Nuclear structure function F_2^A: Moments $M_n(F_2^A)$ and kinematics beyond $x = 1$

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An analysis of the behavior of nuclear structure functions F_2^A at large x and their moments M_n^A at large n has been performed within two theoretical approaches: (i) the QCD-motivated Q^2-rescaling model and (ii) the operator product expansion method within an effective meson-nucleon theory which is prompted by nuclear physics. Our theoretical estimates of the nuclear structure function at $x \geq 1$ are in good agreement with existing data. The moments, derived from experimental data, are found to depend essentially on the behavior of the respective structure functions beyond $x = 1$. A relation between the Q^2-rescaling parameter ξ_A and nuclear averages, i.e., mean kinetic energy and chemical potential of nucleons, as well as a dependence of ξ_A on n are established.

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I. INTRODUCTION

The study of deep inelastic scattering (DIS) of leptons off hadrons is devoted to investigating the internal structure of hadrons. The formerly accumulated experimental data in particle physics persuade us that the original quark-parton picture needs refinements. The experimental results of the European Muon Collaboration (EMC) [1] and BCDMS [2, 3] collaboration point to a more complicate quark-parton structure of nucleons than it has been assumed originally. Experiments with polarized beams and precision data on the $F_2^{\rm D}(x)/F_2^p(x)$ ratio at small Bjorken scale variable x led to the "spin crisis," and gave indications that possibly the SU(3) and isospin symmetries of quarks in nucleons are violated. These observations initiated a number of theoretical attempts to reconcile data with the standard model. It is clear that accurate neutron structure function data are helpful to clarify the situation. And this information is predominantly obtained by means of nuclear targets. This is why the experimental DIS programs of present and planned accelerators widely use nuclear targets. The respective measurements are anticipated impatiently and they will be carried out by several collaborations at SLAC, CERN, and the DESY *ep* collider HERA in reactions with protons, deuterons (D), and ^{3}He, including polarized particles as well.

In addition, it is necessary to mention that nuclear data are traditionally used as a source of the information about quark distributions in the *isoscalar nucleon*. Such type of information is a proper part of the input into the "global fit" of the parton distribution functions and QCD analysis of the deep-inelastic experiments.

The study of nuclear structure functions led to the discovery of the EMC effect; i.e., it was found that the quark distribution inside the nucleus differs from that of a free nucleon. This fact shows another aspect of DIS reactions as a tool for investigating the influence of a nuclear medium on the properties of nucleons. In addition to the experiments mentioned above, it is worth emphasizing the research program of the new Continuous Electron Beam Accelerator Facility (CEBAF). In particular it includes the study of DIS of electrons on nuclei at large values of Bjorken variable x ($x \sim 1$) where nuclear structure effects are expected to show up clearly.

These experimental and theoretical aspects require refinements of the quantitative description of DIS of leptons off nuclei. For a survey on theoretical descriptions we refer the interested reader to Ref. [4]. So far there are two theoretical approaches, which allow for a relative thorough analysis and interpretation of the existing data. The first one is the Q^2-rescaling model [5]. It is based on perturbative QCD and makes use of one, A-dependent, free parameter. With appropriate adjustment of this parameter, a good agreement with experimental data in the region of intermediate values of x can be achieved. The other approach is the recently proposed model [6, 7], which relies on the operator product expansion (OPE) method for the hadronic tensor $W_{\mu\nu}$ and one-boson-exchange (OBE) approximation for nucleon-nucleon interaction (we shall refer to this method as OPE-OBE). This model has been obtained in a quite self-consistent

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way. It includes also mesonic exchange contributions and other nuclear structure corrections and appears to be one of the most rigorous nuclear physics-inspired approaches. Originally, the OPE-OBE method has been developed to describe DIS processes on the deuteron at intermediate values of x. In the present paper, we extend our model [6, 7], that is, we generalize it for heavy nuclei and apply it to the large x region, even at $x > 1$.

Both the Q^2-rescaling model and the OPE-OBE model deal rather with moments as basic quantities than with the corresponding structure functions. In order to compare the two models, in this paper we particularly pay attention to the investigation of the behavior of moments as a function of their order. We derive moments from experimental data and compute their values within the two models. The comparison of the moments in the interval, where they give approximatively the same results, allows us to establish a relation between the QCD-motivated Q^2-rescaling model and our nuclear-physics based model.

Our paper is organized as follows. In Sec. II we present the basic relations and calculate moments of experimental structure functions. In Sec. III a short consideration of QCD analysis of moments is given within the Q^2-rescaling model. In Sec. IV we use our model to calculate the structure functions at large x, and consider the moments, and find a relation to Q^2-rescaling model. Concluding remarks can be found in Sec. V, and the summary is contained in Sec. VI.

II. MOMENTS OF NUCLEAR STRUCTURE FUNCTIONS

The OPE shows that at high transferred momenta, $Q^2 \to \infty$, one can factorize the forward Compton amplitude $T^A_{\mu\nu}(p_A, q)$ (or the corresponding hadronic tensor $W^A_{\mu\nu} \propto \mathrm{Im}T^A_{\mu\nu}$) into two parts $\sum C_n O_n$ describing the short- and large-distance physics [8]. More explicitly,

$$T^A_{\mu\nu}(p_A,q) = \sum_{a,n=2,4,\ldots}^{\infty} C^{(1)}_{a,n}\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\frac{2^n q_{\mu_1}\ldots q_{\mu_n}}{(-q^2)^n}\langle p_A|O^{\mu_1\cdots\mu_n}_a(0)|p_A\rangle$$
$$+\sum_{a,n=2,4,\ldots}^{\infty} C^{(2)}_{a,n}\left(g_{\mu\mu_1}-\frac{q_\mu q_{\mu_1}}{q^2}\right)\left(g_{\nu\mu_2}-\frac{q_\nu q_{\mu_2}}{q^2}\right)\frac{2^n q_{\mu_3}\ldots q_{\mu_n}}{(-q^2)^{n-1}}\langle p_A|O^{\mu_1\cdots\mu_n}_a(0)|p_A\rangle, \tag{2.1}$$

where $q^2 \equiv -Q^2$, $O^{\mu_1\cdots\mu_n}_a(0)$ are twist-two operators; a runs over all types of fields; $C^{(i)}_{a,n}$ denote the Wilson coefficients; μ, ν are Lorentz indices. The index A refers to nuclei with mass number A, and N is reserved to nucleons. We employ here the standard notation with q as transferred momentum of the scattered lepton, $p_{A(N)}$ as momentum of the nucleus (nucleon), and $g_{\mu\nu}$ stands for the metrics.

