Screening approach to address relativistic species as sources of cosmological perturbations

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joint work with
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and
Maksym Brilenkov (University of Oslo)

arXiv:2206.13495 [gr-qc]

Dynamics of the homogeneous background & the perturbed metric in Poisson gauge

$$ds^2 = a(\eta)^2\left(d\eta^2 - \delta_{\alpha\beta}dx^\alpha dx^\beta\right)$$

$$g_{ik} = \bar{g}_{ik} + \delta g_{ik}$$

$$ds^2 = a^2(\eta)\left[(1+2\Psi)d\eta^2 + 2B_\alpha dx^\alpha d\eta - \delta_{\alpha\beta}(1-2\Phi)dx^\alpha dx^\beta\right]$$

Friedmann eqn.s for the **Λ**CDM model for NR species only* :

$$\frac{3\mathcal{H}^2}{a^2} = \kappa\bar{\varepsilon} + \Lambda$$

$$\frac{2\mathcal{H}' + \mathcal{H}^2}{a^2} = \Lambda$$

- η: conformal time; $cdt = ad\eta$
- $a(\eta)$: scale factor
- x^α: comoving coordinates; $\alpha, \beta = 1,2,3$
- $\mathcal{H} \equiv a'/a$; $\ ' \equiv d/d\eta$
- $\kappa \equiv 8\pi G_N/c^4$;
  G_N: Newtonian gravitational constant
- $\bar{\varepsilon}$: average energy density

Stress-energy perturbations

We employ a system of point-like particles* described by the EMT

$$T^{ik} = \sum_n m_{(n)} c^2 \frac{\delta\left(\boldsymbol{r} - \boldsymbol{r}_n\right)}{\sqrt{-g}} \left(g_{ml} \frac{dx^m_{(n)}}{d\eta} \frac{dx^l_{(n)}}{d\eta} \right)^{-1/2} \frac{dx^i_{(n)}}{d\eta} \frac{dx^k_{(n)}}{d\eta}$$

$$T^0_0 = \frac{c^2}{a^3} \sum_n m_{(n)} \delta\left(\boldsymbol{r} - \boldsymbol{r}_n\right)(1 + 3\Phi)$$

$$T^\alpha_\beta = 0$$

Stress-energy perturbations

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$$T_0^0 = \frac{c^2}{a^3} \sum_n m_{(n)} \delta\left(\mathbf{r}-\mathbf{r}_n\right)(1+3\Phi)$$

1st order

$$\rho(1+3\Phi) = \bar{\rho} + \delta\rho + (\bar{\rho} + \cancel{\delta\rho})3\Phi$$

$$T_\beta^\alpha = 0$$

Eingorn, M., ApJ **825**, 84 (2016)

Stress-energy perturbations

$$\delta T^0_\alpha = -\frac{c^2}{a^3}\sum_n m_{(n)}\delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\tilde{v}_{(n)\,\alpha} + \frac{\bar{\rho}c^2}{a^3}B_\alpha$$

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Stress-energy perturbations

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$$\delta T^0_\alpha = -\frac{c^2}{a^3}\sum_n m_{(n)}\delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\tilde{v}_{(n)\alpha} + \frac{\bar{\rho}c^2}{a^3}B_\alpha$$

- $\rho_{(n)} \equiv m_{(n)}\delta(\boldsymbol{r}-\boldsymbol{r}_n)$
- $\rho \equiv \bar{\rho} + \delta\rho$
- $\bar{\varepsilon} = \frac{\bar{\rho}c^2}{a^3}$
- $\frac{dx^\alpha_{(n)}}{d\eta} \equiv \tilde{v}^\alpha_{(n)}$

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Stress-energy perturbations

$$\delta T^0_\alpha = -\frac{c^2}{a^3}\sum_n m_{(n)}\delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\tilde{v}_{(n)\alpha} + \frac{\bar{\rho}c^2}{a^3}B_\alpha$$

$$\rho_{(n)}\tilde{v}_{(n)\alpha}$$

$$(\bar{\rho} + \delta\rho)B_\alpha$$

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Eingorn, M., ApJ **825**, 84 (2016)

