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Dictionary-free handling of all-scale cosmological perturbations

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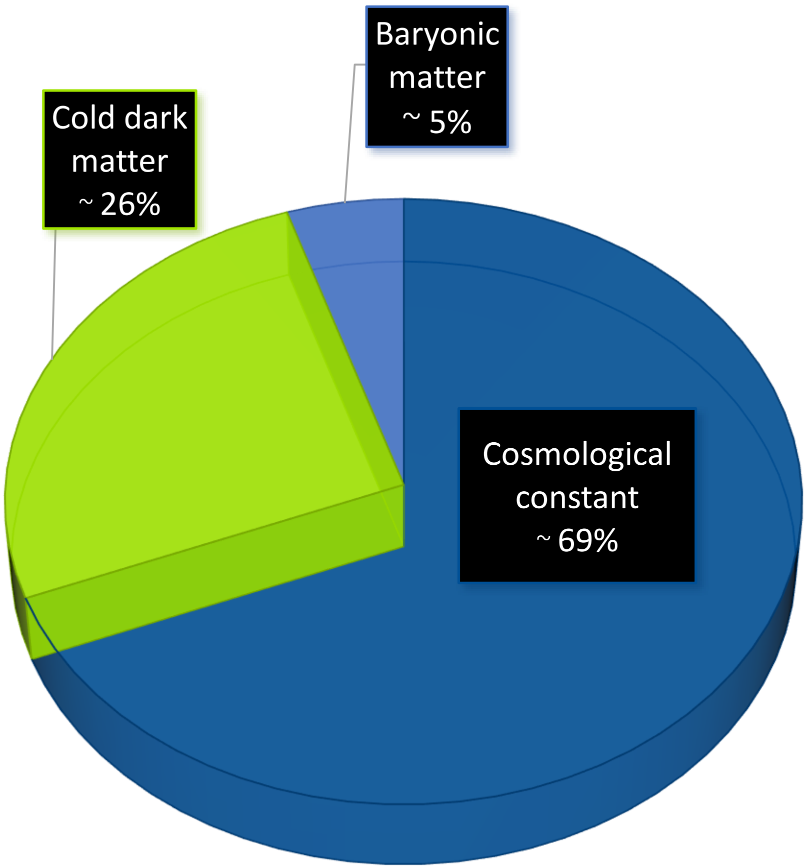
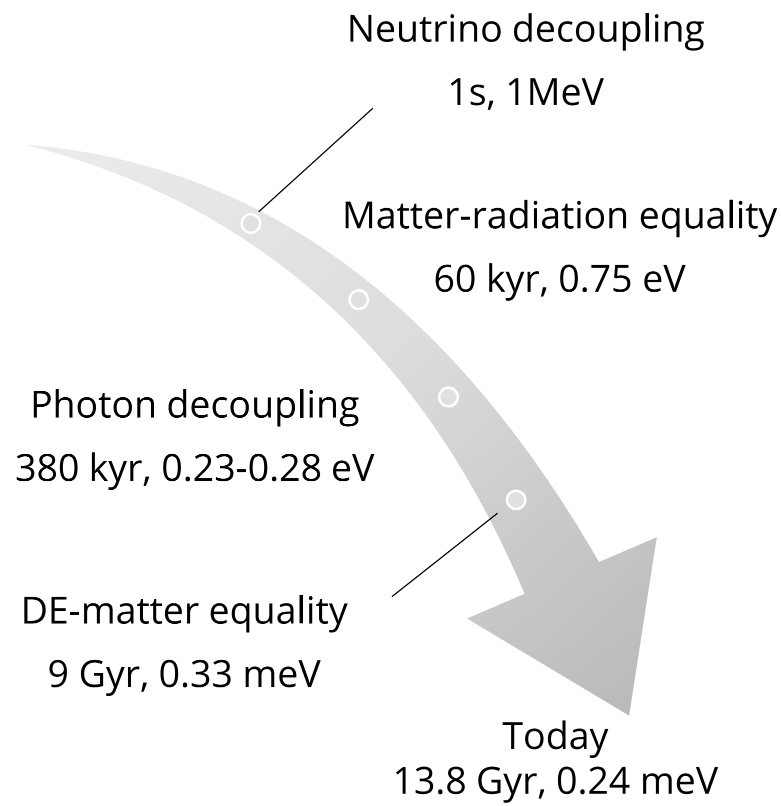
The ΛCDM model