Via the standard OPE technique the Wilson coefficients and matrix elements are related to the moments of the structure function F_2^A by

$$M_{n-1}(F_2^A) = \frac{1}{A}\left(\frac{M_A}{m}\right)^{n-1}\sum_a C^{(2)}_{a,n}\bar{a}^A_{a,n},$$
$$M_n(F_2^A) = \frac{1}{A}\int_0^{M_A/m} F_2^A(x)x^{n-1}dx \tag{2.2}$$

(cf. Ref. [5][1]), where $x \equiv x_N = -q^2/2p_N q$, and the reduced matrix elements $\bar{a}^A_{a,n}$ are defined as

$$\langle p_A|O^{\mu_1\cdots\mu_n}_a|p_A\rangle = p^{\mu_1}_A\ldots p^{\mu_n}_A\,\bar{a}^A_{a,n}. \tag{2.3}$$

The nuclear moments $M_n^A(F_2^A)$ defined by Eqs. (2.2) and (2.3) are normalized so that $M_n^A(F_2^A) \sim M_n^N(F_2^N)$ with an accuracy of the nuclear corrections. Therefore, we use below the ratio $M_n^A(F_2^A)/M_n^N(F_2^N) \sim 1$ to demonstrate the influence of nuclear medium on the nucleon structure function moments, i.e., the "EMC effect in the space of moments." This ratio has a similar shape as the usual ratio of structure functions, however, it is expected to be more sensitive to model assumptions and, hence, more informative. Indeed, reconstructing structure functions as the inverse Mellin transform of Eq. (2.2), one needs moments defined in the whole interval of n, i.e., there appears here the necessity for a model to give a reliable n dependence of the moments, in particular at large n. Note, that the requirement to give a good description of the EMC ratio of structure functions is not sufficient here, because the intermediate region of x corresponds to the few lowest values of n in the space of moments. The higher-order moments correspond to the region of large x, where the structure functions are too small and both the theoretical and experimental investigations are subtle. The behavior of the moments, as a function of n, depends essentially on properties of the structure function at $x \sim 1$, and vice versa, slightly different model predictions for M_n may lead to drastically different behavior of structure functions at $x \geq 1$.

The experimental values of the moments ought to be extracted from the experimental data on structure functions by making use of Eq. (2.2) and carrying out the integral numerically. Obviously, for this purpose one needs precision measurements of structure functions in large intervals of x, Q^2, and A. At present the set of experimental data at large enough x is rather poor, and only the BCDMS data on carbon [9] can be seriously

[1]Note the change of definition of the moments in comparison with Refs. [6, 7].

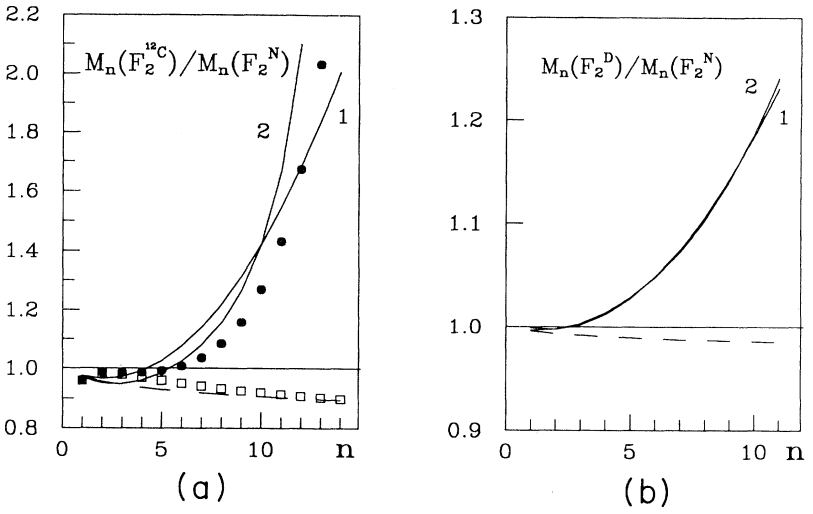

FIG. 1. The ratio $M_n^A(F_2^A)/M_n^N(F_2^N)$ of the moments of nuclear and nucleon structure functions. The dashed curve represents the result of the Q^2-rescaling model. (a) A=12, i.e., the carbon nucleus. Solid curves represent results of our consideration [1 – calculation by Eq. (4.19); 2 – using Eq. (4.10)]. Experimental data are obtained by integrating fits of data (error bars are not depicted; solid circles - set B; empty squares - set A). The nucleon structure function moments are calculated with our parametrization of the structure function of the isoscalar free nucleon (A10)–(A13). (b) A=2, i.e., the deuteron. Solid curves represent the results of our consideration [1 – calculation by Eq. (4.20); 2 – using Eqs. (4.5) and (4.6)].

taken into account when deriving moments from the experimental data. Moreover, even in this case the resulting integrals contain ambiguities related to the form of chosen parametrization of $F_2^A(x,Q^2)$ at large x. The conventional parametrization of the structure function at $x \to 1$, being inspired by the quark-parton picture, has the form $\sim (1-x)^\gamma$. Perhaps this is a reliable form for nucleon structure functions, but it needs modifications in the case of nuclei. Indeed, the actual variable x of the nuclear structure function $F_2^A(x,Q^2)$ is $x_A = m/M_A\, x_N$, with $0 < x_A < 1$ (m stands for nucleonic mass). Consequently, the nuclear structure function is defined at $0 < x_N < M_A/m$. Because of the Fermi motion, one expects a tail of the structure function $F_2^A(x,Q^2)$ beyond single nucleon kinematics, i.e., $F_2^A(x \geq 1) \geq 0$. Since the large-x region is enhanced by a factor x^{n-1} it may strongly influence the corresponding moments. In accordance with these facts, we use two methods for computations of the moments. In the first one we include into the parametrization the early nuclear data ($x \leq 0.75$) and assume that the structure function vanishes as $x \to 1$ (see Appendix A, set A). In this case the behavior of the resulting moments is almost linear [see Fig. 1(a), empty squares]. It seems that often such a treatment of experimental data has been implicitly used when determining the parameters of models.