Field equations $\delta G_i^k = \kappa \delta T_i^k$ for determining metric perturbations

$00-$

$$\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Psi = \frac{\kappa a^2}{2}\delta T_0^0$$

$0\alpha-$

$$\frac{1}{4}\Delta B_\alpha + \Phi'_{,\alpha} + \mathcal{H}\Psi_{,\alpha} = \frac{\kappa a^2}{2}\delta T_\alpha^0$$

- $\Delta \equiv \frac{\delta^{\alpha\beta}\partial^2}{\partial x^\alpha \partial x^\beta}$
- $\Pi_{\alpha\beta} \equiv \left(\delta_{\gamma\alpha}T_\beta^\gamma - \delta_{\alpha\beta}T_\gamma^\gamma/3\right)$
- $\chi \equiv \Phi - \Psi$

$\alpha\beta-$

$$B'_{(\gamma,\sigma)} + 2\mathcal{H}B_{(\sigma,\gamma)} = 0$$

Field equations $\delta G_i^k = \kappa \delta T_i^k$ for determining metric perturbations

$00-$

$$\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Psi = \frac{\kappa a^2}{2}\delta T_0^0$$

$$(1+4\Phi)\Delta\Phi - 3\mathcal{H}\Phi' + 3\mathcal{H}^2(\chi - \Phi) + \frac{3}{2}\delta^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta} = -\frac{\kappa a^2}{2}\delta T_0^0$$

$0\alpha-$

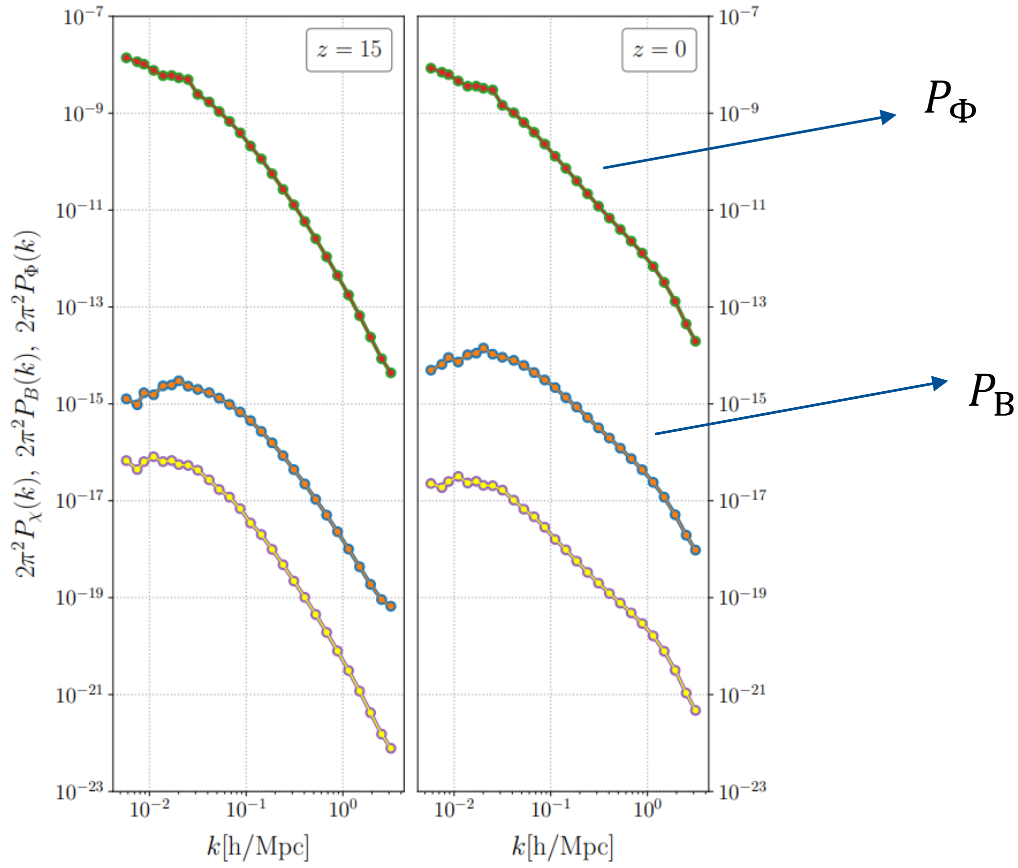
$$\frac{1}{4}\Delta B_\alpha + \Phi'_{,\alpha} + \mathcal{H}\Psi_{,\alpha} = \frac{\kappa a^2}{2}\delta T_\alpha^0$$

