ of the

448   relation.

449   The Maxwell-Garnett relation is derived from the effective-medium approximation

450   homogenization theory for approximately spherical dielectric particles that are uniformly

451   distributed on a background host medium [26] and provides better prediction accuracy

452   when large numbers of inclusions are present. Further, the inclusions are not required to be

453   the same size as long as they are much smaller than the wavelength of the operating field.

454   Additionally, the effective permittivity depends on the degree of polydispersivity of the

455   spherical particles [30], which leads to a percolation threshold of $P = 1$, i.e., the ceramic

456   phase is no longer connected when everything is entirely composed of air. This holds for

457   the air block inclusions because the ceramic remains interconnected throughout the porosity

458   range and thus, agrees well with the Maxwell-Garnett relation, which approaches the upper

459   HS bound because $\varepsilon'_{\text{cord}} > \varepsilon'_{\text{air}}$. In comparison, the cells of the simulated foams tend to not

460   be as well-represented by spherical particles isolated in a host medium because they are

461   interconnected through open pores, resulting in less agreement with the Maxwell-Garnett

462   relation (see Table 9). Moreover, the measured data were better represented by the air block

463   inclusions, possibly due to the semiconductive silicon effectively connecting the entire

464   skeleton.

465   The DEM and DEM-complement relations are non-symmetric. These relations always

466   address the spherical inclusion phase as a dilute concentration with respect to the effective

467   medium, which is the best approximation of the ceramic inclusions embedded in a

468   continuous medium, i.e., the ceramic block inclusions.

469   For the block inclusions, non-equal contributions generally arise due to the significant

470   difference between their dielectric properties. When the host medium was air ($\varepsilon'_{air}/\varepsilon'_{SiSiC} =$

471   0.058), charge carrier accumulation near the interphase boundaries was promoted, which

472   caused the mixing to be a non-conducting network. Conversely, when the host medium was

473   the SiSiC ($\varepsilon'_{SiSiC}/\varepsilon'_{air} = 17.20$), the mixing is a conducting network [34], which increases

474   the leak current and results in increased dielectric loss. Consequently, the dielectric loss

475   factor of the air block inclusions (SiSiC host medium) was higher than the loss factor of the

476   ceramic block inclusions (air host medium), as shown in Table 6.

477   **Table 6.** Comparison of the effective dielectric permittivities of the SiSiC, as predicted by

478   the Bruggeman-Landauer, Maxwell-Garnett, and DEM-complement relations, with the

479   results of the block and foam simulations.

480   **5   Conclusions and outlook**

481   In this study, the dielectric properties of SiC-based solid foams, namely SBSiC, SSiC, and

482   SiSiC, as well as cordierite were analyzed. The complex permittivity was of particular

483   interest because it provides the imaginary part or loss factor, which is a key parameter for

484   predicting and designing systems heated by microwaves. Both the effective dielectric

485   constant and the effective loss factor of the SiSiC and SBSiC foams had remarkably

26

486   different values as compared to the pure SiC, which could be attributed to the additives or

487   dopants used during the sintering of the foams. The primary conclusions from this study

488   are:

489   ▪   The dielectric properties of open-cell solid foams based on SiC were strongly

490   correlated with the foam porosity. In contrast, no correlation was observed for the

491   pore density.

492   ▪   Satisfactory agreement was obtained for cordierite foams by applying the mixing

493   relations. Notably, the general power mean relation (fitted to the experimental data)

494   returned an exponent $n \approx 1/3$, which agrees with that of the well-known Landau

495   mixing model prediction exponential value of $n = 1/3$.

496   ▪   The effective permittivities predicted by the Bruggeman-Landauer relation

497   reasonably agreed with the numerical simulations that were based on the real foam

498   morphology because this model accounted for the phase-inversion symmetry of the

499   foam skeleton as its porosity increased. By comparison, the Maxwell-Garnett and

500   DEM-complement relations were suggested to predict the numerical simulation

501   results based on simple block non-interconnected (air and ceramic) inclusions,

502   respectively.

503   Future work will focus on numerical simulations and experimental predictions of the

504   dielectric properties of other foam materials and the disclosure of SiSiC, SBSiC, and SSiC

505   fully dense permittivities. Furthermore, the effects of dopants and sintering additives on the

506   dielectric properties of SiC foams will be studied systematically.

507   **Acknowledgment**

511   **Conflict of interest**

512   The authors declare that they have no known competing financial interests or personal

513   relationships that could have appeared to influence the work reported in this paper.

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Figure captions

636   **Figure 1**. (A) Image of the SiSiC foam morphology obtained by SEM; micro X-ray

637   tomography (µCT) cross-sectional images of the SiSiC foams (B) 30 ppi and (C) 45 ppi.