The second method uses also the recent BCDMS data on carbon at $x > 1$ [9] (Appendix A, set B). This leads to a dramatic change of the moments derived from the data because their behavior, as a function of n, becomes polynomial [see solid circles in Fig. 1(a)]. This important fact should be kept in mind when analyzing the moments. As seen from Fig. 1(a), only the first few moments are nearly independent of the behavior of $F_2^A(x,Q^2)$ at large x.

For the theoretical analysis it is necessary to rely on a consistent calculation of both the coefficient functions $C_{a,n}^{(i)}$ and the matrix elements of the twist-two operators $O_a^{\mu_1\cdots\mu_n}$ sandwiched between nuclear ground state vectors $|p_A\rangle$. So far there does not exist a rigorous theoretical method for computing simultaneously both these pieces. Actually only one part of expansion (2.1) can be calculated in a more or less self-consistent way, the other remains to be fixed from experiment. Concerning the two models considered in the present paper, this situation is as follows.

(i) Within the QCD-based Q^2-rescaling model [5], the coefficient functions are perturbatively calculable, while the nuclear matrix elements have nonperturbative origin in QCD and consequently are connected with some phenomenology.

(ii) The OPE-OBE method [6, 7] operates with the OPE within effective meson-nucleon models in the one-boson exchange approximation. In contrast with (i), the matrix elements of the operators $O_a^{\mu_1\cdots\mu_n}$ are calculable within this method, whereas the coefficient functions $C_{a,n}^{(i)}$ remain to be determined from experiment.

In spite of the apparent difference between the two approaches, both of them give a good description of the nuclear data in the intermediate region of the Bjorken variable x, $0.2 \leq x \leq 0.7$ (EMC effect region, see Fig. 2). In this region the results of calculation do not de-

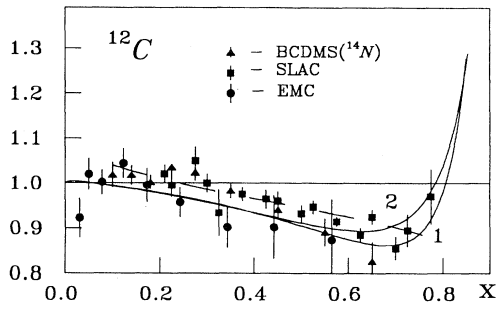

FIG. 2. The ratio of the carbon and isoscalar nucleon structure functions. Solid curves are results of our model (1 – the OPE analysis within the meson-nucleon model [6, 7]; 2 – the "old x-rescaling formulas" with the CDFM nuclear spectral function). The dashed line uses the Q^2-rescaling model [5]. Experimental data are taken from Ref. [11].

pend so sensitively on model parameters and, possibly, they can be interrelated in this interval. The distinction should appear at the boundary of the validity of the models, i.e., at low Q^2 and large or small x. In this paper we shall not consider the region of very small x, where it has been experimentally found that the ratio of nuclear and isoscalar nucleon structure functions is less than unity for heavy nuclei [10]. Here an interference of nuclear shadowing effects and meson exchange contribution is expected [7], and the physics of this effect requires special investigations.

III. QCD ANALYSIS OF NUCLEAR MOMENTS

We proceed now with a QCD analysis of the behavior of the nuclear structure function moments. To begin with, recall that in QCD both the pieces of OPE in expansion (2.1) depend on a regularization parameter μ_A. The coefficient functions $C_{a,n}^{(i)}$ are calculable within perturbative QCD. The hadronic matrix elements $\langle p_A|O_a^{\mu_1\cdots}|p_A\rangle$ are of nonperturbative character due to their long-range behavior and, therefore, not yet calculable in QCD. Their dependence on the QCD-regularization parameter, however, is constrained by the condition that, together with the coefficient functions, the physical observables (e.g., the moments M_n^A) are independent of the choice of this (free) parameter. Applying the renormalization group equations to the coefficient functions, the moments can be written in the form

$$M_n^A(Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu_A^2)}\right)^{d_{n+1}} M_n^A(\mu_A^2), \tag{3.1}$$

where $\alpha_s(Q^2)$ denotes the QCD coupling constant, and d_n stands for the familiar anomalous dimension in the leading order. Within the Q^2-rescaling approach a uniform (i.e., n-independent) change of scale μ_A from one nucleus to another is assumed [5]. Therefore, the nuclear moments are related with the nucleon ones through the relation

$$M_n^A(Q^2) = M_n^N(\xi_A Q^2), \tag{3.2}$$

where ξ_A is the A-dependent rescaling parameter. For example, known values used for carbon and deuteron targets are $\xi_{^{12}\mathrm{C}} = 1.6$, $\xi_{\mathrm{D}} = 1.07$, respectively [5]. Equation (3.2) can be rewritten in leading order of α_s in the form

$$M_n^A(Q^2)/M_n^N(Q^2) = \left(\frac{\alpha_s\left(\xi_A Q^2\right)}{\alpha_s\left(Q^2\right)}\right)^{d_{n+1}} \tag{3.3}$$

Relation (3.3) describes the EMC effect in the space of moments within the Q^2-rescaling model. For concrete calculations it is necessary to compute the anomalous dimension d_n, connected with the regularization scheme. Following the authors of the Q^2-rescaling model, we work in the leading order in α_s, and in this case both the α_s and anomalous dimension are scheme independent. Thus, d_n and α_s are of the form

$$d_n = \frac{16}{33 - 2N_f}[S(n) - 3/4 - 1/2n(n+1)], \tag{3.4}$$

$$S(n) = \sum_{j=1}^{n} \frac{1}{j};$$

$$\alpha_s = 4\pi/\beta_0 \ln(Q^2/\Lambda^2), \tag{3.5}$$

with the QCD parameter $\Lambda = 200$ MeV/c.

We compute the ratio $M_n^A(Q^2)/M_n^N(Q^2)$ for two targets: carbon and deuteron. The results are depicted in Figs. 1(a) and 1(b) (dashed lines). It is seen that the Q^2-rescaling model gives a linear dependence of the ratio $M_n^A(Q^2)/M_n^N(Q^2)$ on n which is in good agreement with experimental data obtained when assuming $F_2^A = 0$ at $x > 1$, but does not describe ($n \geq 5$) the data if the nuclear structure function leaks out beyond $x = 1$. This is a quite understandable effect because in the Q^2-rescaling model the Fermi motion is neglected and the parameter ξ_A is n independent. Comparing the behavior of the ratio $M_n^A(Q^2)/M_n^N(Q^2)$ for carbon and deuteron (see Fig. 1) and bearing in mind that nuclear structure function does not vanish at $x > 1$ it is clear that ξ_A must depend on n. Moreover, it must decrease as n increases and even become less than unity for large n. Note, that originally in the Q^2-rescaling model the independence of ξ_A of n was considered as an approximation [5] and estimates, based on the quark bag model and early nuclear data ($x < 0.7$), show a validity of this assumption up to $n \sim 8 - 10$. The new BCDMS data allow the conclusion that the Q^2-rescaling model describes the nuclear moments only up to $n \sim 5$ for heavy nuclei.