$\alpha\beta-$

Adamek, J., Daverio, D., Durrer, R., Kunz, M., JCAP **07,** 053 (2016)

$$\left(\delta_\alpha^\gamma \delta_\beta^\sigma - \frac{1}{3}\delta^{\gamma\sigma}\delta_{\alpha\beta}\right) \times \left[B'_{(\gamma,\sigma)} + 2\mathcal{H}B_{(\sigma,\gamma)} + \chi_{,\sigma\gamma} - 2\chi\Phi_{,\sigma\gamma} + 2\Phi_{,\sigma}\Phi_{,\gamma} + 4\Phi\Phi_{,\gamma\sigma}\right] = \kappa a^2 \Pi_{\alpha\beta}$$

The cosmic screening approach



L=1680 Mpc/h with
1 Mpc/h res.
starting from z=100

Eingorn, M., Yukselci, A.E. and Zhuk, A., Phys. Lett. B **826,** 136911 (2022)

Helmholtz equations for Φ and B_α

$$\Delta\Phi - \frac{3\kappa\bar{\rho}c^2}{2a}\Phi = \frac{\kappa c^2}{2a}\delta\rho - \frac{3\kappa c^2\mathcal{H}}{2a}\Xi$$

$$\Delta\mathbf{B} - \frac{2\kappa\bar{\rho}c^2}{a}\mathbf{B} = -\frac{2\kappa c^2}{a}\left(\sum_n \rho_n\tilde{\boldsymbol{v}}_n - \nabla\Xi\right)$$

$$\Delta\Xi = \nabla\sum_n \rho_n\tilde{\boldsymbol{v}}_n \rightarrow \ \Xi = \frac{1}{4\pi}\sum_n m_n\frac{\tilde{\boldsymbol{v}}_n(\boldsymbol{r}-\boldsymbol{r}_n)}{|\boldsymbol{r}-\boldsymbol{r}_n|^3}$$

Eingorn, M., ApJ **825**, 84 (2016)

The cosmic screening approach

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$$\Delta\Phi - \cancel{\frac{3\kappa\bar{\rho}c^2}{2a}}\Phi = \frac{\kappa c^2}{2a}\delta\rho - \cancel{\frac{3\kappa c^2\mathcal{H}}{2a}}\Xi$$

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Eingorn, M., ApJ **825**, 84 (2016)

Solutions in the cosmic screening approach

$$\Phi = \frac{1}{3} - \frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\boldsymbol{r}-\boldsymbol{r}_n|}\exp(-q_n)$$
$$+\frac{3\kappa c^2\mathcal{H}}{8\pi a}\sum_n \frac{m_n[\tilde{\boldsymbol{v}}_n(\boldsymbol{r}-\boldsymbol{r}_n)]}{|\boldsymbol{r}-\boldsymbol{r}_n|}\frac{1-(1+q_n)\exp(-q_n)}{q_n^2}$$

$$\mathbf{B} = \frac{\kappa c^2}{8\pi a}\sum_n \Bigg[\frac{m_n\tilde{\boldsymbol{v}}_n}{|\boldsymbol{r}-\boldsymbol{r}_n|}\frac{\left(3+2\sqrt{3}q_n+4q_n^2\right)\exp(-2q_n/\sqrt{3})-3}{q_n^2}$$
$$+\frac{m_n[\tilde{\boldsymbol{v}}_n(\boldsymbol{r}-\boldsymbol{r}_n)]}{|\boldsymbol{r}-\boldsymbol{r}_n|^3}(\boldsymbol{r}-\boldsymbol{r}_n)\frac{9-\left(9+6\sqrt{3}q_n+4q_n^2\right)\exp(-2q_n/\sqrt{3})}{q_n^2}\Bigg]$$

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Solutions in the cosmic screening approach