Concordance cosmology



Planck 2018 results. VI. Cosmological parameters, A&A **641**, A6 (2020)

Baumann, D., *Lecture Notes on Cosmology, Part III Math Tripos,* Cambridge (2014)

Superclusters & voids

~ 100 Mpc across

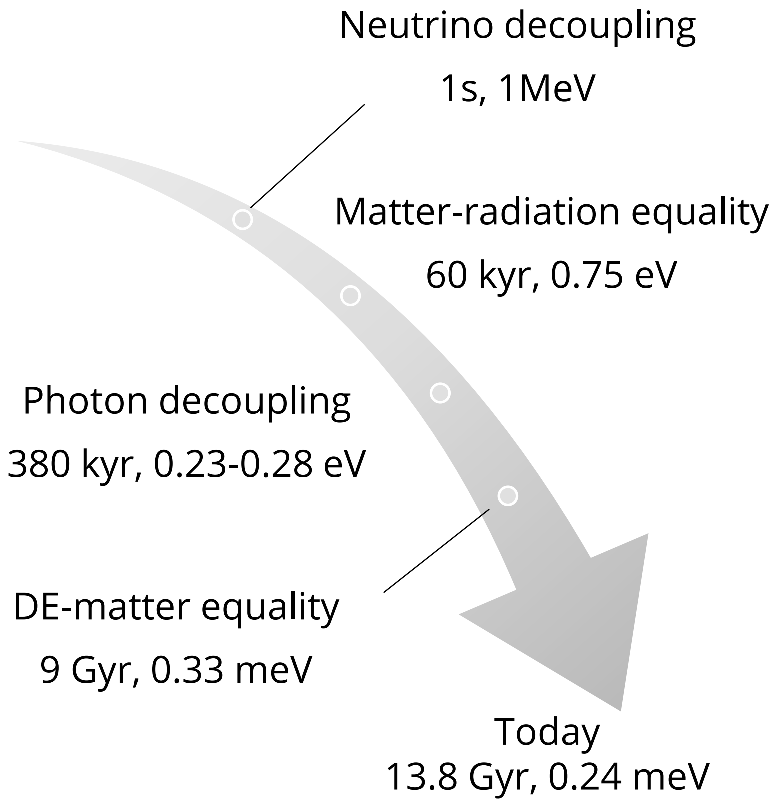
The nonlinear scale

-from tens of megaparsecs

Typical galaxy clusters have sizes of several megaparsecs

The Milky Way

~ 30 kpc



Baumann, D., *Lecture Notes on Cosmology, Part III Math Tripos,* Cambridge (2014)

Metric perturbations in an arbitrary gauge – the scalar sector

$$ds^2 = a(\eta)^2\left(-d\eta^2 + \delta_{\alpha\beta}dx^\alpha dx^\beta\right) \longrightarrow g_{ik} = \bar{g}_{ik} + \delta g_{ik}$$

$$ds^2 = a^2(\eta)$$
$$\times\left[-(1+2A)d\eta^2 - 2\partial_\alpha B dx^\alpha d\eta + \left[\delta_{\alpha\beta}(1+2H_L) - 2\left(\partial_\alpha\partial_\beta - \delta_{\alpha\beta}\Delta/3\right)H_T\right]dx^\alpha dx^\beta\right]$$

$$\Phi = H_L + \frac{\Delta}{3}H_T - \mathcal{H}\left(B - \dot{H}_T\right)$$ – the Bardeen potential

The stress-energy tensor

$$T_0^0 = -\bar{\varepsilon}(1+\delta),$$

$$T_0^\alpha = -\,(\bar{\varepsilon} + \bar{p})\partial^\alpha v, \quad T_\beta^\alpha = (\bar{p} + \delta p)\delta_\beta^\alpha + \bar{p}\Pi_\beta^\alpha$$