IV. ANALYSIS OF MOMENTS WITHIN A NUCLEAR PHYSICS MOTIVATED APPROACH

Approaches motivated by nuclear physics and accounting for the Fermi motion of nucleons should describe the moments of nuclear structure function, including large n. Below we apply the OPE-OBE method [6, 7] to the high-order moments of the nuclear structure functions. In spite of the quite rigorous field-theoretical formalism and utilization of the OPE, it is certainly a "nuclear" method, since it reproduces traditional nuclear physics notions, namely, the Schrödinger equation for bound nucleons and the NN potential in the OBE.

Unfortunately, only in case of the deuteron the explicit expressions for the structure function and its moments can be obtained in a completely consistent way. For the heavier nuclei, some approximations are needed.

A. Deuteron moments and structure function F_2^D

In accordance of the theoretical OPE-OBE method the explicit expression of moments of the deuteron structure function is of the form

$$M_n(F_2^{\rm D}) = M_n(F_2^N)\int \frac{d^3\mathbf{p}}{(2\pi)^3}|\Psi_{\rm D}(\mathbf{p})|^2\left(1+\frac{p_z}{m}\right)\left(1+\frac{p_z}{m}+\frac{\mathbf{p}^2}{2m^2}\right)^n \tag{4.1}$$

$$+M_n(F_2^N)\int \frac{d^3\mathbf{p}\,d^3\mathbf{k}}{(2\pi)^6}\Psi_{\rm D}^\dagger(\mathbf{p})V(\mathbf{k})\Psi_{\rm D}(\mathbf{p}+\mathbf{k})\frac{1}{k_z}\left[\left(1+\frac{k_z}{2m}\right)^n-\left(1-\frac{k_z}{2m}\right)^n\right] \tag{4.2}$$

$$+M_n(F_2^M)\int \frac{d^3\mathbf{p}\,d^3\mathbf{k}}{(2\pi)^6}\Psi_{\rm D}^\dagger(\mathbf{p})V(\mathbf{k})\Psi_{\rm D}(\mathbf{p}+\mathbf{k})\frac{m}{\omega^2(\mathbf{k})}\frac{(-1)^n-1}{2}\left(\frac{k_z}{m}\right)^{n+1}, \tag{4.3}$$

where $\Psi_{\rm D}$ is the conventional deuteron wave function; $\omega^2(\mathbf{k}) = (\mathbf{k}^2+\mu^2)$; μ stands for mesonic mass. $V(\mathbf{k})$ denotes the one-boson-exchange potential generated by the interaction term in the Hamiltonian of the theory [6]. $M_n(F_2^M)$ denote moments of mesons, which are relevant to determine the OBE potential [12]. Being obtained in a self-consistent theoretical way, Eqs. (4.1)–(4.3) include also the meson exchange corrections that assure all the sum rules to be satisfied exactly in this approach [6,7]. It has been found [6,13] that the mesonic corrections contribute only at very small n and also rapidly decrease with increasing n, so they can be neglected in our consideration of heavy nuclei at large n. Nevertheless the meson exchange contributions are necessary for satisfying of the energy-momentum sum rule at $n=1$; their contribution in the deuteron structure function is about 0.5%. Please note that the contributions (4.1)–(4.3) are obtained in a usual nuclear physics approximation: they are accurate up to g^2 in the nucleon-meson coupling constant g, so that all our calculations should be performed with this accuracy.

The sum of the terms (4.1) and (4.2) is the contribution of the Fermi motion of interacting nucleons. Applying the inverse Mellin transform to expressions (4.1) and (4.2) we reconstruct the nucleon contribution to the deuteron structure functions in the convolution form

$$F_2^{N/\rm D}(x) = \int_x^{M_{\rm D}/m} f^{N/\rm D}(y)\,F_2^N(x/y)dy, \tag{4.4}$$

with $f^{N/\rm D} = f_{IA}^{N/\rm D} + f_{\rm int}^{N/\rm D}$ given by

$$f_{IA}^{N/\rm D}(y) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}|\Psi_{\rm D}(\mathbf{p})|^2\left(1+\frac{p_z}{m}\right)\delta\left(y-\left[1+\frac{\mathbf{p}^2}{2m^2}+\frac{p_z}{m}\right]\right), \tag{4.5}$$

$$f_{\rm int}^{N/\rm D}(y) = \int \frac{d^3\mathbf{p}\,d^3\mathbf{k}}{(2\pi)^6}\Psi_{\rm D}^\dagger(\mathbf{p})V(\mathbf{k})\Psi_{\rm D}(\mathbf{p}+\mathbf{k})\frac{1}{k_z}\left\{\delta\left(y-\left|1+\frac{k_z}{2m}\right|\right)-\delta\left(y-\left|1-\frac{k_z}{2m}\right|\right)\right\}. \tag{4.6}$$

The distribution function $f_{IA}^{N/\rm D}$ describes the Fermi motion of the on-mass-shell nucleons and is quite similar to the conventional formula of nuclear physics usually referred to as the "impulse approximation." When taking into account only this type of "Fermi smearing" one gets wrong nuclear structure functions. In particular, it breaks the sum rule for the four-momentum ($\langle y\rangle > 1$) and does not give the known EMC-like A dependence.

Instead of the modification of the impulse approximation by a redefinition of the variable y as used, e.g., in Ref. [14], we get here the pure interaction term (4.6) of exchange origin. The sum of $f_{IA}^{N/\rm D}$ and $f_{\rm int}^{N/\rm D}$ gives the final result for the nucleon contribution to the deuteron. Formulas (4.5) and (4.6) may be written in a more compact form. Expanding the δ-function in (4.5) around "on-mass shell" y ($y = 1+\mathbf{p}^2/2m^2+p_z/m$), and keeping formally only the g^2 terms, and substituting in Eq. (4.6) the difference of two δ-functions by its first derivative we get

$$f^{N/\rm D}(y) = \int \frac{d\mathbf{p}}{(2\pi)^3}\,|\Psi_{\rm D}(\mathbf{p})|^2\left(1+\frac{p_z}{m}\right) \times\delta\left(y-\left[1+\frac{\varepsilon_{\rm D}}{m}-\frac{\mathbf{p}^2}{2m^2}+\frac{p_z}{m}\right]\right). \tag{4.7}$$

The quantity $\varepsilon(\mathbf{p}) = \varepsilon_{\rm D} - \mathbf{p}^2/2m$ in Eq. (4.7) is exactly the "separation energy" for the deuteron. In this context one observes that the obtained formula (4.7) is very similar to that suggested in the previous x-rescaling model [14].