638   **Figure 2.** Schematic of the dielectric measurement system based on the cavity perturbation

639   technique. $S_{21}(f)$ is the frequency-dependent S-parameter response at port 2 to the signal at

640   port 1.

641   **Figure 3.** Schematic structure of (A) SBSiC, (B) SSiC and (C) SiSiC.

642   **Figure 4.** Foam morphology reconstruction and meshing.

643   **Figure 5.** Examples of the foam morphology inclusions (upper row), ceramic block

644   inclusions (middle row), and air block inclusions (lower row), within the boxes. Ceramic

645   and air materials are color-coded in red and blue, respectively.

646   **Figure 6.** Sequential procedure from foam morphology (upper row) and block morphology

647   (lower row) inclusions towards computational modeling for computing scattering

648   parameters using numerical simulations.

649   **Figure 7.** Effective dielectric constants (left) and effective loss factors (right) of cordierite

650   (first row), SiSiC (second row), SBSiC (third row) and SSiC (fourth row) at a frequency of

651   2.45 GHz.

652   **Figure 8.** Comparison of measured and predicted effective dielectric constants using

653   various effective medium approximation relations for SiSiC (upper-row, left), SBSiC

654   (upper-row, right), SSiC (lower-row, left) and cordierite (lower-row, right) at 2.45 GHz and

655   100 °C (SSiC permittivities from power extrapolated to 100°C powder* [5] and foam** [6]

656   were used for comparison).

657   **Figure 9.** Comparison of the measured and predicted effective dielectric constants using the

658   weighted mean relations with Hashin-Shtrikman bounds for SiSiC (upper-row, left), SBSiC

659   (upper-row, right), SSiC (lower-row, left), and cordierite (lower-row, right) at 2.45 GHz

660   and 100 °C.

661   **Figure 10**. Comparison of the measured and predicted effective dielectric constants using

662   the general sigmoidal mean relations for different N-parameters and bounds for SiSiC

663   (upper-row, left), SBSiC (upper-row, right), SSiC (lower-row, left), and cordierite (lower-

664   row, right) at 2.45 GHz and 100 °C.

665   **Figure 11**. Comparison of predicted and measured effective dielectric constants using the

666   general power mean relations (solid line: $n = 0$, dashed line: $n \neq 0$); and bounds for the

667   SiSiC (upper-row, left), SBSiC (upper-row, right), SSiC (lower-row, left), and cordierite

668   (lower-row, right) at 2.45 GHz and 100 °C.

669   **Figure 12**. Comparison of the effective dielectric constants for the SiSiC open-cell foams

670   and blocks at 2.45 GHz and 100 °C obtained via simulations with the experimental data and

671   effective medium approximation relations.

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675   **Table captions**

676   **Table 2**. Complex permittivity of SiC powder and cordierite.

677   **Table 2.** Material compositions.

678   **Table 3.** Bulk and foam sample properties.

679   **Table 4.** Maximum inclusion size parameter (x) of foams; 30 ppi foams were used for the

680   calculations because they have the largest cell diameter.

681   **Table 5.** Mixing relations for the description of effective permittivities. ([1] The exponential

682   relation proposed by Pabst and Gregorová [29] is not identical to the simple exponential

683   relation promoted by Rice [36]).

684   **Table 6.** Comparison of the effective dielectric permittivities of the SiSiC, as predicted by

685   the Bruggeman-Landauer, Maxwell-Garnett, and DEM-complement relations, with the

686   results of the block and foam simulations.

687

688 **Table 3**. Complex permittivity of SiC powder and cordierite.

Material	Temperature (°C)	Frequency (GHz)	Complex permittivity (ε)	Reference
SiC powder	30	2.45	$11.45 - j2.00$	[5]
	50		$11.72 - j1.65$	
	70		$11.87 - j1.36$	
Cordierite	~20	1×10^{-6}	$6 - j0.06$	[31]
	~20	8.52	$4.77 - j0.008$	[32]
	20	10	$4.30 - j8.6x10^{-4}$	[7]
	~20	18.3	$4.89 - j11.6x10^{-4}$	[8]

689

690 **Table 2.** Material compositions.

Material	Ingredients
SBSiC	Bi-modal SiC powder composition, silica sources and alumina powder as inorganic binder matrix.
SSiC	Fine SiC powder, organic temporary binder acting as carbon source after pyrolysis, and boron as a sintering aid.
SiSiC	Fine SiC powder, organic temporary binder acting as carbon source after pyrolysis to react with the infiltrated silicon.
Cordierite	Composition of cordierite chamotte, bentonite, alumina, clay, and soapstone.