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$$+\frac{3\kappa c^2\mathcal{H}}{8\pi a}\sum_n \frac{m_n[\tilde{\boldsymbol{v}}_n(\boldsymbol{r}-\boldsymbol{r}_n)]}{|\boldsymbol{r}-\boldsymbol{r}_n|}\frac{1-(1+q_n)\exp(-q_n)}{q_n^2}$$

$$\boldsymbol{q}_n(\eta,\boldsymbol{r}) \equiv \sqrt{\frac{3\kappa\bar{\rho}c^2}{2a}}(\boldsymbol{r}-\boldsymbol{r}_n) = \frac{a(\boldsymbol{r}-\boldsymbol{r}_n)}{\lambda}\ , \qquad \lambda \equiv \sqrt{\frac{2a^3}{3\kappa\bar{\rho}c^2}}$$

Solutions in the cosmic screening approach

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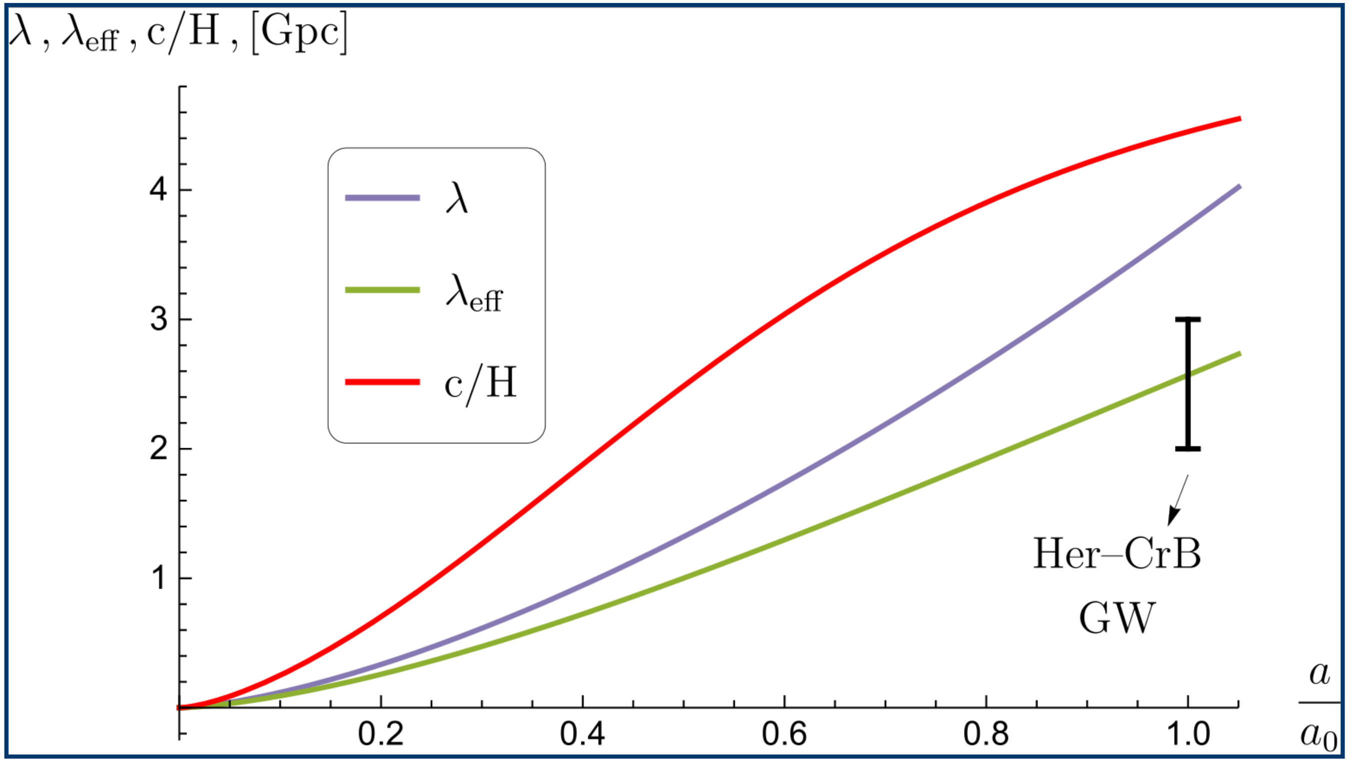
$$\boldsymbol{q}_n(\eta,\boldsymbol{r}) \equiv \sqrt{\frac{3\kappa\bar{\rho}c^2}{2a}}(\boldsymbol{r}-\boldsymbol{r}_n) = \frac{a(\boldsymbol{r}-\boldsymbol{r}_n)}{\lambda}\ , \qquad \lambda \equiv \sqrt{\frac{2a^3}{3\kappa\bar{\rho}c^2}}$$