B. Generalization to heavy nuclei

Defining the "deuteron spectral function" as

$$S_{\rm D}(\mathbf{p},\varepsilon) \equiv |\Psi_{\rm D}(\mathbf{p})|^2\,\delta(\varepsilon-\varepsilon(\mathbf{p})), \tag{4.8}$$

Eq. (4.7) may be rewritten in the form

$$f^{N/\rm D}(y) = \int \frac{d\mathbf{p}}{(2\pi)^3}\,d\varepsilon\,S_{\rm D}(\mathbf{p},\varepsilon)\left(1+\frac{p_z}{m}\right) \times\delta\left(y-\left[1+\frac{\varepsilon}{m}+\frac{p_z}{m}\right]\right), \tag{4.9}$$

and can be generalized to the case of any nuclear mass number $A>2$ by

$$f^{N/A}(y) = \int \frac{d\mathbf{p}}{(2\pi)^3}\,d\varepsilon\,S_A(\mathbf{p},\varepsilon)\left(1+\frac{p_z}{m}\right) \times\delta\left(y-\left[1+\frac{\varepsilon}{m}+\frac{p_z}{m}\right]\right), \quad A>2, \tag{4.10}$$

where $S_A(\mathbf{p},\varepsilon)$ is the nuclear spectral function. Observe that the present approach coincides with the x-rescaling model only concerning the nucleon contribution part (Fermi motion of bound nucleons in the impulse approximation). Hence, the nuclear structure function is defined as

$$F_2^{N/A}(x) = \int_x^{M_A/m} dy\, F_2^N(x/y) \int \frac{d\mathbf{p}}{(2\pi)^3} d\varepsilon\, S_A(\mathbf{p},\varepsilon)\Big(1+\frac{p_z}{m}\Big)\delta\Big(y-\Big[1+\frac{\varepsilon}{m}+\frac{p_z}{m}\Big]\Big). \tag{4.11}$$

For numerical calculations one needs a realistic parametrization of the isoscalar nucleon structure function $F_2^N(x,Q^2)$. Usually for this aim one uses the deuteron structure function, assuming that it can be represented with high accuracy as a sum of two quasifree nucleons. In our approach the deuteron is considered as a nucleus with internal nuclear structure (boundness, Fermi motion, etc.), so that the isoscalar structure function ought to be given as a sum of free proton and neutron ones. We construct the neutron structure function by fitting the combined BCDMS data on proton and deuteron. The fit needs to be consistent with the present approach; therefore, the nuclear structure effects are taken into account in a minimization procedure (see Appendix B and Ref. [15]).

For the analysis of the deuteron moments we use directly formulas (4.1)–(4.3), whereas for the nuclear moments it is more convenient to carry out integral (2.2) and make use of Eq. (4.11). As has been shown above (see Fig. 1), the nuclear moments are sensitive to the behavior of the corresponding structure function at large x. Note that in this region other nuclear effects (e.g., effects of nucleon-nucleon correlations in the spectral function) and other degrees of freedom (Δ isobars [16,17], possible multiquark states, etc.) may become relevant. In our consideration we include only nucleonic and mesonic degrees of freedom. In this context an investigation of the behavior of the structure function defined by Eq. (4.11) in the neighborhood of $x = 1$ is worthwhile in order to show that it gives a satisfactory description of the data.

C. Nuclear structure functions $F_2^A(x)$ beyond $x = 1$

Let us estimate the behavior of the nuclear structure function (4.11) at $x \geq 1$ namely, its absolute value and its slope (i.e., the logarithmic derivative) at $x = 1$. For this aim we use in Eq. (4.11) a simple, but reasonable, approximation of the isoscalar nucleon structure function at large x:

$$F_2^N(x) \sim B\,(1-x)^3, \tag{4.12}$$

and find $B \approx 0.58$ at Q^2=60 GeV$^2/c^2$ from an analysis of the BCDMS data on the deuteron and proton.

At $x \to 1$ we have

$$F_2^{N/A}(1) \approx B \int_1^{M_A/m} dy \left(1-\frac{1}{y}\right)^3 \int \frac{d\mathbf{p}}{(2\pi)^3} d\varepsilon\, S_A(\mathbf{p},\varepsilon)\Big(1+\frac{p_z}{m}\Big)\delta\Big(y-\Big[1+\frac{\varepsilon}{m}+\frac{p_z}{m}\Big]\Big), \tag{4.13}$$

$$\frac{dF_2^{N/A}(1)}{dx} \approx -3B \int_1^{M_A/m} \frac{dy}{y}\left(1-\frac{1}{y}\right)^2 \int \frac{d\mathbf{p}}{(2\pi)^3} d\varepsilon\, S_A(\mathbf{p},\varepsilon)\Big(1+\frac{p_z}{m}\Big)\delta\Big(y-\Big[1+\frac{\varepsilon}{m}+\frac{p_z}{m}\Big]\Big). \tag{4.14}$$

Integrating over y and keeping the leading term in p/m ($p^2/m^2 \sim \varepsilon/m$) we can obtain the simplest estimate of the nuclear structure function in our model near the boundary of single-particle kinematics

$$F_2^{N/A}(1) \sim \frac{B}{8}\frac{\langle p^3\rangle}{m^3}, \tag{4.15}$$

$$\frac{dF_2^{N/A}(1)}{dx} \sim -\frac{3B}{8}\frac{\langle p^2\rangle}{m^2}, \tag{4.16}$$

where angular brackets means averaging with nuclear momentum distribution. In deriving these expressions we have assumed that the spectral function is of spherical symmetry. For the numerical estimate we employ $\langle p^3\rangle \approx \bar{p}^3$ and $\langle p^2\rangle \approx \bar{p}^2$ with some mean value $\bar{p}$, $\bar{p}/m \approx 0.2$, and therefore

$$F_2^{N/A}(1) \sim \frac{B}{8}\frac{\bar{p}^3}{m^3} \approx 6\times 10^{-4}. \tag{4.17}$$