691

692 **Table 3.** Bulk and foam sample properties.

Material	Description	Averaged cell diameter (µm)	Density (g/cm^3)	Porosity (%)	Diameter / height of samples (mm)
SBSiC	Bulk	-	2.14	29.1	7.9 / 10.0
	30 ppi	≈ 1460	0.18 - 0.28	90.7 – 94.0	7.2 ± 0.7 / 9.3 ± 0.7
	45 ppi	≈ 890	0.16 - 0.22	92.6 - 94.7	
	60 ppi	≈ 600	0.22	92.7	
SSiC	Bulk	-	2.88	8.7	7.9 / 10.0
	30 ppi	≈ 1460	0.19 - 0.21	93.3 - 94.1	6.9 ± 1.0 / 9.0 ± 1.0
	45 ppi	≈ 890	0.11 - 0.15	95.2 - 96.4	
	60 ppi	≈ 600	0.11 - 0.17	94.7 - 96.5	
SiSiC	Bulk	-	2.69	0.0	7.9 / 10.0
	30 ppi	≈ 1460	0.27 - 0.35	88.0 - 88.9	7.4 ± 0.5 / 9.5 ± 0.5
	45 ppi	≈ 890	0.20 - 0.22	90.9 - 91.8	
	60 ppi	≈ 600	0.18 - 0.20	92.2 - 93.0	
Cordierite	Bulk	-	1.80	29.3	7.9 / 10.0
	30 ppi	≈ 1460	0.33 - 0.37	85.3 - 87.2	7.4 ± 0.5 / 9.5 ± 0.5
	45 ppi	≈ 890	0.27 - 0.49	80.7 - 89.5	
	60 ppi	≈ 600	0.27	89.5	

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694   **Table 4.** Maximum inclusion size parameter (x) of foams; 30 ppi foams were used for the

695   calculations because they have the largest cell diameter.

Foams	SBSiC	SSiC	SiSiC	Cordierite
$\varepsilon'_{eff,30ppi}$	1.85	1.67	7.10	1.45
λ (mm)	90.7	99.3	46.4	101.32
x_{30ppi}	0.05	0.05	0.10	0.05

696

697 **Table 5.** Mixing relations for the description of effective permittivities. ([1] The exponential

698 relation proposed by Pabst and Gregorová [29] is not identical to the simple exponential

699 relation promoted by Rice [36]).

Effective-medium approximations	
Maxwell-Garnett approximation (Maxwell-type relation non-symmetric) [30]	$\dfrac{\varepsilon'_{\text{eff}} - \varepsilon'_{\text{cer}}}{\varepsilon'_{\text{eff}} + (d-1)\varepsilon'_{\text{cer}}} = (P)\dfrac{\varepsilon'_{\text{air}} - \varepsilon'_{\text{cer}}}{\varepsilon'_{\text{air}} + (d-1)\varepsilon'_{\text{cer}}}$
Bruggeman-Landauer approximation (self-consistent relation symmetric) [30, 35]	$(1-P)\dfrac{\varepsilon'_{\text{cer}} - \varepsilon'_{\text{eff}}}{\varepsilon'_{\text{cer}} + (d-1)\varepsilon'_{\text{eff}}} = -(P)\dfrac{\varepsilon'_{\text{air}} - \varepsilon'_{\text{eff}}}{\varepsilon'_{\text{air}} + (d-1)\varepsilon'_{\text{eff}}}$
Differential Effective Medium (DEM) approximation (Bruggeman relation non-symmetric) [30, 35]	$\dfrac{\varepsilon'_{\text{air}} - \varepsilon'_{\text{eff}}}{\varepsilon'_{\text{air}} - \varepsilon'_{\text{cer}}}\left(\dfrac{\varepsilon'_{\text{cer}}}{\varepsilon'_{\text{eff}}}\right)^{1/d} = 1 - P$
Exponential relation [36]	$\dfrac{\varepsilon'_{\text{eff}}}{\varepsilon'_{\text{cer}}} = \exp(-B \cdot P)$
Pabst-Gregorová [1] exponential relation [29]	$\dfrac{\varepsilon'_{\text{eff}}}{\varepsilon'_{\text{cer}}} = \exp\left(-B\dfrac{P}{1-P}\right)$