The «N-body» gauge

$$B = v$$

$$k^2\Phi^N = 4\pi G_N a^2 \bar{\varepsilon}_c^N \delta_c^N$$

$$\partial_\eta \delta_c^N + k v_c^N = 0$$

$$\left[\partial_\eta + \mathcal{H}\right] v_c^N = -k\Phi^N$$

$$k^2\Phi = 4\pi G_N a^2 \bar{\varepsilon}\delta$$

$$\partial_\eta \delta_c + k v_c = -3\dot{H}_L$$

$$\left[\partial_\eta + \mathcal{H}\right] v_c = -k(\Phi + \gamma)$$

$$\varepsilon = (1 - 3H_L)\left[\frac{1}{a^3}\sum_n m_n \delta(\boldsymbol{x} - \boldsymbol{x}_n)\right]$$

C. Fidler, C. Rampf, T. Tram, R. Crittenden, K. Koyama, D. Wands, Phys. Rev. D **92,** 123517 (2015)

C. Fidler, T. Tram, C. Rampf, R. Crittenden, K. Koyama, D. Wands, JCAP **09,** 031 (2016)

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!

$$k^2\Phi = 4\pi G_N a^2 \bar{\varepsilon}\delta$$

$$\partial_\eta \delta_c + k v_c = 0$$

$$\left[\partial_\eta + \mathcal{H}\right] v_c = -k(\Phi + \gamma)$$

vanishing for constant H_T, when $\delta p = \Pi = 0$:

$$\gamma \equiv \ddot{H}_T + \mathcal{H}\dot{H}_T - 8\pi G_N a^2 p\Pi$$

C. Fidler, T. Tram, C. Rampf, R. Crittenden, K. Koyama, D. Wands, JCAP **09,** 031 (2016)

The «N-body» gauge

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$$k^2\Phi = 4\pi G_N a^2 \bar{\varepsilon}\delta$$

$$\partial_\eta \delta_c + k v_c = 0$$

$$[\partial_\eta + \mathcal{H}] v_c = -k(\Phi + \gamma)$$

2 $H_L = 0$

C. Fidler, T. Tram, C. Rampf, R. Crittenden, K. Koyama, D. Wands, JCAP **09,** 031 (2016)

«N-motion» gauges

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$(B = v)$

$$k^2 \Phi^N = 4\pi G_N a^2 \bar{\varepsilon}_c^N \delta_c^N$$

$$\partial_\eta \delta_c^N + k v_c^N = 0$$

$$\left[\partial_\eta + \mathcal{H}\right] v_c^N = -k \Phi^N$$

!

$$\partial_\eta \delta_c + k v_c = -3\dot{H}_L$$

$$\left[\partial_\eta + \mathcal{H}\right] v_c = -k\left(\Phi + \gamma^{Nm}\right) = -k\Phi^N$$

$$\delta_c^N \equiv \delta_c^{Nm} + 3H_L^{Nm}$$

C. Fidler, T. Tram, C. Rampf, R. Crittenden, K. Koyama, D. Wands, JCAP **09,** 031 (2016)

«N-motion» gauges

Metric potentials - evolved separately in an Einstein-Boltzmann code*
up to first order to keep track of relativistic corrections

& nonlinear evolution of CDM
is simulated with the help of
Newtonian N-body codes

*such as CLASS
(Cosmic Linear Anisotropy Solving System):
D. Blas, J. Lesgourges, T. Tram, JCAP **07,** 04 (2011)