For the slope s we have

$$s \equiv -\frac{1}{F_2^{N/A}(1)}\frac{dF_2^{N/A}(1)}{dx} \sim 3\left(\frac{\bar{p}}{m}\right)^{-1} \approx 15. \tag{4.18}$$

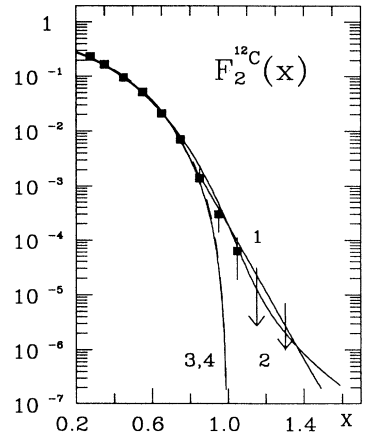

FIG. 3. The absolute value of the carbon structure function. Curves 1(3) – our fit of the carbon structure function [set B (A)], curve 2 – the present approach with Fermi motion and boundness effects taken into account in the CDFM spectral function, curve 4 – our fit of the isoscalar free nucleon . Experimental data are taken from Ref. [9].

Our estimates are in good agreement with the experimental data of the BCDMS collaboration [9] (see Fig. 3). A more accurate calculation of nuclear structure function conforming with Eq. (4.11) requires a nuclear model for the spectral function $S_A(\mathbf{p},\varepsilon)$. In standard shell model it is nothing else but the single-particle momentum distribution. In our case we have to use a more complicated nuclear model for calculation of $S_A(\mathbf{p},\varepsilon)$, since for explanations of the DIS nuclear data it is necessary to take into account more complicated nuclear collective effects and excitations, as is shown in Ref. [14]. For this purpose we use the spectral function in the framework of coherent density fluctuation model (CDFM) [18, 19] which describes the main characteristics of nuclei and includes nucleon-nucleon correlations and gives a good description of the data in the region $0.1 < x < 0.7$ (see Fig. 2). Results of numerical calculations are displayed in Fig. 3. It can be seen that the theoretical calculation which uses Eq. (4.11) practically coincides with the phenomenological fit (set B) and, hence it is in agreement with available experimental data. This is a nontrivial result since our theoretical approach has been inspired by early DIS data (i.e., the EMC effect) and it is not obvious that it must work at large x too. Further, it seems that in this region the manifestation of other degrees of freedom are not yet visible in DIS processes on nuclei.

D. Behavior of the moments $M_n(F_2^A)$ at large n

In accordance with our adopted model accuracy, for estimating the nuclear moments in Eqs. (2.2) and (4.11) we keep only $\langle \mathbf{p}^2\rangle_A/m^2 \sim \langle V\rangle_A/m \sim g^2$ terms, and thus we get

$$M_n(F_2^A)/M_n(F_2^N) \simeq 1+\frac{1}{3}n^2\frac{\langle T\rangle_A}{m}+\frac{4}{3}n\frac{\langle T\rangle_A}{m}+n\frac{\langle V\rangle_A}{m}, \tag{4.19}$$

where $\langle T\rangle_A = \langle \mathbf{p}^2\rangle_A/2m$ is the mean kinetic energy of nucleon in nucleus A; $\langle V\rangle_A = 2(\varepsilon_A - \langle T\rangle_A)$ denotes mean potential energy of nucleon; $\varepsilon_A \approx -8$ MeV is the average binding energy (i.e., the chemical potential) of nucleus per nucleon [do not confuse this with the separation energy ε in (4.11)]. Analogously for the deuteron we get

$$M_n(F_2^D)/M_n(F_2^N) \simeq 1+\frac{1}{6}n^2\frac{\langle T\rangle_{\rm D}}{m}+\frac{2}{3}n\frac{\langle T\rangle_{\rm D}}{m}+n\frac{\langle V\rangle_{\rm D}}{m}, \tag{4.20}$$

where $\langle T\rangle_{\rm D} = \langle \mathbf{p}^2\rangle_{\rm D}/m$ is mean kinetic energy of nucleons in deuteron; $\langle V\rangle_{\rm D} = \varepsilon_{\rm D} - \langle T\rangle_{\rm D}$ is mean potential energy of nucleon; $\varepsilon_{\rm D} \approx -2.22$ MeV is the deuteron binding energy. In spite of the simple approximative character of Eqs. (4.19) and (4.20) they show explicitly the dependence of nuclear moments on n. It is also seen that, within the present approach moments and their n dependence are essentially determined by factors of a pure nuclear structure origin, i.e., nuclear potential, kinetic energy, binding effect, etc. The results of calculations based on formulas (4.19) and (4.20) are presented in Figs. 1(a) and 1(b) (solid curves labeled by 1). There are also presented results of exact calculations relying on formulas (4.4), (4.5), (4.6), and (4.10) (solid curves labeled by 2). Both calculations are in agreement with experimental data.

E. Q^2-rescaling parameter ξ_A from the nuclear structure point of view

When comparing the results within the two models (i.e., the Q^2-rescaling model and the OPE-OBE method) it is seen that they give almost the same behavior for the first few moments. Namely, this interval of low n plays the predominant role in the determination of the structure function in the intermediate region of x. However, for the general n dependence the two models give different results: our model predicts a more sharp n dependence of moments. It seems, such a dependence is more preferable since it describes the experimental data in the large region of x, even at $x > 1$, whereas the Q^2 rescaling has been designated originally to describe only the data in the intermediate region of x. Nevertheless the performed analysis persuades us that there exists a tight relation between the Q^2 rescaling and our model. Since they give close results for the first moments the parameters of the models should be interrelated. Therefore it is interesting to analyze the parameter ξ_A in terms of the nuclear structure. For this purpose we formally equate the moments calculated in two approaches (3.3) and (4.19) and (4.20):

$$\left(\frac{\alpha_s(\xi_A Q^2)}{\alpha_s(Q^2)}\right)^{d_{n+1}} \simeq 1+\Delta_n,$$

with

$$\Delta_n = \frac{1}{3}n^2\frac{\langle T\rangle_A}{m}+\frac{4}{3}n\frac{\langle T\rangle_A}{m}+n\frac{\langle V\rangle_A}{m}, \quad \text{for} \quad {}^{12}\text{C}, \tag{4.21}$$

$$\Delta_n = \frac{1}{6}n^2\frac{\langle T\rangle_{\rm D}}{m}+\frac{2}{3}n\frac{\langle T\rangle_{\rm D}}{m}+n\frac{\langle V\rangle_{\rm D}}{m}, \quad \text{for} \quad \text{D}.$$