Empirical mixing relations	
Weighted arithmetic mean of the upper and lower bounds [28]	$\varepsilon'_{\text{eff}} = (1 - \Psi_{\text{arithm}})\varepsilon_{\text{B}}^+ + \Psi_{\text{arithm}}\varepsilon_{\text{B}}^-$
Weighted harmonic mean of the upper and lower bounds [28]	$\varepsilon'_{\text{eff}} = \left[\dfrac{(1 - \Psi_{\text{harm}})}{\varepsilon_{\text{B}}^+} + \dfrac{\Psi_{\text{harm}}}{\varepsilon_{\text{B}}^-}\right]^{-1}$
Weighted geometric mean of the upper and lower bounds [28]	$\varepsilon'_{\text{eff}} = \exp\left[(1 - \Psi_{\text{geom}})\ln(\varepsilon_{\text{B}}^+) + \Psi_{\text{geom}}\ln(\varepsilon_{\text{B}}^-)\right]$
General sigmoidal mean of the upper and lower bounds [28]	$\varepsilon'_{\text{eff}} = (1 - \delta_{0,N})[(1-P)(\varepsilon_{\text{B}}^+)^N + P(\varepsilon_{\text{B}}^-)^N]^{\frac{1}{N}}$ $+ \delta_{0,N}\{\exp[(1-P)\ln(\varepsilon_{\text{B}}^+) + P\ln(\varepsilon_{\text{B}}^-)]\}$ where $\delta_{0,N} = \begin{cases} 1 & \text{if } N = 0 \\ 0 & \text{otherwise} \end{cases}$
General power mean of the upper and lower dielectric constants [28]	$\varepsilon'_{\text{eff}} = (1 - \delta_{0,n})[(1-P)(\varepsilon'_{\text{cer}})^n + P(\varepsilon'_{\text{air}})^n]^{\frac{1}{n}}$ $+ \delta_{0,n}\{\exp[(1-P)\ln(\varepsilon'_{\text{cer}}) + P\ln(\varepsilon'_{\text{air}})]\}$ where $\delta_{0,n} = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}$

Wiener bounds ($\varepsilon_B = \varepsilon_W$)		Hashin–Shtrikman bounds ($\varepsilon_B = \varepsilon_{SH}$)	
Upper Wiener bound [28]	$\varepsilon_{\text{W}}^+ = (1-P)\varepsilon'_{\text{cer}} + P\varepsilon'_{\text{air}}$	Upper Hashin–Shtrikman bound [28]	$\varepsilon_{\text{HS}}^+ = (1-P)\varepsilon'_{\text{cer}} + P\varepsilon'_{\text{air}} - \dfrac{(1-P)P(\varepsilon'_{\text{cer}} - \varepsilon'_{\text{air}})^2}{3\varepsilon'_{\text{cer}} - (1-P)(\varepsilon'_{\text{cer}} - \varepsilon'_{\text{air}})}$
Lower Wiener bound [28]	$\varepsilon_{\text{W}}^- = \left[\dfrac{(1-P)}{\varepsilon'_{\text{cer}}} + \dfrac{P}{\varepsilon'_{\text{air}}}\right]^{-1}$	Lower Hashin–Shtrikman bound [28]	$\varepsilon_{\text{HS}}^- = (1-P)\varepsilon'_{\text{cer}} + P\varepsilon'_{\text{air}} - \dfrac{(1-P)P(\varepsilon'_{\text{cer}} - \varepsilon'_{\text{air}})^2}{3\varepsilon'_{\text{air}} + P(\varepsilon'_{\text{cer}} - \varepsilon'_{\text{air}})}$

700 **Table 6.** Comparison of the effective dielectric permittivities of the SiSiC, as predicted by

701 the Bruggeman-Landauer, Maxwell-Garnett, and DEM-complement relations, with the

702 results of the block and foam simulations.

Porosity	Effective dielectric permittivity $\varepsilon'_{\text{eff}}$ (-)					
	Simulation with foam morphology	Simulated blocks – air inclusions	Simulated blocks –ceramic inclusions	Bruggeman-Landauer	Maxwell-Garnett	DEM-complement
0.11	$79.83 - j2.99$	$78.30 - j3.94$	$20.74 - j0.25$	$79.62 - j4.27$	$80.41 - j4.32$	$55.59 - j2.40$
0.30	$51.03 - j1.43$	$56.80 - j2.96$	$7.99 - j0.04$	$53.12 - j2.82$	$58.37 - j3.12$	$19.35 - j0.43$
0.50	$26.42 - j0.72$	$37.49 - j1.97$	$4.29 - j0.01$	$25.85 - j1.28$	$38.74 - j2.05$	$6.65 - j0.05$
0.70	$8.96 - j0.39$	$21.62 - j1.09$	$2.49 - j0.00$	$5.11 - j0.09$	$21.98 - j1.13$	$2.75 - j0.01$
0.85	$5.40 - j0.01$	$0.00 - j0.00$	$1.64 - j0.00$	$1.75 - j0.00$	$10.95 - j0.54$	$1.71 - j0.00$

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