C. Fidler, T. Tram, C. Rampf, R. Crittenden, K. Koyama, D. Wands, JCAP **09,** 031 (2016)

A sample *dictionary* based on a weak-field expansion

$$ds^2 = a^2(\eta)$$
$$\times \left[-(1+2A)d\eta^2 - 2\partial_\alpha B dx^\alpha d\eta + \left[\delta_{\alpha\beta}(1+2H_L) - 2\left(\partial_\alpha\partial_\beta - \delta_{\alpha\beta}\Delta/3\right)H_T\right]dx^\alpha dx^\beta\right]$$

$$A = -\Phi^N - \left(\partial_\eta + \mathcal{H}\right)k^{-2}\dot{H}_T$$
$$H_L = \Phi^N - \frac{1}{3}H_T - \gamma$$

$$v = v^N$$
$$\delta = \delta^N - 3H_L$$

$$\Phi^N - \Phi = \gamma$$

C. Fidler, T. Tram, C. Rampf, R. Crittenden, K. Koyama, D. Wands, JCAP **12,** 022 (2017)

Six physical degrees of freedom of the perturbed metric in Poisson gauge

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$$ds^2 = a(\eta)^2\left(-d\eta^2 + \delta_{\alpha\beta}dx^\alpha dx^\beta\right) \longrightarrow g_{ik} = \bar{g}_{ik} + \delta g_{ik}$$

In the Poisson gauge, the line element takes on the form

$$ds^2 = a^2(\eta)\left[-(1+2\Phi)d\eta^2 - 2B_\alpha dx^\alpha d\eta + \delta_{\alpha\beta}(1-2\Psi)dx^\alpha dx^\beta + 2h_{\alpha\beta}dx^\alpha dx^\beta\right]$$

Scalar potentials: Gravitational lensing and clustering

Vector potential: Frame dragging

Tensor perturbation: Gravitational waves

Stress-energy perturbations

We consider a system of point-like particles described by the EMT

$$T^{ik} = \sum_n m_{(n)} \frac{\delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)}{\sqrt{-g}} \left(g_{ml} \frac{dx^m_{(n)}}{d\eta} \frac{dx^l_{(n)}}{d\eta} \right)^{-1/2} \frac{dx^i_{(n)}}{d\eta} \frac{dx^k_{(n)}}{d\eta}$$

- $\frac{dx^\alpha_{(n)}}{d\eta} \equiv \tilde{v}^\alpha_{(n)}$

$$T^0_0 = -\frac{1}{a^3} \sum_n m_{(n)} \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)(1+3\Phi)$$

$$T^0_0 = -\frac{1}{a^4} \sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right) \sqrt{q^2_{(n)} + m^2_{(n)} a^2} \left(1 + 3\Phi + \frac{q^2_{(n)}}{q^2_{(n)} + a^2 m^2_{(n)}} \Phi \right)$$

$$T^\alpha_\beta = 0$$

$$T^\alpha_\beta = -\frac{\delta^{\alpha\gamma}}{a^4} \sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right) \frac{q_{(n)_\gamma} q_{(n)_\beta}}{\sqrt{q^2_{(n)} + m^2_{(n)} a^2}} \left(1 + 4\Phi + \frac{m^2_{(n)} a^2}{q^2_{(n)} + m^2_{(n)} a^2} \Phi \right)$$

Stress-energy perturbations

$$T^0_\alpha = \frac{1}{a^4}\sum_n \delta\left(\boldsymbol{r} - \boldsymbol{r}_n\right) q_{(n)_\alpha}$$

$$T^0_\alpha = \frac{1}{a^4}\sum_n \delta\left(\boldsymbol{r} - \boldsymbol{r}_n\right) q_{(n)_\alpha}(1 + 2\Phi + \chi)$$

- $\chi \equiv \Phi - \Psi$

Eingorn, M., Yukselci, A.E. and Zhuk, A., Phys. Lett. B **826,** 136911 (2022)

Adamek, J., Daverio, D., Durrer, R., Kunz, M., JCAP **07,** 053 (2016)

Field equations $\delta G_i^k = \kappa \delta T_i^k$ for determining metric perturbations

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$00-$

$$\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Phi = -4\pi G a^2 \delta T_0^0$$

$$(1+4\Phi)\Delta\Phi - 3\mathcal{H}\Phi' + 3\mathcal{H}^2(\chi - \Phi) + \frac{3}{2}\delta^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta} = -4\pi G a^2 \delta T_0^0$$