Further we use Eq. (3.5), Δ_n we find

$$\ln(\xi_A) \approx -\ln\left(\frac{Q^2}{\Lambda^2}\right)\Delta_n/d_{n+1}, \tag{4.22}$$

$$\xi_A \approx 1-\ln\left(\frac{Q^2}{\Lambda^2}\right)\Delta_n/d_{n+1}.$$

In deriving Eqs. (4.21) and (4.22) we assume the small-

TABLE I. The n dependence of parameter ξ and ratio of moments for ^{12}C and D.

n	$\xi_{^{12}\rm C}$	M_n^A/M_n^N	$\xi_{\rm D}$	$M_n^{\rm D}/M_n^N$
1	1.45	0.982	1.09	0.996
2	1.41	0.975	0.85	1.013
3	1.27	0.977	0.65	1.043
4	1.07	0.992	0.47	1.094
5	0.81	1.027	0.28	1.192

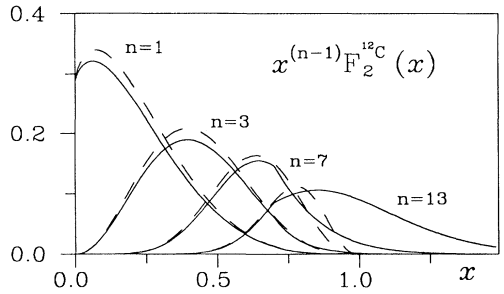

FIG. 4. Illustration of the contribution of the primitive function $x^{n-1}F_2^{^{12}C}$ into integral (2.2) for different values of n [dashed (solid) curves – parametrization of the nuclear structure function by set A (B)]. For convenience, the curves $n = 3, 7$ and 13 are multiplied by factors 10, 100 and 500, respectively.

ness of Δ_n and that the deviation of ξ_A from unity is not too large. This constrains n to be not larger than 4–5 [see Eqs. (4.21) and (4.22)] and $A \leq 20$ [5]. Now we are in a position to calculate the parameter ξ_A for different nuclei and its formal n dependence as well. Taking the CDFM for calculations of nuclear averages and the Bonn deuteron wave function [12] for the deuteron ones, one gets values of the rescaling parameter ξ_A at Q^2=60 GeV$^2/c^2$ which are listed in Table I, together with the ratio (3.3) for each n.

It can be seen that from the nuclear structure point of view, the rescaling parameter ξ_A is monotonically decreasing with n and even becomes less than unity. Notice that such a dependence gives a good description of the experimental behavior of the EMC-like ratio (3.3). It is clear that the values $\xi_A < 1$ simulate the Fermi motion in impulse approximation and, perhaps could not be interpreted in terms of changing scales within QCD-motivated approaches.

Obviously, this estimate is rather rough, and it is valid only for the first moments, but a remarkable moment is worth emphasizing. The absolute values of the parameter ξ_A have been calculated within our OPE-OBE approach, and for small n they are in a reasonable agreement with those obtained in Ref. [5] in the framework of the Q^2-rescaling model. As is seen from Table I (first column), Figs. 1(a) and 4 the n dependence of ξ_A is relevant to determine the behavior of the moments of heavy nuclei. Since only small values of n contribute to the inverse Mellin transform, the parameter ξ_A may be considered as n independent in the space of structure functions. In this case one recovers the assumptions of the Q^2-rescaling model ($x < 0.75$ and $n < 5$).

V. CONCLUDING REMARKS

The performed analysis shows that the space of moments of the DIS structure function is more reliable certifying the validity of theoretical models. However, here one must be careful in deriving moments via the Mellin transform. Even a small change of the behavior of structure function at $x = 1$ leads to a completely different Mellin transform [cf. Fig. 1(a)]. In our opinion the monotonical decrease of moments as a function of n does not correspond to the reality. To get a reliable n dependence of moments one has to take into account all the available DIS data also including the region $x > 1$. With this in mind we can affirm that our approach describes rather good all the peculiarities of available nuclear data on structure functions and nuclear moments as well. This fact lets us argue that other degrees of freedom are not necessarily to be included into the considerations in the neighborhood of $x = 1$.

The Q^2-rescaling model describes data only in restricted intervals of x and n. For enlarging these intervals one needs a modification of the model. The straightforward way is to declare ξ_A as n dependent. In this case the parameters of two models are to be in a tight relation which should allow a calculation of QCD-motivated parameters through the well-known parameters (static characteristics) of nonrelativistic nuclear physics. We have estimated such a relation for the first few values of n at which both approaches give almost the same result. In spite of simplicity of the estimate, it seems to catch the basic feature of the effect: the parameter ξ_A being a function of n has to decrease as n increases and has to be less than unity in the region where the EMC ratio is above unity. As $n \to 1$ the absolute values of ξ_A correspond to those computed in Ref. [5].

VI. SUMMARY

In summary, we use a recently formulated approach for a self-consistent calculation of nuclear modifications of parton structure functions. We analyze the large x region and find good agreement with existing experimental data. By analyzing the moments we can establish a relation to the phenomenological parameter of the Q^2-rescaling model.

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APPENDIX: PARAMETRIZATION OF STRUCTURE FUNCTIONS

Below we present the explicit parametrizations, which are used as input into our calculations in Eqs. (2.2), (4.4), and (4.11) in deriving moments from experimental data, and computing the nuclear structure functions. Note that, in general, the procedure of fitting of data is a rather complicated problem and several uncertainties here are inevitable. For example, in addition to the necessity to conciliate data obtained by different experi-

mental groups, the poverty of experiment at the boundary values of variables x and Q^2, i.e., $x \to 0$, $x \to 1$ and low Q^2, compels one to make assumptions about the behavior of structure functions in these regions. In addition, the parametrization should be in accordance with QCD predictions. Therefore, a suitable Q^2 dependence which, for instance, follows the Q^2- evolution equations of Altarelli-Parisi, is preferable. And, at last, extracting parameters for the neutron structure function from nuclear experiments, the nuclear structure effects should be included into the consideration as well. More details about parametrization of DIS experimental data can be found in Refs. [20].