$$\Delta\Phi - \frac{a^2}{\lambda_{\text{eff}}^2}\Phi = \frac{\kappa c^2}{2a}\delta\rho\ ,\ \Phi = \frac{1}{3}\left(\frac{\lambda_{\text{eff}}}{\lambda}\right)^2 - \frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\boldsymbol{r}-\boldsymbol{r}_n|}\exp\left(-\frac{a|\boldsymbol{r}-\boldsymbol{r}_n|}{\lambda_{\text{eff}}}\right)$$

$$\frac{1}{\lambda_{\text{eff}}^2} = \frac{3}{c^2a^2H}\left(\int\frac{da}{a^3H^3}\right)^{-1}$$

Canay, E., Eingorn, M., PDU **29**, 100565 (2020)



I. Horváth, D. Szécsi, J. Hakkila , Á. Szabó, I. I. Racz, L. V. Tóth, S. Pinter , Z. Bagoly, MNRAS, 498(2), 2544 (2020)

Canay, E., Eingorn, M., PDU **29**, 100565 (2020)

Stress-energy perturbations - revisited

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* $c = 1, \;\; \tilde{v}^{\alpha}_{(n)} \rightarrow q^{\alpha}_{(n)}$

$$T^0_0 = \frac{1}{a^3} \sum_n m_{(n)} \delta\left(\boldsymbol{r} - \boldsymbol{r}_n\right)(1 + 3\Phi)$$

Stress-energy perturbations - revisited

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SYSTEMS UNDERSTANDING

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$$T^0_0 = \frac{1}{a^3} \sum_n m_{(n)} \delta\left(\boldsymbol{r} - \boldsymbol{r}_n\right)(1 + 3\Phi)$$

$$T^0_0 = \frac{1}{a^4} \sum_n \delta\left(\boldsymbol{r} - \boldsymbol{r}_n\right) \sqrt{q^2_{(n)} + m^2_{(n)} a^2} \left(1 + 3\Phi + \frac{q^2_{(n)}}{q^2_{(n)} + a^2 m^2_{(n)}} \Phi\right)$$

$$= \frac{1}{a^4} \sum_n \delta\left(\boldsymbol{r} - \boldsymbol{r}_n\right) \sqrt{q^2_{(n)} + m^2_{(n)} a^2} + \frac{1}{a^4} \sum_n \delta\left(\boldsymbol{r} - \boldsymbol{r}_n\right) \frac{4 q^2_{(n)} + 3 m^2_{(n)} a^2}{\sqrt{q^2_{(n)} + m^2_{(n)} a^2}} \Phi$$

Stress-energy perturbations - revisited

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$* \; c = 1, \quad \tilde{v}^{\alpha}_{(n)} \to q^{\alpha}_{(n)}$

$$T^0_0 = \frac{1}{a^3}\sum_n m_{(n)}\delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)(1+3\Phi)$$

$$T^0_0 = \frac{1}{a^4}\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\sqrt{q^2_{(n)}+m^2_{(n)}a^2}\left(1+3\Phi+\frac{q^2_{(n)}}{q^2_{(n)}+a^2m^2_{(n)}}\Phi\right)$$

$$= \frac{1}{a^4}\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\sqrt{q^2_{(n)}+m^2_{(n)}a^2} + \frac{1}{a^4}\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\frac{4q^2_{(n)}+3m^2_{(n)}a^2}{\sqrt{q^2_{(n)}+m^2_{(n)}a^2}}\Phi$$

$$T^0_0 = -\frac{1}{a^4}\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\sqrt{q^2_{(n)}+m^2_{(n)}a^2}\left(1+3\Phi+\frac{q^2_{(n)}}{q^2_{(n)}+a^2m^2_{(n)}}\Phi+q^{\alpha}_{(n)}B_{\alpha}\right)$$