$0\alpha-$

$$-\frac{1}{4}B_\alpha - \Phi'_{,\alpha} - \mathcal{H}\Phi_{,\alpha} = -4\pi G a^2 T_\alpha^0$$

$$-\frac{1}{4}B_\alpha - \Phi'_{,\alpha} - \mathcal{H}\left(\Phi_{,\alpha} - \chi_{,\alpha}\right) = -4\pi G a^2 T_\alpha^0$$

- $\Delta \equiv \delta^{\alpha\beta}\partial^2/\partial x^\alpha \partial x^\beta$
- $\Pi_{\alpha\beta} \equiv \left(\delta_{\gamma\alpha}T_\beta^\gamma - \delta_{\alpha\beta}T_\gamma^\gamma/3\right)$

$\alpha\beta-$

$$\left(\delta_\alpha^\gamma \delta_\beta^\sigma - \frac{1}{3}\delta^{\gamma\sigma}\delta_{\alpha\beta}\right)$$

$$\times\left[B'_{(\gamma,\sigma)} + 2\mathcal{H}B_{(\sigma,\gamma)} + \chi_{,\sigma\gamma} - 2\chi\Phi_{,\sigma\gamma} + 2\Phi_{,\sigma}\Phi_{,\gamma} + 4\Phi\Phi_{,\gamma\sigma}\right] = 8\pi G a^2 \Pi_{\alpha\beta}$$

Evolving the particle ensemble

$$q'_{(n)_\alpha} = -\sqrt{q_{(n)}^2 + m_{(n)}^2 a^2}\left[\left(1 + \frac{q_{(n)}^2}{q_{(n)}^2 + a^2 m_{(n)}^2}\right)\Phi_{,\alpha} - \chi_{,\alpha} + \frac{\delta^{\beta\gamma} q_{(n)_\gamma} B_{\beta,\alpha}}{\sqrt{q_{(n)}^2 + m_{(n)}^2 a^2}}\right]$$

$$v_{(n)_\alpha} = \frac{q_{(n)_\alpha}}{\sqrt{q_{(n)}^2 + m_{(n)}^2 a^2}}\left[\left(3 - \frac{q_{(n)}^2}{q_{(n)}^2 + a^2 m_{(n)}^2}\right)\Phi - \chi\right] + B_\alpha$$

Adamek, J., Daverio, D., Durrer, R., Kunz, M., JCAP **07,** 053 (2016)

The cosmic screening approach

$$T_0^0 = -\frac{1}{a^3}\sum_n m_{(n)}\delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right)(1+3\Phi) \longrightarrow \delta T_0^0 \equiv T_0^0 - \bar{T}_0^0 = -\frac{c^2}{a^3}\delta\rho - \frac{3\bar{\rho}c^2}{a^3}\Phi$$

$$T_\alpha^0 = \frac{1}{a^4}\sum_n \delta\left(\boldsymbol{r}-\boldsymbol{r}_n\right) q_{(n)\alpha} \longrightarrow \delta T_\alpha^0 = \frac{c^2}{a^3}\sum_n \rho_n \tilde{v}_n^\alpha - \frac{\bar{\rho}c^2}{a^3}B_\alpha$$

$$T_\beta^\alpha = 0$$

- $\rho_{(n)} \equiv m_{(n)}\delta(\boldsymbol{r}-\boldsymbol{r}_n)$
- $\rho \equiv \bar{\rho} + \delta\rho$

★ $\bar{\varepsilon} = \frac{\bar{\rho}c^2}{a^3}$

Eingorn, M., ApJ **825**, 84 (2016)

Field equations $\delta G_i^k = \kappa \delta T_i^k$ for determining metric perturbations

$00-$

$$\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Phi = -4\pi G a^2 \delta T_0^0$$

$$(1+4\Phi)\Delta\Phi - 3\mathcal{H}\Phi' + 3\mathcal{H}^2(\chi - \Phi) + \frac{3}{2}\delta^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta} = -4\pi G a^2 \delta T_0^0$$