1. Fit of the carbon structure function $F_2^{^{12}\mathrm{C}}(x)$

In order to calculate the moments of the nuclear (carbon) structure function $F_2{}^{^{12}\mathrm{C}}$ from experimental data we use two kinds of parametrization of the BCDMS data on carbon. The first of them is the parametrization limited at $x = 1$ (set A) by the factor $(1-x)^\gamma$. This corresponds to the popular method of parametrization used for a global fit of world data [20] and to the parametrization used in QCD analysis [21] when determining corrections to the Q^2 dependence of experimental data. In this case only data [2] at $x \le 0.75$ are included into analysis. The second parametrization (set B) has a $\exp(-sx)$ tail at $x > 0.7$, and the data set also includes the BCDMS data at $x > 1$ [9].

Set A: $(x \le 1)$

$$F_2{}^{^{12}\mathrm{C}}(x,Q^2) = f_v(x,Q^2) + f_s(x,Q^2), \tag{A1}$$

$$f_v = A_v x^\alpha (1-x)^{\gamma_v}\,(1 + B_v(1-x)^\beta), \tag{A2}$$

$$f_s = A_s(1-x)^{\gamma_s}, \tag{A3}$$

where all parameters are Q^2 dependent. After χ^2 minimization of the parameters we get, at $Q^2 = 60$ GeV$^2/c^2$,

$$A_v = 0.514, \quad \alpha = 0.899, \quad \gamma_v = 3.225, \tag{A4}$$
$$B_v = 2.668, \quad \beta = 1.467;$$
$$A_s = 0.293, \quad \gamma_s = 4.352. \tag{A5}$$

Set B: $(x \le M_A/m)$

$$F_2{}^{^{12}\mathrm{C}}(x,Q^2) = \begin{cases} f_v(x,Q^2) + f_s(x,Q^2), & x \le 0.7 \\ \left(f_v(0.7,Q^2) + f_s(0.7,Q^2)\right) e^{-s(x-0.7)}, & x > 0.7 \end{cases} \tag{A6}$$

where f_v and f_s have the functional form (A2) and (A3), respectively; all parameters are also Q^2 dependent. At Q^2=60 GeV$^2/c^2$ we get:

$$A_v = 0.394, \quad \alpha = 0.900, \quad \gamma_v = 3.011, \tag{A7}$$
$$B_v = 3.910, \quad \beta = 1.666,$$
$$A_s = 0.290, \quad \gamma_s = 4.393, \tag{A8}$$
$$s = 14.085. \tag{A9}$$

The result of the fit for two sets of parametrization is depicted on Fig. 3 (curves 1 and 3, respectively). In Fig. 4 is shown the behavior of the primitive function that determines the nuclear moments for two sets of parametrization (dashed and solid curves). It illustrates the sensitivity of the moments (for large n) on the structure function $F_2^A(x)$ at $x \to 1$ (see also Fig. 1).

2. Fit of the isoscalar free nucleon $F_2^N(x)$

First, we define the isoscalar free nucleon structure function

$$F_2^N(x,Q^2) = \tfrac{1}{2}[F_2^p(x,Q^2) + F_2^n(x,Q^2)], \tag{A10}$$

where F_2^p and F_2^n are the free proton and neutron structure functions, respectively. Both structure functions $F_2^p(x,Q^2)$ and $F_2^n(x,Q^2)$ were obtained in Ref. [15] from the combined proton and deuteron data of the BCDMS collaboration [22]. We use, for $F_2^p(x,Q^2)$ the parametrization,

$$F_2^p(x,Q^2) = f_v^p + f_s^p,$$
$$f_v^p = A_v x^{a_1}(1-x)^{a_2}(1 + a_3 x + a_4 x^2),$$
$$f_s^p = a_5(1-x)^{a_6}, \tag{A11}$$
$$a_i = a_{i_0} + a_{i_1}\ln(Q^2/Q_0^2), \quad Q_0^2 = 0.04 \ \mathrm{GeV}^2/c^2.$$

The function f_v is interpreted as a contribution of the valence quark to the proton structure function. Therefore, the parameter A_v is fixed by the baryon conservation law. The values of the parameters a_i after minimization of χ^2 ($\chi^2 = 287.4/252$ points are given in Table II). No errors on the parameters are displayed since they are strongly correlated. Further, for neutron structure function we suppose that the sea part f_s of either neutron and pro-

TABLE II. Values of the parameters a_{i_k}.

i	1		2		3	
k	0	1	0	1	0	1
a_{i_k}	0.67611	-0.03873	3.27950	-0.00757	3.1431	0.10993
i	4		5		6	
k	0	1	0	1	0	1
a_{i_k}	-0.50675	-0.46720	0.11330	0.02712	6.0298	0.06582

TABLE III. Values of the parameters b_{i_k}.

i	1		2		3		4	
k	0	1	0	1	0	1	0	1
b_{i_k}	0.5712	-0.2149	2.6976	-0.00232	-12.547	0.6880	6.0195	0.01146

ton structure functions is the same, i.e., $f_s^p = f_s^n$ and we chose the neutron structure function in the form

$$F_2^n(x,Q^2) = R_v(x,Q^2) f_v^p(x,Q^2) + f_s^p(x,Q^2),$$
$$R_v(x,Q^2) = R_0\left(1 + \sum_{i=1}^{4} b_i x^i\right), \qquad \text{(A12)}$$
$$b_i = b_{i_0} + b_{i_1}\ln(Q^2/Q_0^2),$$

where the condition for proton and neutron structure functions to obey the Gottfried sum rule determines the parameter R_0 via

$$\int_0^1 R_v(x,Q^2) f_v(x,Q^2)\frac{dx}{x} = \frac{2}{3}. \qquad \text{(A13)}$$

The remaining eight parameters b_{i_k} in (A12) are found by the accurate exclusion of the nuclear structure effects in the deuteron [15] and their values are given in Table III.

Note that the present parametrization of the structure function of the isoscalar-free nucleon is valid in the regions of x and Q^2 which are determined by the corresponding region of the BCDMS experimental data, i.e., $0.07 \le x \le 0.75$ and $7.5\ \text{GeV}^2 \le Q^2 \le 230\ \text{GeV}^2$. However, the obtained parametrization of nucleon structure function is extracted in a quite self-consistent way from experiments and can be used as a good approximation of the isoscalar free nucleon structure function in the theoretical analysis of deep-inelastic scattering on nuclei.

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