Brilenkov, M., Canay, E. and Eingorn, M., arXiv:2206.13495v1 [gr-qc]

Adamek, J., Daverio, D., Durrer, R., Kunz, M., JCAP **07,** 053 (2016)

Stress-energy perturbations - revisited

$$\overline{\frac{1}{a^4}\sum_n \delta(\boldsymbol{r}-\boldsymbol{r}_n)\sqrt{q_{(n)}^2+m_{(n)}^2a^2}} \equiv \bar{\varepsilon} \quad \rightarrow \quad \frac{3\mathcal{H}^2}{a^2} = \kappa\bar{\varepsilon}+\Lambda$$

$$\frac{1}{a^4}\sum_n \delta(\boldsymbol{r}-\boldsymbol{r}_n)\sqrt{q_{(n)}^2+m_{(n)}^2a^2}-\bar{\varepsilon}$$

$$=\frac{1}{a^4}\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\sqrt{q_{(n)}^2+m_{(n)}^2a^2}+\overline{\frac{1}{a^4}\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\frac{4q_{(n)}^2+3m_{(n)}^2a^2}{\sqrt{q_{(n)}^2+m_{(n)}^2a^2}}}\,\Phi$$

Stress-energy perturbations - revisited

$$\frac{1}{a^4}\overline{\overline{\sum_n \delta(\boldsymbol{r}-\boldsymbol{r}_n)\sqrt{q_{(n)}^2+m_{(n)}^2 a^2}}} \equiv \bar{\varepsilon} \quad \rightarrow \quad \frac{3\mathcal{H}^2}{a^2} = \kappa\bar{\varepsilon} + \Lambda$$

1st order

$$\frac{1}{a^4}\sum_n \delta(\boldsymbol{r}-\boldsymbol{r}_n)\sqrt{q_{(n)}^2+m_{(n)}^2 a^2} - \bar{\varepsilon}$$

$$= \frac{1}{a^4}\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\sqrt{q_{(n)}^2+m_{(n)}^2 a^2} + \frac{1}{a^4}\overline{\overline{\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\frac{4q_{(n)}^2+3m_{(n)}^2 a^2}{\sqrt{q_{(n)}^2+m_{(n)}^2 a^2}}}}\,\Phi$$

1st order

$$\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Psi = \frac{\kappa a^2}{2}\delta T_0^0$$

$$\frac{1}{a^4}\sum_n \delta(\boldsymbol{r}-\boldsymbol{r}_n)\sqrt{q_{(n)}^2 + m_{(n)}^2 a^2} - \bar{\varepsilon}$$

$$= \frac{1}{a^4}\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\sqrt{q_{(n)}^2 + m_{(n)}^2 a^2} + \frac{1}{a^4}\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\frac{4q_{(n)}^2 + 3m_{(n)}^2 a^2}{\sqrt{q_{(n)}^2 + m_{(n)}^2 a^2}}\Phi$$

$$T_{\alpha}^{0} = -\frac{1}{a^4}\sum_{n} \delta\left(\boldsymbol{r} - \boldsymbol{r}_n\right) q_{(n)_{\alpha}}$$

$$\frac{1}{4}\Delta B_{\alpha} + \Phi'_{,\alpha} + \mathcal{H}\Psi_{,\alpha} = \frac{\kappa a^2}{2}\delta T_{\alpha}^{0}$$

$$T^{\alpha}_{\beta} = 0$$

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$$T^{\alpha}_{\beta} = -\frac{\delta^{\alpha\gamma}}{a^4}\sum_n \delta\left(\boldsymbol{r} - \boldsymbol{r}_n\right)\frac{q_{(n)_\gamma} q_{(n)_\beta}}{\sqrt{q^2_{(n)} + m^2_{(n)} a^2}}$$