$0\alpha-$

$$-\frac{1}{4}B_\alpha - \Phi'_{,\alpha} - \mathcal{H}\Phi_{,\alpha} = -4\pi G a^2 T_\alpha^0$$

$$-\frac{1}{4}B_\alpha - \Phi'_{,\alpha} - \mathcal{H}\left(\Phi_{,\alpha} - \chi_{,\alpha}\right) = -4\pi G a^2 T_\alpha^0$$

- $\Delta \equiv \delta^{\alpha\beta}\partial^2/\partial x^\alpha \partial x^\beta$
- $\Pi_{\alpha\beta} \equiv \left(\delta_{\gamma\alpha}T_\beta^\gamma - \delta_{\alpha\beta}T_\gamma^\gamma/3\right)$

$\alpha\beta-$

$$\left(\delta_\alpha^\gamma\delta_\beta^\sigma - \frac{1}{3}\delta^{\gamma\sigma}\delta_{\alpha\beta}\right)$$

$$\times\left[B'_{(\gamma,\sigma)} + 2\mathcal{H}B_{(\sigma,\gamma)} + \chi_{,\sigma\gamma} - 2\chi\Phi_{,\sigma\gamma} + 2\Phi_{,\sigma}\Phi_{,\gamma} + 4\Phi\Phi_{,\gamma\sigma}\right] = 8\pi G a^2 \Pi_{\alpha\beta}$$

The cosmic screening approach

$$\Delta\Phi - \frac{3\kappa\bar{\rho}c^2}{2a}\Phi = \frac{\kappa c^2}{2a}\delta\rho - \frac{3\kappa c^2\mathcal{H}}{2a}\Xi$$

$$\Delta\mathbf{B} - \frac{2\kappa\bar{\rho}c^2}{a}\mathbf{B} = -\frac{2\kappa c^2}{a}\left(\sum_n \rho_n \tilde{\boldsymbol{v}}_n - \nabla\Xi\right)$$

$$\Delta\Xi = \nabla\sum_n \rho_n\tilde{\boldsymbol{v}}_n \rightarrow \; \Xi = \frac{1}{4\pi}\sum_n m_n \frac{\tilde{\boldsymbol{v}}_n(\boldsymbol{r}-\boldsymbol{r}_n)}{|\boldsymbol{r}-\boldsymbol{r}_n|^3}$$

The cosmic screening approach

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$$\Phi = \frac{1}{3} - \frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\boldsymbol{r}-\boldsymbol{r}_n|}\exp(-q_n)$$
$$+\frac{3\kappa c^2\mathcal{H}}{8\pi a}\sum_n \frac{m_n[\tilde{\boldsymbol{v}}_n(\boldsymbol{r}-\boldsymbol{r}_n)]}{|\boldsymbol{r}-\boldsymbol{r}_n|}\frac{1-(1+q_n)\exp(-q_n)}{q_n^2}$$

$$\boldsymbol{q}_n(\eta,\boldsymbol{r}) \equiv \sqrt{\frac{3\kappa\bar{\rho}c^2}{2a}}(\boldsymbol{r}-\boldsymbol{r}_n) = \frac{a(\boldsymbol{r}-\boldsymbol{r}_n)}{\lambda}\ , \qquad \lambda \equiv \sqrt{\frac{2a^3}{3\kappa\bar{\rho}c^2}}$$

$$\Delta\Phi - \frac{a^2}{\lambda_{\text{eff}}^2}\Phi = \frac{\kappa c^2}{2a}\delta\rho\ ,\ \Phi = \frac{1}{3}\left(\frac{\lambda_{\text{eff}}}{\lambda}\right)^2 - \frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\boldsymbol{r}-\boldsymbol{r}_n|}\exp\left(-\frac{a|\boldsymbol{r}-\boldsymbol{r}_n|}{\lambda_{\text{eff}}}\right)$$

$$\frac{1}{\lambda_{\text{eff}}^2} = \frac{3}{c^2a^2H}\left(\int\frac{da}{a^3H^3}\right)^{-1}$$

Canay, E., Eingorn, M., PDU **29**, 100565 (2020)

Her-CrB GW

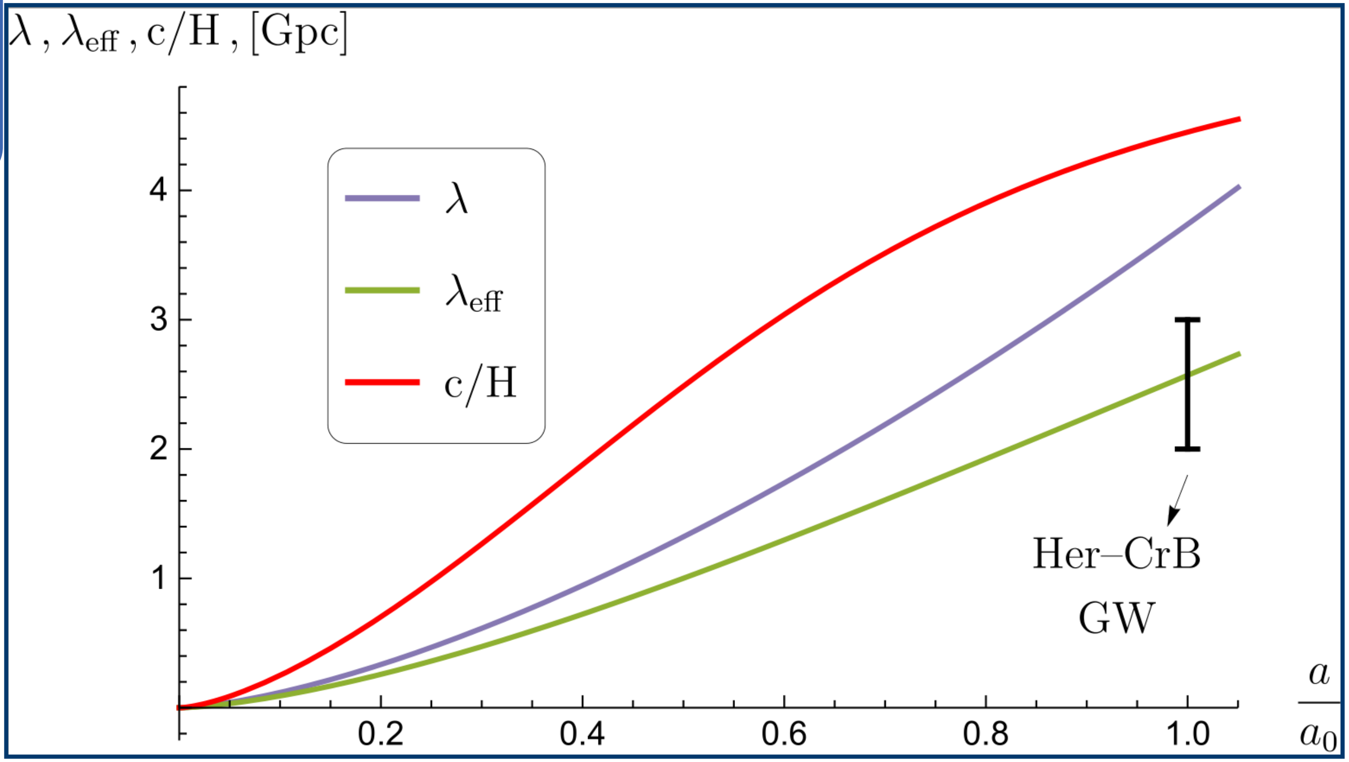
~ 2-3 Gpc

Superclusters & voids

~ 100 Mpc across

The nonlinear scale

-from tens of megaparsecs



Horvath, I., Hakkila, J., Bagoly, Z., A&A **561,** L12 (2014)

Horvath, I., Bagoly, Z., Hakkila, J., Toth L.V., A&A **584,** A48 (2015)

Canay, E., Eingorn, M., PDU **29**, 100565 (2020)

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