$$-\frac{1}{3a^4}\sum_n \delta\left(\boldsymbol{r} - \boldsymbol{r}_n\right)\frac{q^2_{(n)}\left(4q^2_{(n)} + 5m^2_{(n)} a^2\right)}{\left(q^2_{(n)} + m^2_{(n)} a^2\right)^{3/2}}\delta^{\alpha}_{\beta}\Phi$$

$$T^{\alpha}_{\beta} = 0$$

$$T^{\alpha}_{\beta} = -\frac{\delta^{\alpha\gamma}}{a^4}\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\frac{q_{(n)_\gamma}q_{(n)_\beta}}{\sqrt{q_{(n)}^2+m_{(n)}^2a^2}}$$

$$-\frac{1}{3a^4}\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)\frac{q_{(n)}^2\left(4q_{(n)}^2+5m_{(n)}^2a^2\right)}{\left(q_{(n)}^2+m_{(n)}^2a^2\right)^{3/2}}\delta^{\alpha}_{\beta}\Phi$$

- $\chi \equiv \Phi - \Psi$

$$\Psi = \Phi - \frac{3\kappa}{16\pi a^2}\sum_n \frac{q_{(n)_\gamma}q_{(n)_\beta} - q_{(n)}^2\delta_{\beta\gamma}/3}{\sqrt{q_{(n)}^2+m_{(n)}^2a^2}}\frac{\left(x^\gamma - x^\gamma_{(n)}\right)\left(x^\beta - x^\beta_{(n)}\right)}{|\boldsymbol{r}-\boldsymbol{r}_n|^3}$$

Helmholtz equations for Φ and B_α

$$\Delta\Phi - \frac{a^2}{\lambda^2}\Phi = \frac{\kappa a^2}{2}\left(\frac{1}{a^4}\sum_n \delta(\boldsymbol{r}-\boldsymbol{r}_n)\sqrt{q_{(n)}^2 + m_{(n)}^2 a^2} - \bar{\varepsilon}\right) - \frac{3\kappa\mathcal{H}}{2a}\Xi$$

$$\Xi = \frac{1}{4\pi a}\sum_n \frac{q_{(n)}^\alpha\left(x^\alpha - x_{(n)}^\alpha\right)}{|\boldsymbol{r}-\boldsymbol{r}_n|^3}$$

Helmholtz equations for Φ and B_α

$$\Delta\Phi - \frac{a^2}{\lambda^2}\Phi = \frac{\kappa a^2}{2}\left(\frac{1}{a^4}\sum_n \delta(\boldsymbol{r}-\boldsymbol{r}_n)\sqrt{q_{(n)}^2 + m_{(n)}^2 a^2} - \bar{\varepsilon}\right) - \frac{3\kappa\mathcal{H}}{2a}\Xi$$

$$\lambda \equiv \sqrt{\frac{2a^3}{3\kappa\bar{\rho}}}$$

$$\frac{a}{\sqrt{3(\mathcal{H}^2 - \mathcal{H}')}}$$

$$\Xi = \frac{1}{4\pi a}\sum_n \frac{q_{(n)}^\alpha\left(x^\alpha - x_{(n)}^\alpha\right)}{|\boldsymbol{r}-\boldsymbol{r}_n|^3}$$

Evolving the particle ensemble

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$$q'_{(n)_\alpha} = -\sqrt{q_{(n)}^2 + m_{(n)}^2 a^2}\Psi_{,\alpha} - \frac{q_{(n)}^2 \Phi_{,\alpha}}{\sqrt{q_{(n)}^2 + m_{(n)}^2 a^2}} - q_{(n)_\gamma} B_{\gamma,\alpha}$$

$$v_{(n)_\alpha} = \frac{q_{(n)_\alpha}}{\sqrt{q_{(n)}^2 + m_{(n)}^2 a^2}}(1+\Psi) + \frac{q_{(n)_\alpha}}{\sqrt{q_{(n)}^2 + m_{(n)}^2 a^2}}\left(2 - \frac{q_{(n)}^2}{q_{(n)}^2 + a^2 m_{(n)}^2}\right)\Phi + B_\alpha$$

Brilenkov, M., Canay, E. and Eingorn, M., arXiv:2206.13495v1 [gr-qc]

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